Putting Energy in Control

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1. Network modeling of physical systems
2. Interconnection structures and finite-dimensional port-Hamiltonian systems
3. Interconnections and control
4. Systems theory of complex physical systems
5. Distributed-parameter port-Hamiltonian systems

In collaboration with B. Maschke, R. Ortega, G. Blankenstein, G. Golo, S. Stramigioli, V. Talasila, R. Pasumarthy, ...

Modeling for control

Points for discussion:

- Choice of model class is crucial
- ‘Data-based modeling’ needs a model class
- Model class of linear systems is very useful, but has severe limitations
- For physical systems we need model classes which are also ’physics-based’
- Which capture basic conservation laws (whose existence is independent of any numerical values!)
- Model class must be ‘closed under interconnection’, in order to build up models for complex systems

Prevailing trend in *modeling and simulation* of complex (lumped-parameter) physical systems:

Network or object-oriented modeling

Advantages:

- ‘Handle complexity by modularity’.
- Modularity and flexibility.
- Re-usability (‘libraries’)
- Suited to design/control
- Multi-domain physics (electrical, mechanical, thermal, chemical, ..)

Disadvantage of current network modeling: it generally leads to a large set of differential and *algebraic* equations (DAE's), *seemingly without any structure*.
(In contrast with 'global' physical modeling methods.)

Aim: to identify the underlying physical structure of network models in order to obtain *models suited for analysis, simulation and control* of broad classes of (nonlinear) physical systems.

Network modeling leads to

open dynamical systems

that interact with other systems via a set of *external variables*.

Basic starting point for this talk:

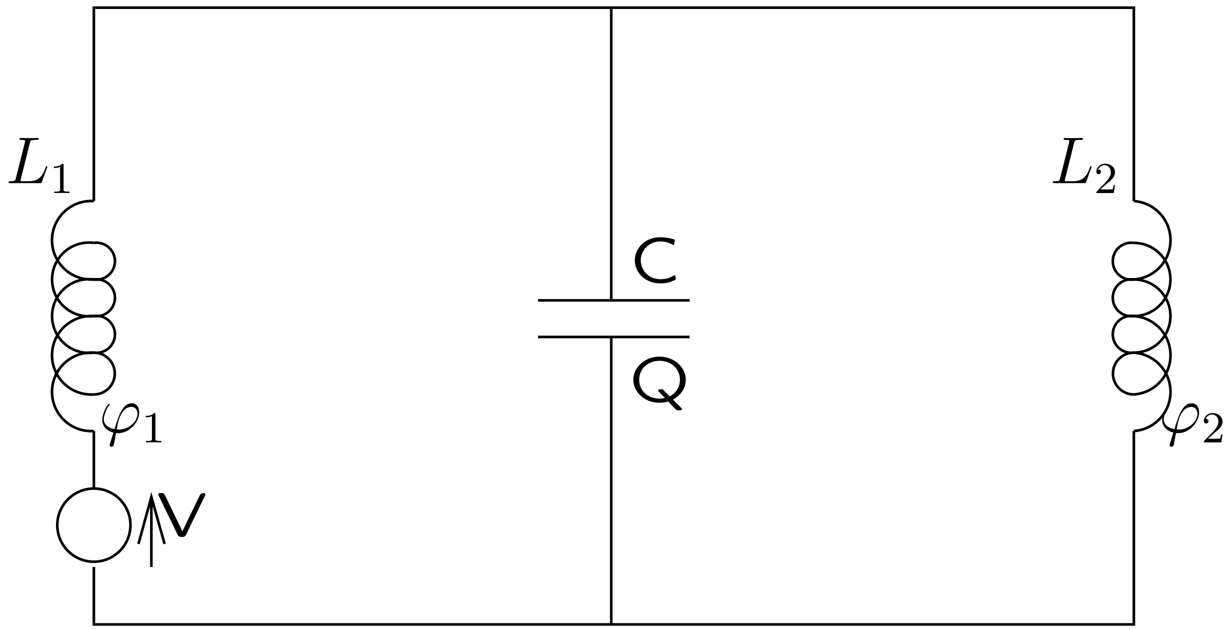
Interaction between system components is modeled by variables that describe *energy exchange* **between components:**

the power-port point of view

Two main reasons:

- For lumped systems with components from *different physical domains* this has proved to be successful; e.g., bond graphs.
- Assumption naturally leads to a (generalized) Hamiltonian description of the system components and the complex system, also capturing other conservation laws.

Motivating example Two inductors with magnetic energies $H_1(\varphi_1), H_2(\varphi_2)$ (φ_1 and φ_2 magnetic flux linkages), and capacitor with electric energy $H_3(Q)$ (Q charge). V denotes the voltage of the source.



Question: *How to write this simple network as a "Hamiltonian system" in a modular way?*

Equations for the individual components of the LC-circuit:

$$\begin{array}{lrcll} \textit{Inductor } 1 & \dot{\varphi}_1 & = & f_1 & \text{(voltage)} \\ \text{(current)} & e_1 & = & \frac{\partial H_1}{\partial \varphi_1} & \end{array}$$

$$\begin{array}{lrcll} \textit{Inductor } 2 & \dot{\varphi}_2 & = & f_2 & \text{(voltage)} \\ \text{(current)} & e_2 & = & \frac{\partial H_2}{\partial \varphi_2} & \end{array}$$

$$\begin{array}{lrcll} \textit{Capacitor} & \dot{Q} & = & f_3 & \text{(current)} \\ \text{(voltage)} & e_3 & = & \frac{\partial H_3}{\partial Q} & \end{array}$$

If the elements are *linear* then

$$H_1(\varphi_1) = \frac{1}{2L_1}\varphi_1^2, H_2(\varphi_2) = \frac{1}{2L_2}\varphi_2^2, H_3(Q) = \frac{1}{2C}Q^2$$

Kirchhoff's interconnection laws in $f_1, f_2, f_3, e_1, e_2, e_3, f = V, e = I$ are

$$\begin{bmatrix} -f_1 \\ -f_2 \\ -f_3 \\ e \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ f \end{bmatrix},$$

leading to the interconnected Hamiltonian system

$$\begin{aligned} \begin{bmatrix} \dot{\varphi}_1 \\ \dot{\varphi}_2 \\ \dot{Q} \end{bmatrix} &= \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial \varphi_1} \\ \frac{\partial H}{\partial \varphi_2} \\ \frac{\partial H}{\partial Q} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} f \\ e &= \frac{\partial H}{\partial \varphi_1} \end{aligned}$$

with $H(\varphi_1, \varphi_2, Q) := H_1(\varphi_1) + H_2(\varphi_2) + H_3(Q)$ total energy.

General definition of a port-Hamiltonian system

Ingredients $(\mathcal{X}, \mathcal{F}, \mathcal{D}, H)$:

- Energy-storing elements with energy-variables x_i living in a total state space $\mathcal{X}$.
- Flow variables $f \in \mathcal{F} = \mathbb{R}^m$ and conjugated effort variables $e \in \mathcal{F}^* = \mathbb{R}^m$, terminating on dissipative elements and ports/sources.
- **Interconnection structure**: Dirac structure $\mathcal{D}$

$$(f_x, e_x, f, e) \in \mathcal{D}(x) \subset T_x\mathcal{X} \times T_x^*\mathcal{X} \times \mathcal{F} \times \mathcal{F}^*$$

- **Total energy**: $H(x_1, \cdots, x_k)$.

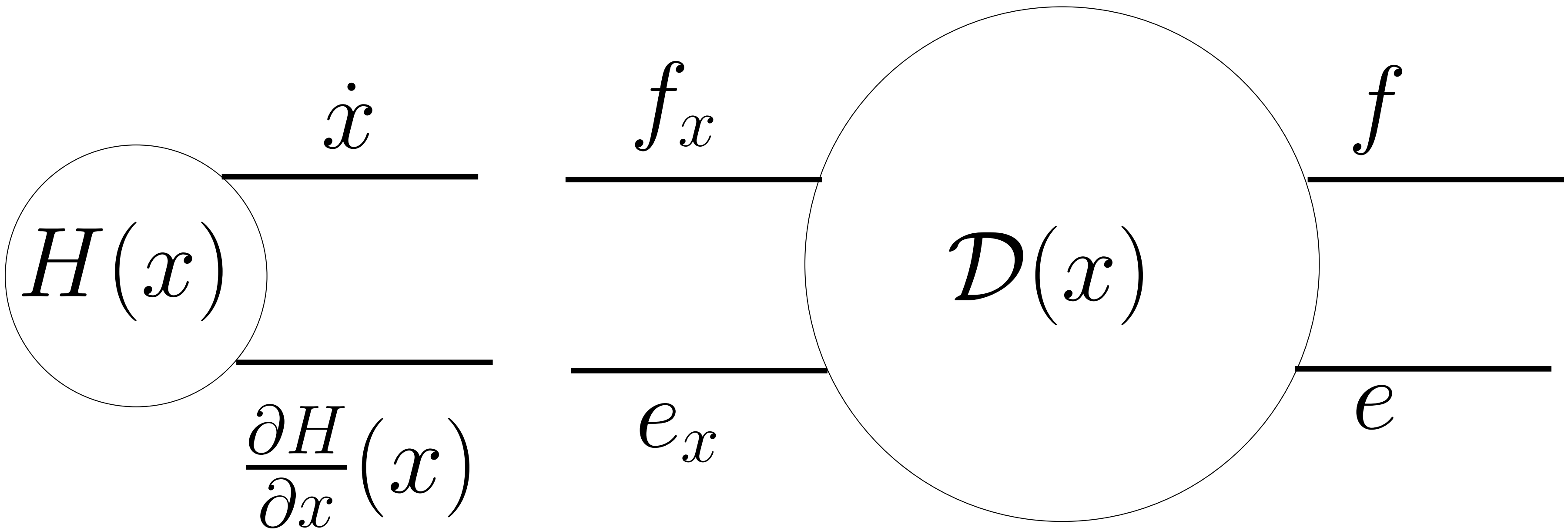


Figure 1: Port-Hamiltonian system

The dynamical system defined by the relations

$$(-\dot{x}(t), \frac{\partial H}{\partial x}(x(t)), f(t), e(t)) \in \mathcal{D}(x(t)), \quad t \in \mathbb{R}$$

is called the **port-Hamiltonian system** $(\mathcal{X}, \mathcal{F}, \mathcal{D}, H)$.

Particular case is a Dirac structure $\mathcal{D}(x) \subset T_x\mathcal{X} \times T_x^*\mathcal{X} \times \mathcal{F} \times \mathcal{F}^*$ given as the graph of the skew-symmetric map

$$\begin{bmatrix} f_x \\ e \end{bmatrix} = \begin{bmatrix} -J(x) & -g(x) \\ g^T(x) & 0 \end{bmatrix} \begin{bmatrix} e_x \\ f \end{bmatrix},$$

leading to a Hamiltonian input-state-output system $(f_x = -\dot{x}, e_x = \frac{\partial H}{\partial x}(x))$

$$\begin{aligned} \dot{x} &= J(x)\frac{\partial H}{\partial x}(x) + g(x)f, \quad x \in \mathcal{X}, f \in \mathbb{R}^m \\ e &= g^T(x)\frac{\partial H}{\partial x}(x), \qquad\qquad e \in \mathbb{R}^m \end{aligned}$$

General LC-circuits

Kirchhoff's current and voltage laws

$$\begin{bmatrix} {A_L}^T & {A_C}^T \end{bmatrix} \begin{bmatrix} I_L \\ I_C \end{bmatrix} = 0, \qquad \begin{bmatrix} V_L \\ V_C \end{bmatrix} = \begin{bmatrix} A_L \\ A_C \end{bmatrix} \lambda$$

defines a Dirac structure between flows and efforts

$$\begin{aligned} f_x &= (I_C, V_L) &= (-\dot{Q}, -\dot{\phi}) \\ e_x &= (V_C, I_L) &= (\tfrac{\partial H}{\partial Q}, \tfrac{\partial H}{\partial \phi}) \end{aligned}$$

with Hamiltonian $H(Q, \phi)$ the total energy.

Leads to port-Hamiltonian system in implicit form

$$\begin{aligned} -\dot{\phi} &= A_L \lambda \\ \tfrac{\partial H}{\partial Q} &= A_C \lambda \\ 0 &= {A_L}^T \tfrac{\partial H}{\partial \phi} - {A_C}^T \dot{Q} \end{aligned}$$

Can be transformed into more convenient form.

Mechanical systems with kinematic constraints

Consider a mechanical system with constraints on the generalized velocities $\dot{q}$, described as

$$A^T(q)\dot{q} = 0.$$

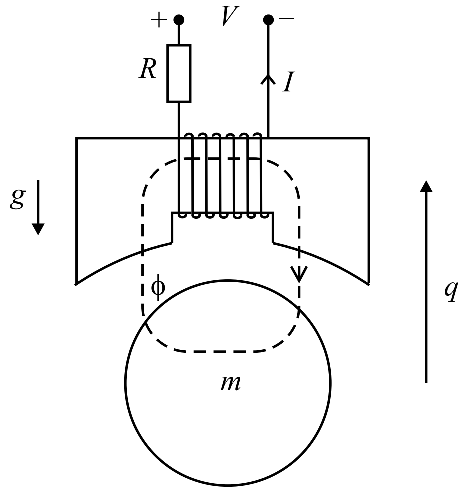
This leads to *constrained Hamiltonian equations*

$$\begin{aligned}
\dot{q} &= \frac{\partial H}{\partial p}(q,p) \\
\dot{p} &= -\frac{\partial H}{\partial q}(q,p) + A(q)\lambda + B(q)f \\
0 &= A^T(q)\frac{\partial H}{\partial p}(q,p) \\
e &= B^T(q)\frac{\partial H}{\partial p}(q,p)
\end{aligned}$$

with $H(q,p)$ total energy, and λ the constraint forces.
Dirac structure is defined by symplectic form on T^*Q together with constraints $A^T(q)\dot{q} = 0$ and force matrix $B(q)$.

Can be extended to general *multi-body systems*.

Electro-mechanical system



$$\begin{bmatrix} \dot{q} \\ \dot{p} \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -\frac{1}{R} \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial q} \\ \frac{\partial H}{\partial p} \\ \frac{\partial H}{\partial \varphi} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} V, \qquad I = \frac{\partial H}{\partial \varphi}$$

Coupling between domains via $H(q,p,\varphi) = mgq + \frac{p^2}{2m} + \frac{\varphi^2}{2k_1(1-\frac{q}{k_2})}$.

Dirac structures, and therefore port-Hamiltonian systems, admit different *equational representations*, with different properties.

Hamiltonian DAE's

Represent the Dirac structure $\mathcal{D}$ in *kernel representation* as

$$\mathcal{D} = \{(f_x, e_x, f, e) \mid F_x(x)f_x + E_x(x)e_x + F(x)f + E(x)e = 0\},$$

with

$$\text{(i)} \qquad E_x F_x^T + F_x E_x^T + EF^T + FE^T = 0,$$

$$\text{(ii)} \qquad \text{rank } [F_x \vdots E_x \vdots F \vdots E] = \dim(\mathcal{X} \times \mathcal{F}).$$

Since the flows f_x and efforts e_x corresponding to the energy-storing elements are given as $f_x = -\dot{x}, \quad e_x = \frac{\partial H}{\partial x}$, the port-Hamiltonian system is described by the DAE's

$$F_x(x(t))\dot{x}(t) = E_x(x(t))\frac{\partial H}{\partial x}(x(t)) + F(x(t))f(t) + E(x(t))e(t)$$

Canonical coordinates

For simplicity take $\mathcal{F} \times \mathcal{F}^*$ to be *void* (no ports).

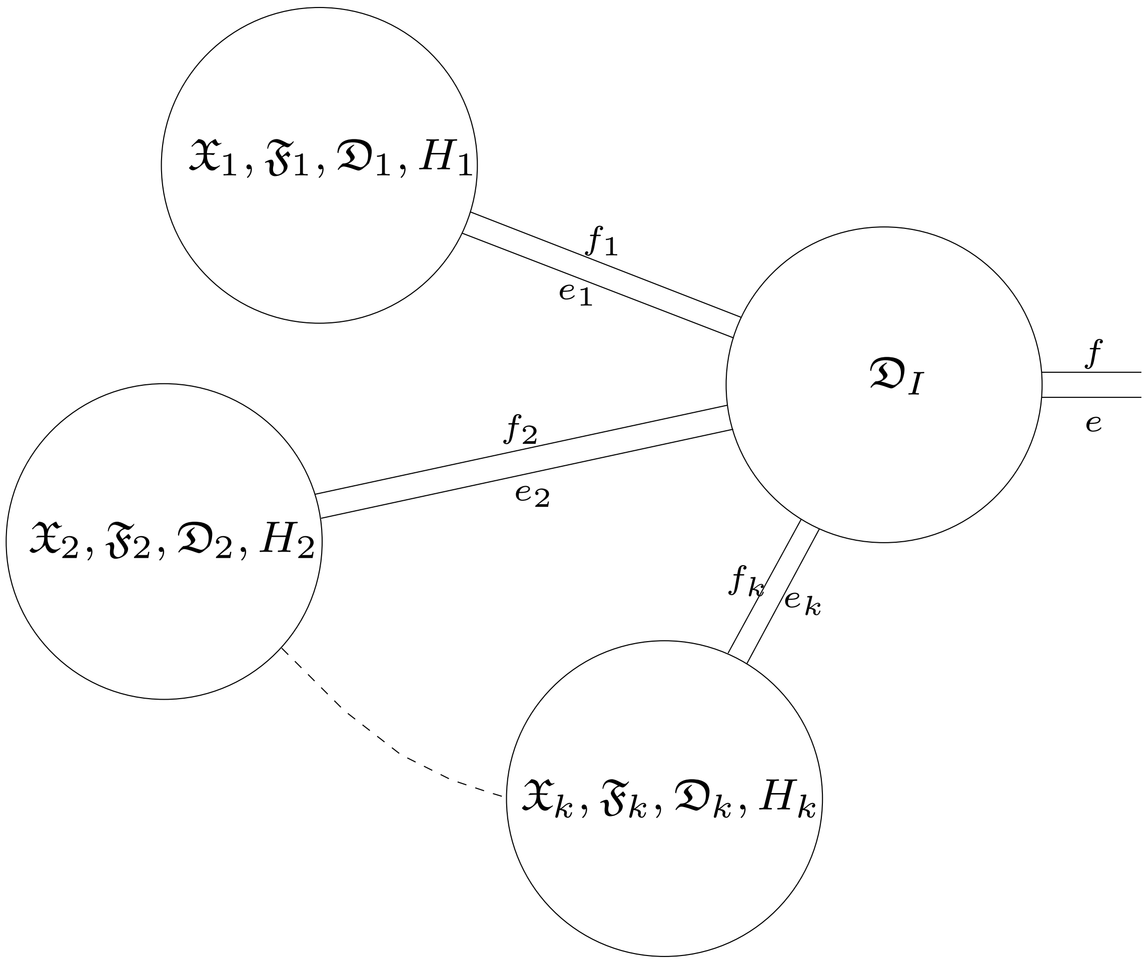
If the generalized Dirac structure on $\mathcal{X}$ is *integrable* then there exist coordinates (q, p, r, s) for $\mathcal{X}$ such that

$$\mathcal{D} = \{(f_q, f_p, f_r, f_s, e_q, e_p, e_r, e_s) \in T_x\mathcal{X} \times T_x^*\mathcal{X}\}$$

$$\left\{ \begin{array}{rclrcl} f_q & = & -e_p, & f_p & = & e_q \\ \\ f_r & = & 0, & 0 & = & e_s \end{array} \right.$$

Hence the port-Hamiltonian system on $\mathcal{X}$ takes the form

$$\begin{aligned} \dot{q} &= \frac{\partial H}{\partial p}(q, p, r, s) \\ \dot{p} &= -\frac{\partial H}{\partial q}(q, p, r, s) \\ \dot{r} &= 0 \\ 0 &= \frac{\partial H}{\partial s}(q, p, r, s) \end{aligned}$$



Interconnected system is a port-Hamiltonian system $(\mathcal{X}, \mathcal{F}, \mathcal{D}, H)$, with $H = H_1 + \cdots + H_k$, and $\mathcal{D}$ based on $\mathcal{D}_1, \cdots, \mathcal{D}_k, \mathcal{D}_I$.

This is a starting point for *control*.

Control by Interconnection

Connect the *plant* port-Hamiltonian system to a *controller* port-Hamiltonian system.

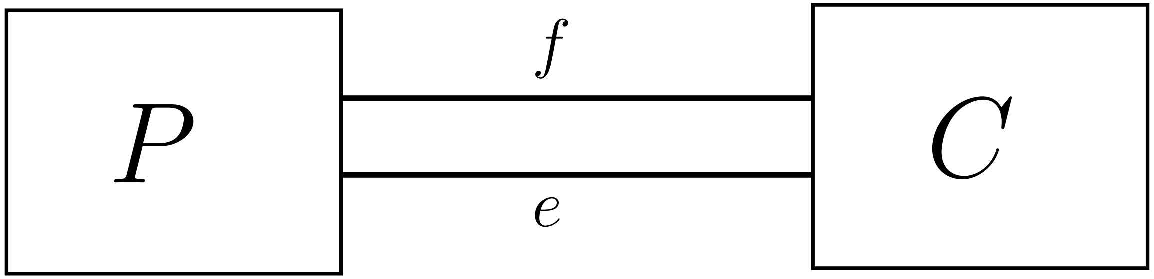


Figure 2: Control by Interconnection

Closed-loop system is again a port-Hamiltonian system with total energy $H_{cl} = H_P + H_C$, and closed-loop Dirac structure D_{cl} based on D_P and D_C.

Port-Hamiltonian systems are more than *energy-conserving* (or energy-dissipating if resistive elements are included): the Dirac structure also determines conserved quantities *independent* of the energy function.

By deliberate choice of D_C we may generate *Casimir functions* K for the closed-loop system, and use the candidate Lyapunov function (even for unstable plant systems!)

$$V := H_P + H_C + K$$

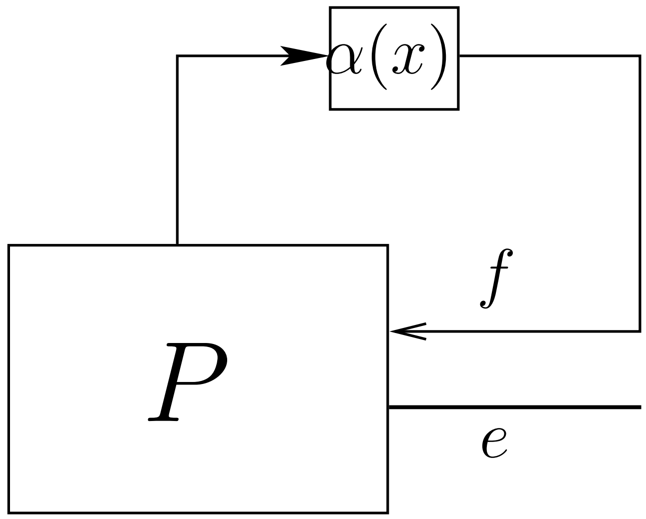
Addition of energy-dissipating elements may result in asymptotic stabilization.

This can be seen as (dynamic) *impedance control*.

Second scheme:

Interconnection-Damping Assignment by State Feedback

Use state feedback to transform the plant port-Hamiltonian system into another port-Hamiltonian system with desired properties: IDA-PBC method (Ortega, vdS, Maschke, Spong, Blankenstein, ..).



When applied to mechanical systems this method is equivalent to the method of *Controlled Lagrangians*, developed by Bloch, Leonard, Marsden, et al..

Systems theory of physical systems

- Theory of composition/interconnection
- Compositional analysis
- Equivalence of components and exact model reduction
- Approximate model reduction and abstraction
- Identification of system parameters
- Coupling of physical systems to discrete transition systems: embedded systems theory

Summary sofar

- Complex lumped systems (from different physical domains) are modeled as port-Hamiltonian systems, in a modular way.
- Models are suited for analysis, design and control.
  Identification of Hamiltonian structure has already shown to be important for stability analysis, derivation of simulation models, model analysis and control.
- Also physical systems with switching topology can be studied within this framework (walking robots, power converters, .. .)

Next question:

How to incorporate distributed-parameter components?

Example

Transmission line

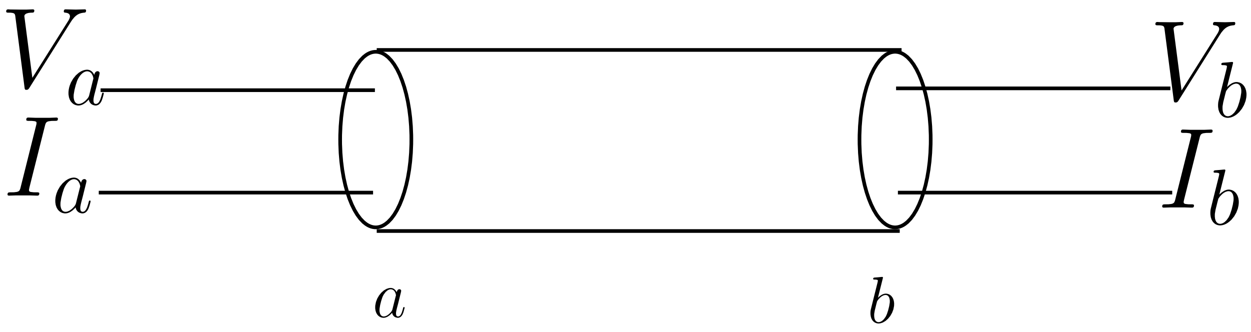


Figure 3: Transmission line

Telegrapher's equations define the boundary control system

$$\begin{aligned}
\frac{\partial Q}{\partial t}(z,t) &= -\frac{\partial}{\partial z}I(z,t) &&= -\frac{\partial}{\partial z}\frac{\phi(z,t)}{L(z)} \\
\frac{\partial \phi}{\partial t}(z,t) &= -\frac{\partial}{\partial z}V(z,t) &&= -\frac{\partial}{\partial z}\frac{Q(z,t)}{C(z)} \\
V_a(t) &= V(a,t), \quad I_a(t) &&= I(a,t) \\
V_b(t) &= V(b,t), \quad I_b(t) &&= I(b,t)
\end{aligned}$$

Transmission line as port-Hamiltonian system

Define flows $f_x = (f_E, f_M)$ and efforts $e_x = (e_E, e_M)$:

electric flow	$f_E : [a,b] \to \mathbb{R}$
magnetic flow	$f_M : [a,b] \to \mathbb{R}$
electric effort	$e_E : [a,b] \to \mathbb{R}$
magnetic effort	$e_M : [a,b] \to \mathbb{R}$

together with boundary flows $f = (f_a, f_b)$ and efforts $e = (e_a, e_b)$.

$$\begin{bmatrix} f_E \\ f_M \end{bmatrix} = \begin{bmatrix} 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial z} & 0 \end{bmatrix} \begin{bmatrix} e_E \\ e_M \end{bmatrix}$$

$$\begin{bmatrix} f_{a,b} \\ e_{a,b} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} e_{E|a,b} \\ e_{M|a,b} \end{bmatrix}$$

defines an infinite-dimensional Dirac structure !

Interconnection

Interconnection of an infinite-dimensional port-Hamiltonian system with a finite-dimensional port-Hamiltonian system leads to a *mixed finite- and infinite-dimensional* port-Hamiltonian system.
"All" techniques of finite-dimensional port-Hamiltonian systems carry over to the infinite-dimensional and mixed case.

Spatial discretization of infinite-dimensional components

First step: *discretization of Dirac structure to finite-dimensional Dirac structure*. How to do this? Discretize the variables in a *different* way, depending on their geometric content: *mixed finite element methods*.

By restriction of the Hamiltonian to the resulting finite-dimensional space of energy variables, this leads to an approximating *finite-dimensional* port-Hamiltonian system.

Conclusions

- Unified framework for *analysis, simulation and control* of complex lumped-parameter linear and *nonlinear* systems with components from different physical domains.
- Port-Hamiltonian description of open distributed-parameter systems (telegrapher's equations, Maxwell's equations, n-dimensional wave equation, compressible ideal fluids, ..).
- Mixed finite-element discretization to finite-dimensional port-Hamiltonian systems, and incorporation in port-based simulation tools.

◦ Analysis and control of infinite-dimensional port-Hamiltonian systems (with Hans Zwart and Javier Villegas).

◦ Extension to discrete/hybrid interaction ... .

See http://www.math.utwente.nl/˜schaftaj