THE RIGHT CONNECTION

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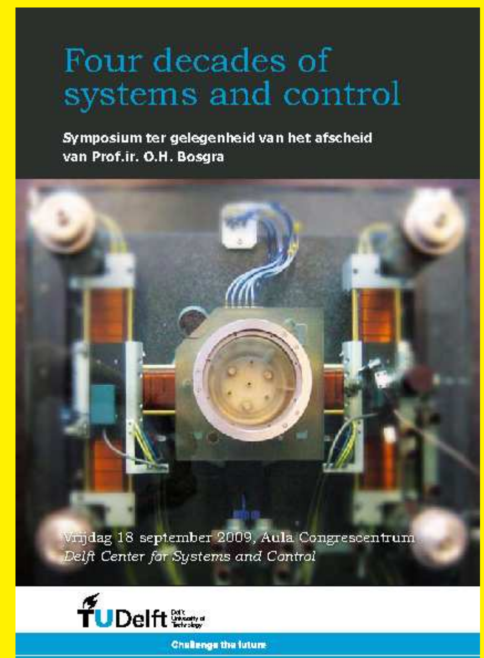
K.U. Leuven

Symposium

Four decades of Systems and Control

September 18, 2009



Ter gelegenheid van het afscheidscollege van Okko Bosgra

In honor of Okko Bosgra

on the occasion of his ‘last lecture’.

De juiste connectie

The right connectie

Theme

How are systems interconnected?

- **Is output-to-input assignment a good way of viewing interconnection of physical systems?**

Theme

How are systems interconnected?

- Is **output-to-input assignment** a good way of viewing interconnection of physical systems?
- Interconnection by **variable sharing.**

Theme

How are systems interconnected?

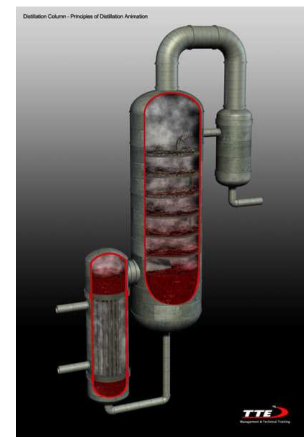
- **Is output-to-input assignment a good way of viewing interconnection of physical systems?**
- **Interconnection by variable sharing.**
- **How is energy transferred between systems?**

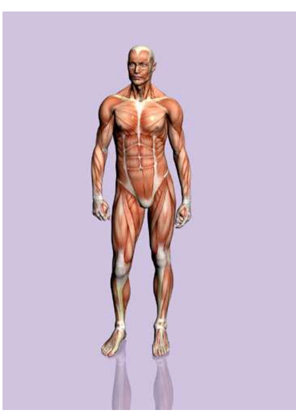
  Are energy transfer and interconnection related?

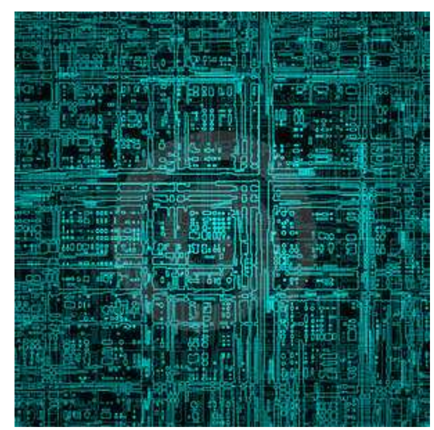
SYSTEMS

Systems

OIL REFINERY (GVG / PD)



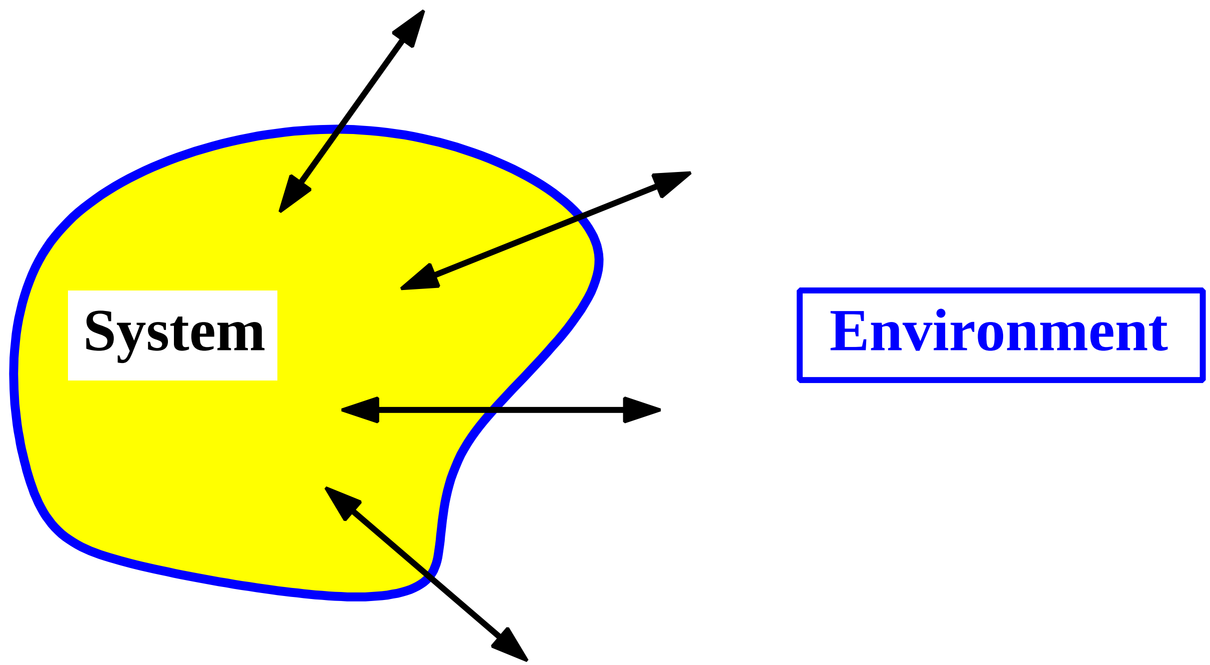




Features

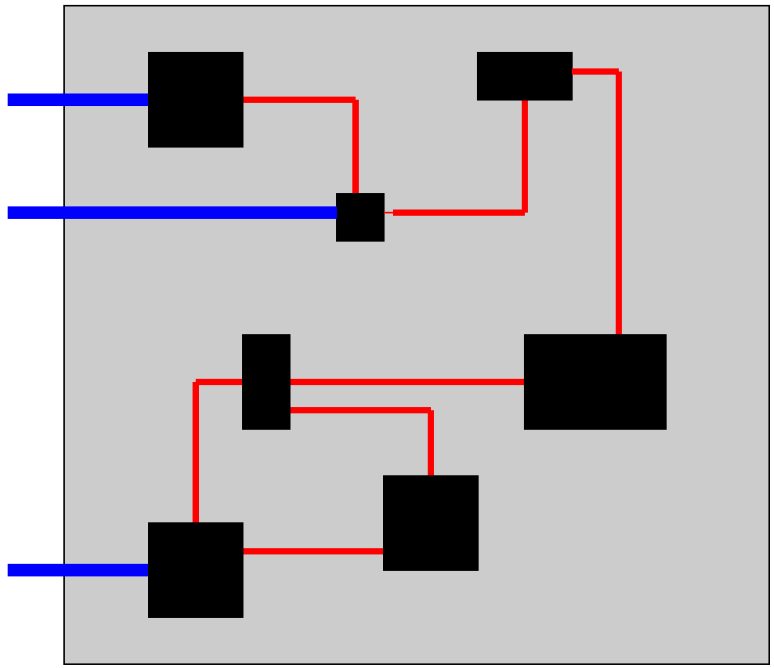
- **Open**
- **Interconnected**
- **Modular**
- **Dynamic**

Open



Systems interact with their environment.

Interconnected



Systems consist of interconnected subsystems.

Modularity

Systems consist of interconnection of standard components.

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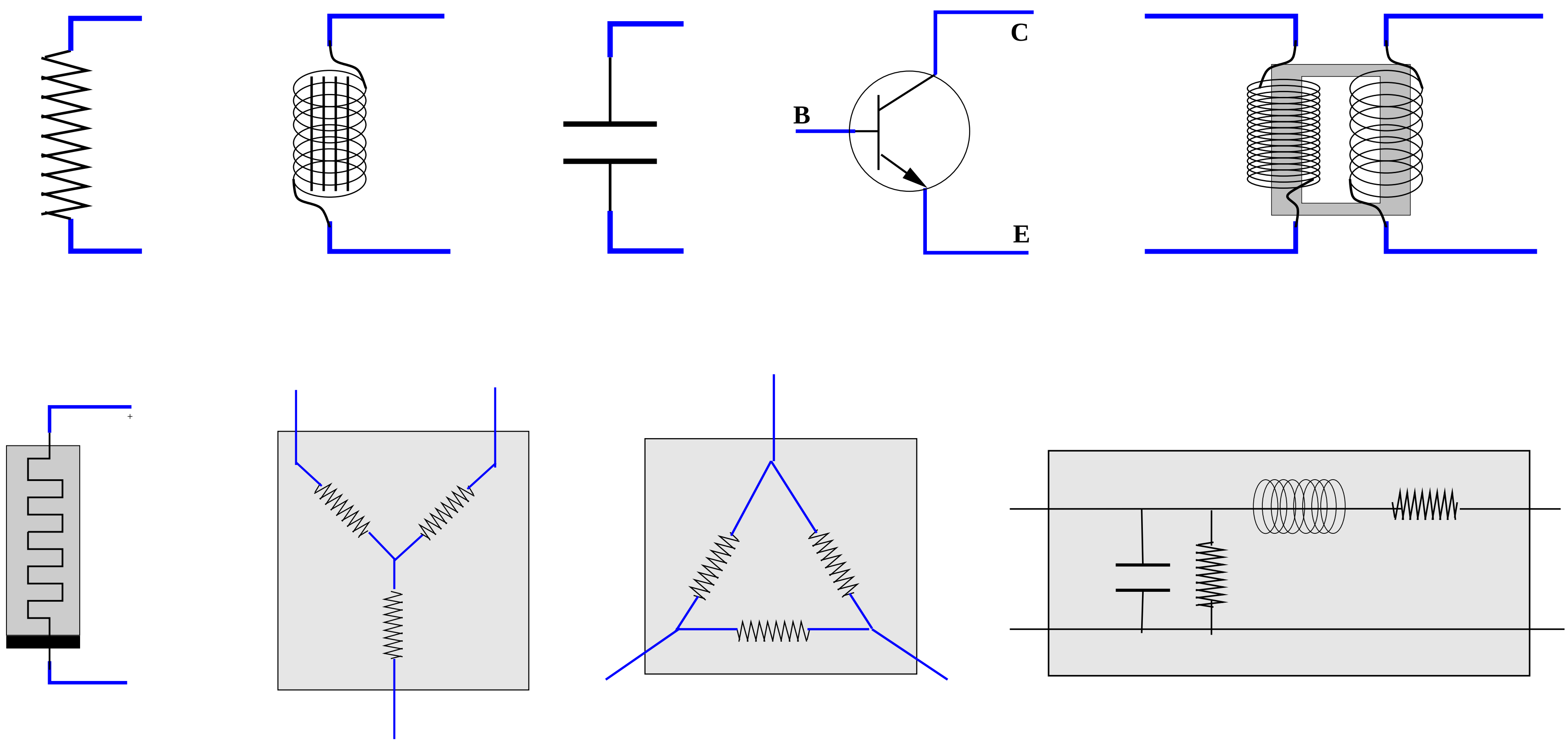
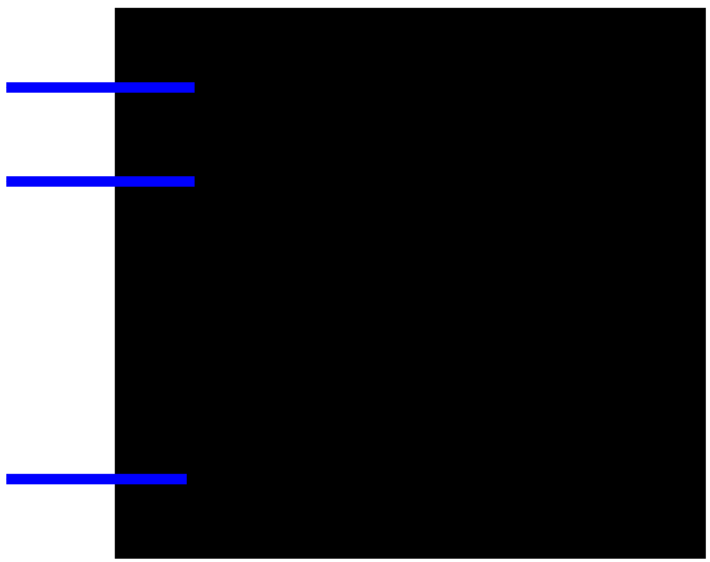


TEARING, ZOOMING, and LINKING

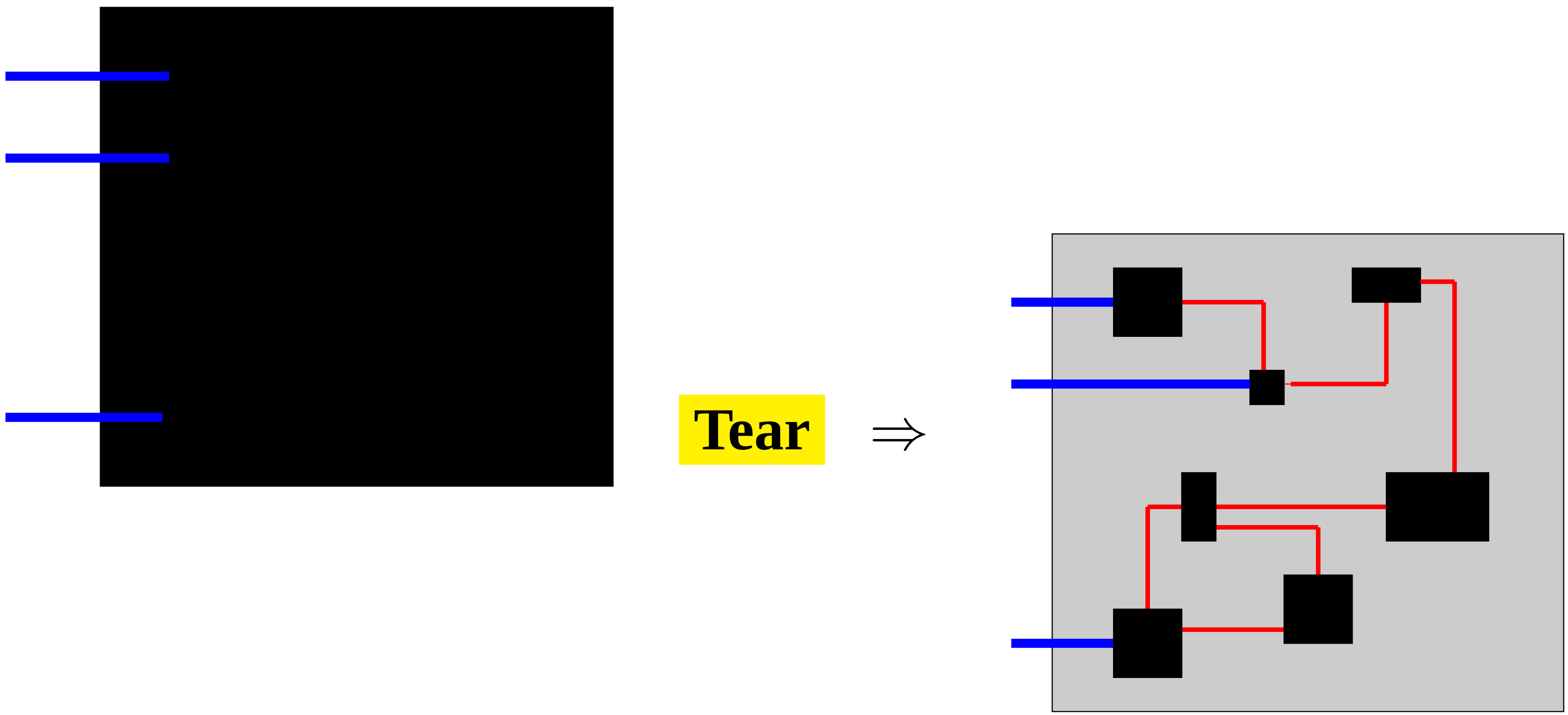
Tearing

¡¡ Model the behavior of selected variables !!

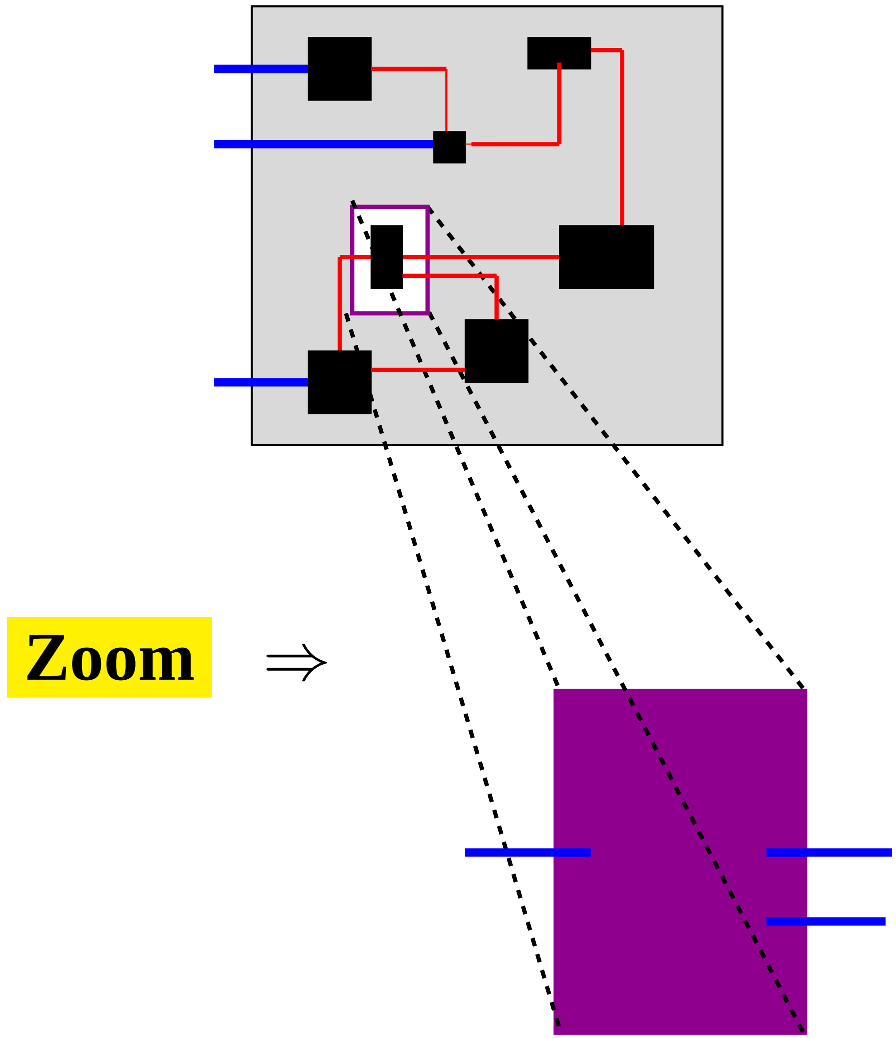


Tearing

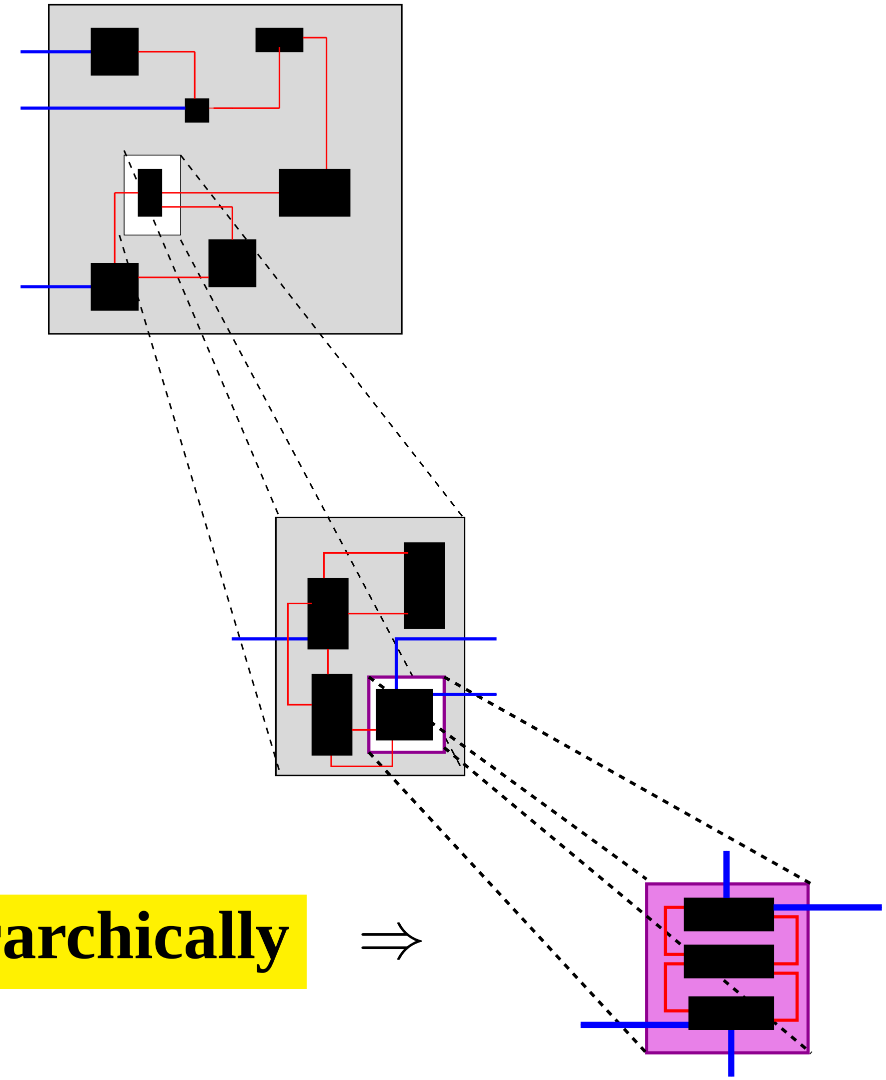
¡¡ Model the behavior of selected variables !!



Zooming



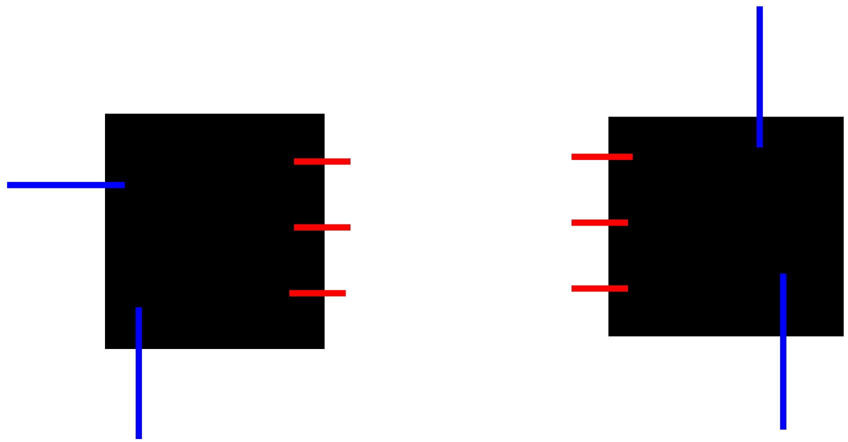
Zooming



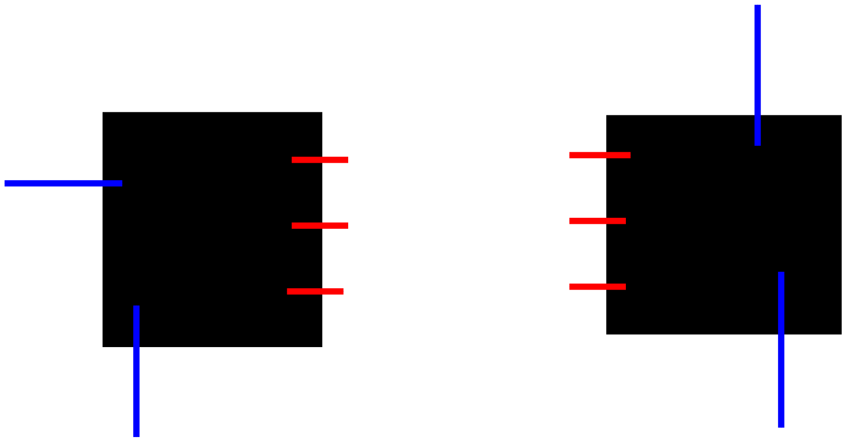
Hierarchically ⇒

Proceed until subsystems (‘modules’) are obtained whose model is known, from first principles, or stored in a database.

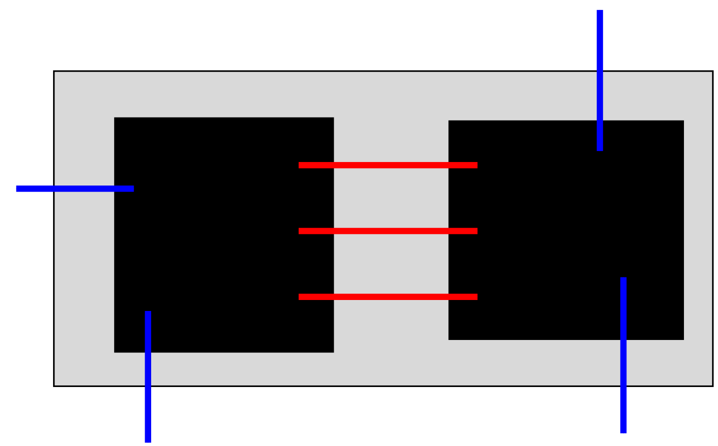
Linking



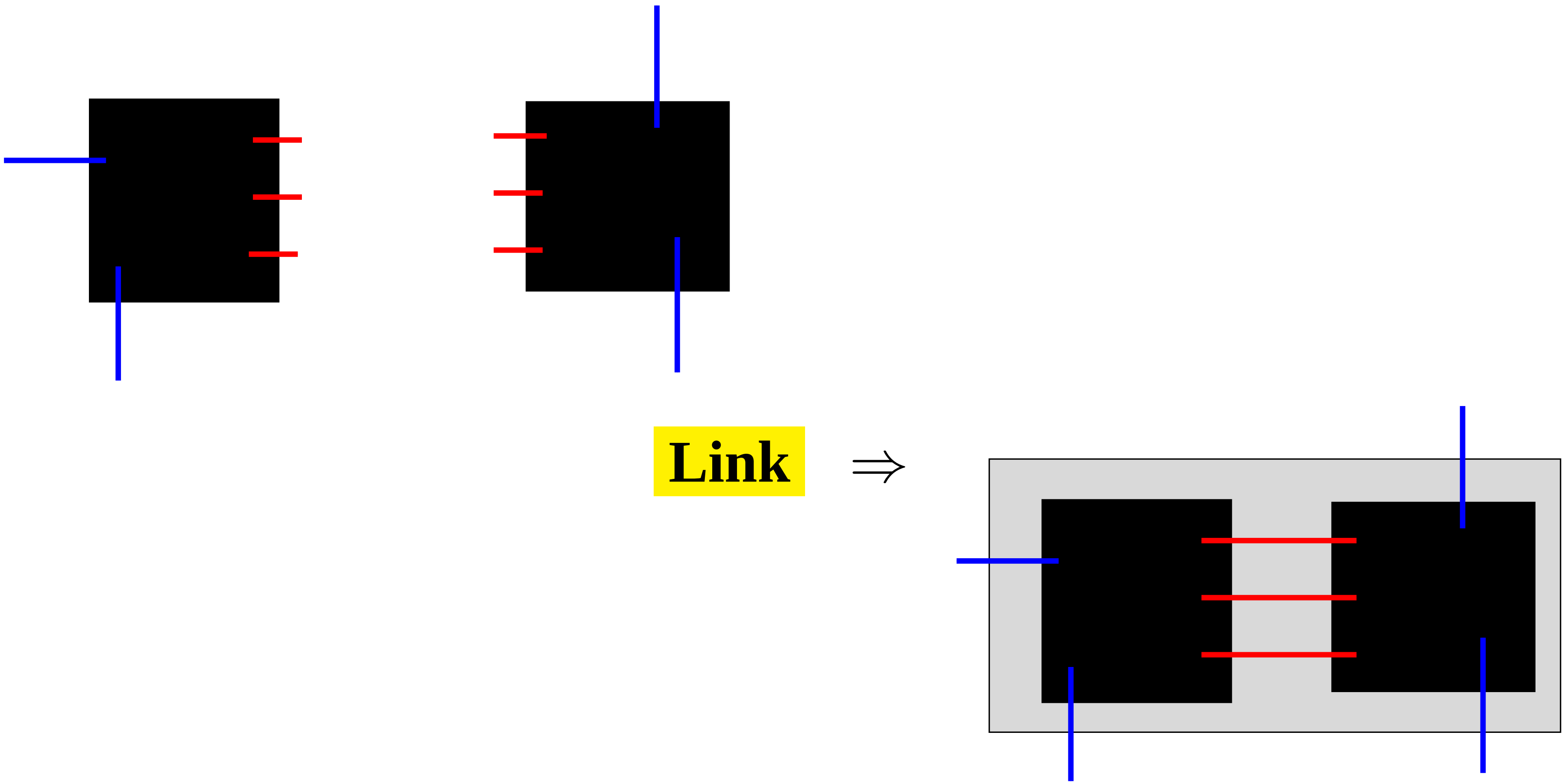
Linking



Link ⇒



Linking



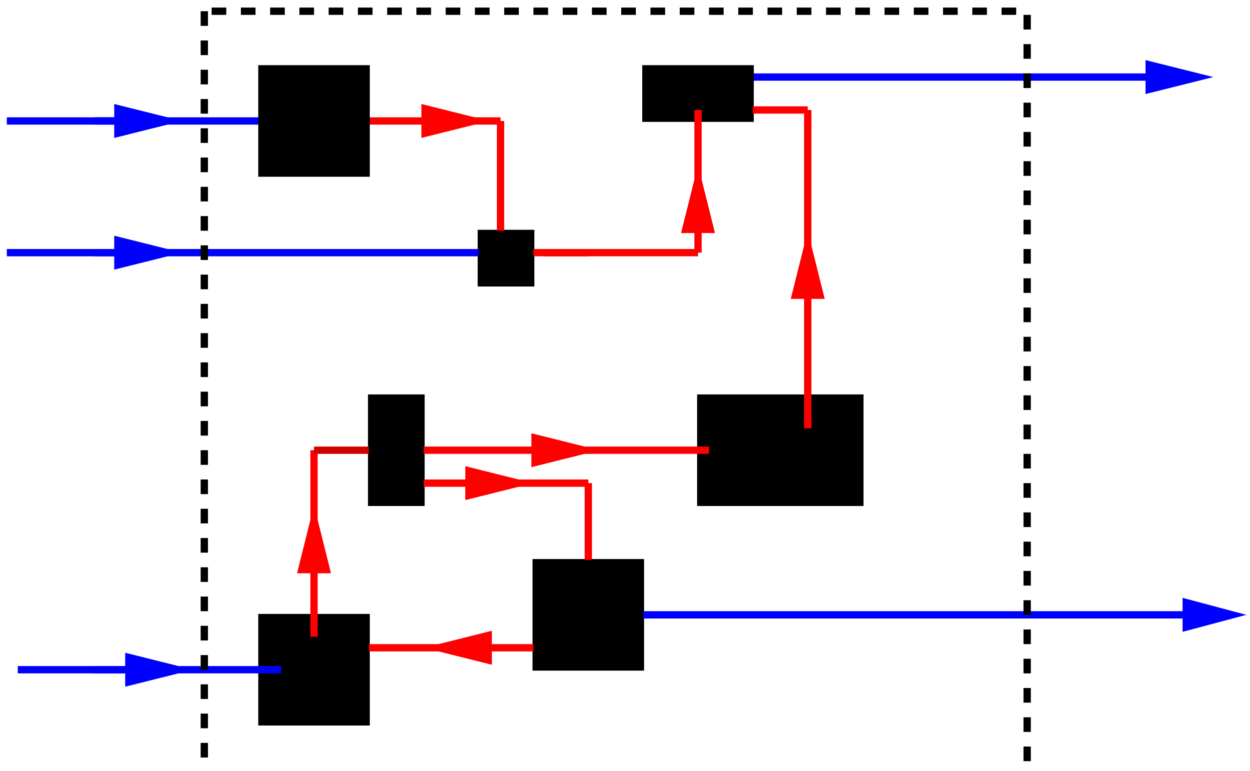
Today's theme: **How should we formalize linking?**

Is there special structure to this linking?

OUTPUT-to-INPUT ASSIGNMENT

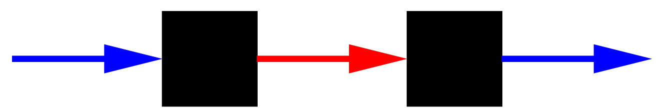
Signal flow graphs

View systems as signal flow graphs:

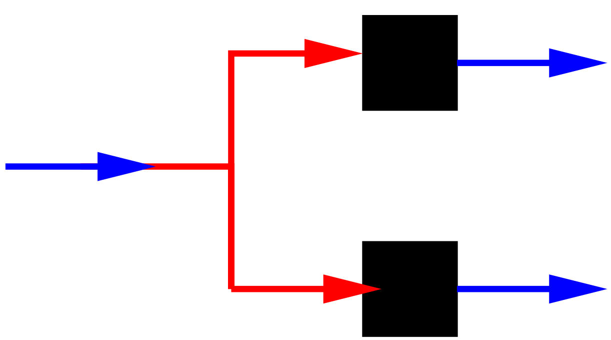


Signal flow graphs

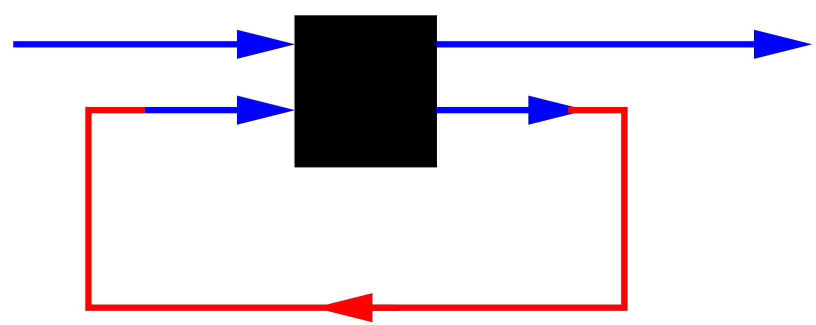
View interconnection as output-to-input assignment:



series

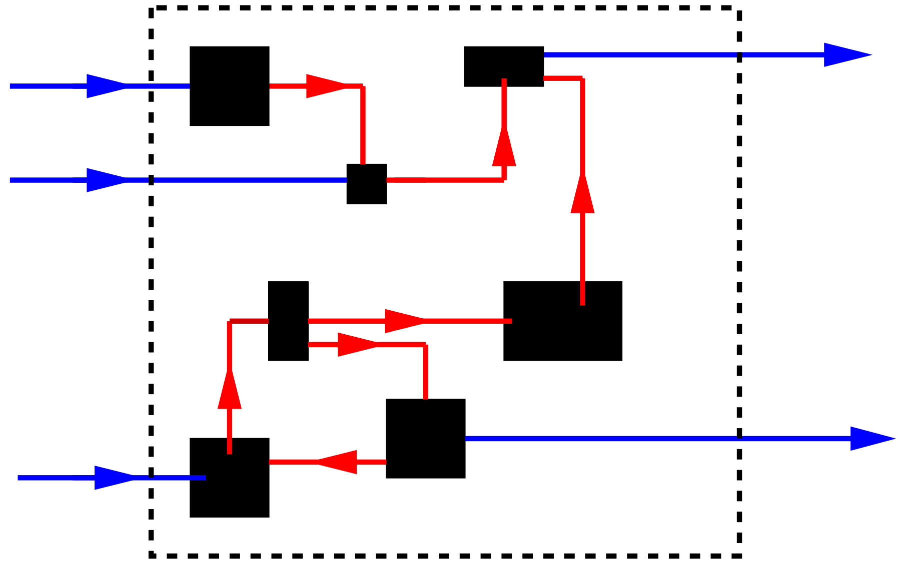


parallel



feedback

Signal flow graphs

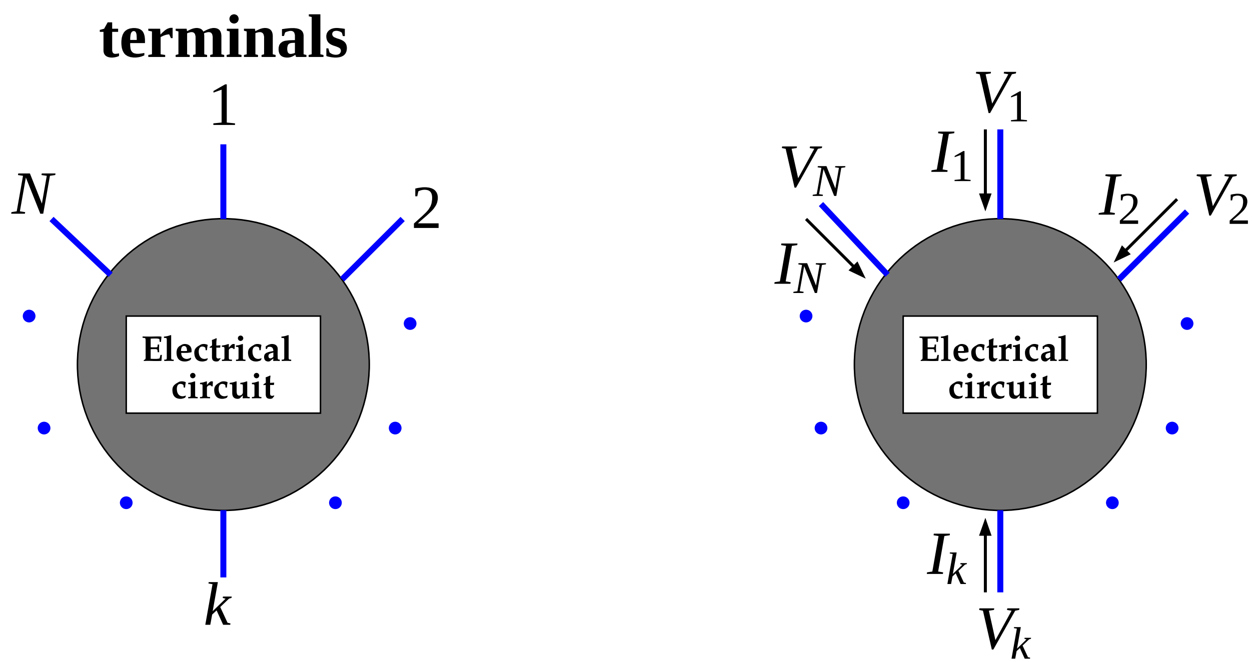


Not appropriate for describing the interaction of physical systems.

A physical system is not a signal processor.

SYSTEMS with TERMINALS

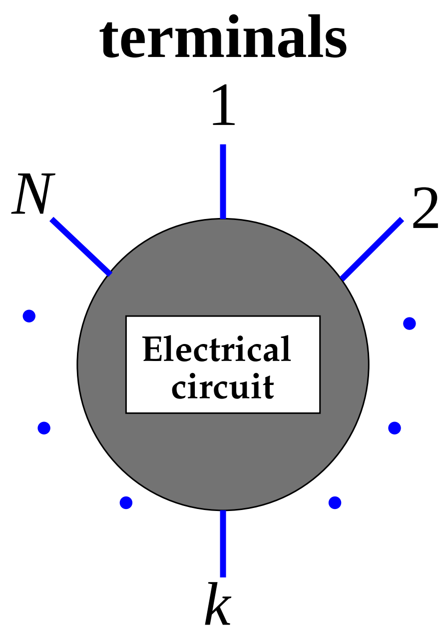
N-terminal circuit

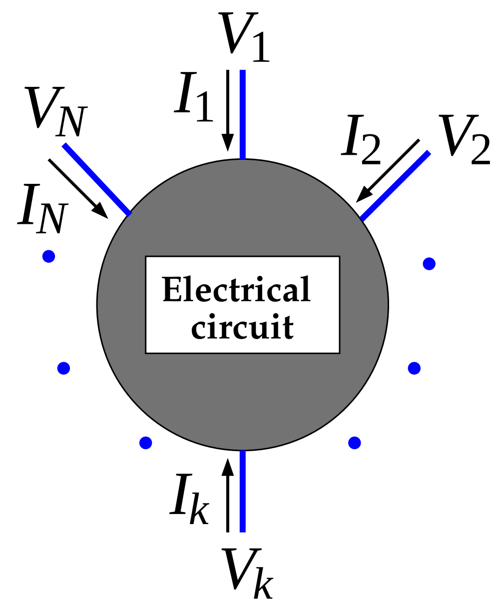


<u>At each terminal</u>:

a potential (!) and a current (counted > 0 into the circuit),

N-terminal circuit





<u>At each terminal</u>:

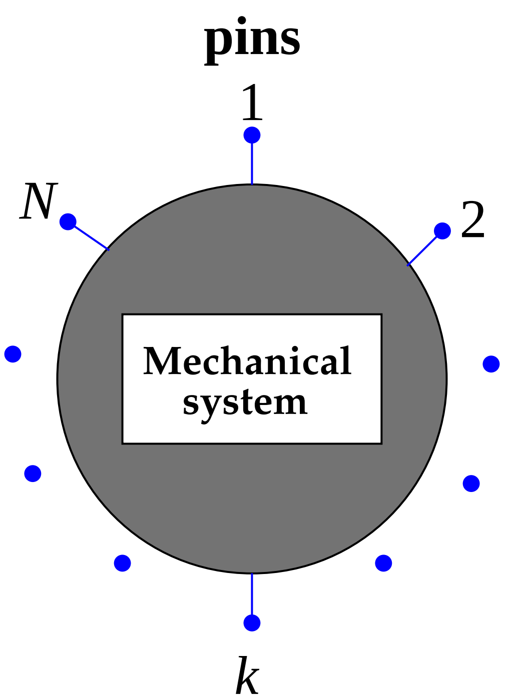
a potential (!) and a current (counted > 0 into the circuit),

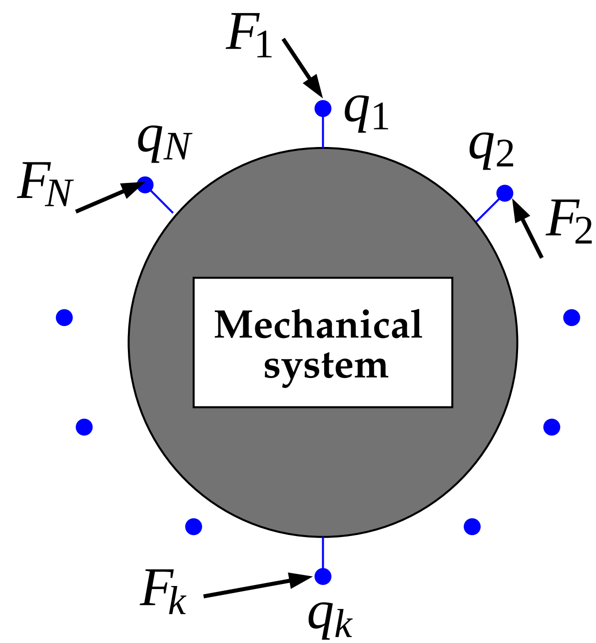
$$\rightsquigarrow \textbf{behavior } \mathscr{B} \subseteq \left(\mathbb{R}^N \times \mathbb{R}^N\right)^{\mathbb{R}}.$$

$(V_1, V_2, \ldots, V_N, I_1, I_2, \ldots, I_N) \in \mathscr{B}$ **means:**

this potential/current trajectory is compatible with the circuit architecture and its element values.

Mechanical system





At each terminal: a position and a force.

$\rightsquigarrow$ **position/force trajectories** $(q, F) \in \mathscr{B} \subseteq ((\mathbb{R}^{\bullet})^{2N})^{\mathbb{R}}$.

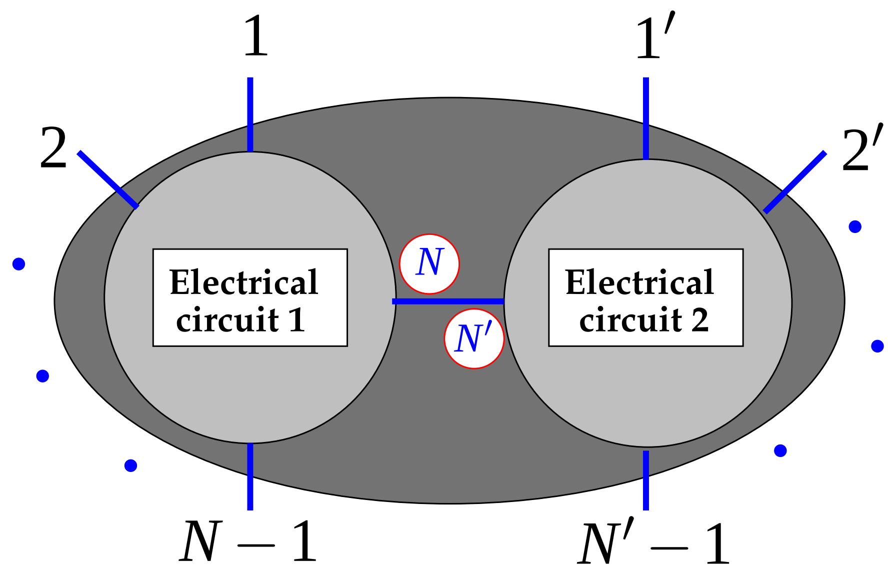
More generally, a position, force, angle, and torque.

Systems

- **Thermal systems:**
  At each terminal: a temperature and a heat flow.
- **Hydraulic systems:**
  At each terminal: a pressure and a mass flow.
- **Multidomain systems:**
  Systems with terminals of different types.
- **...**

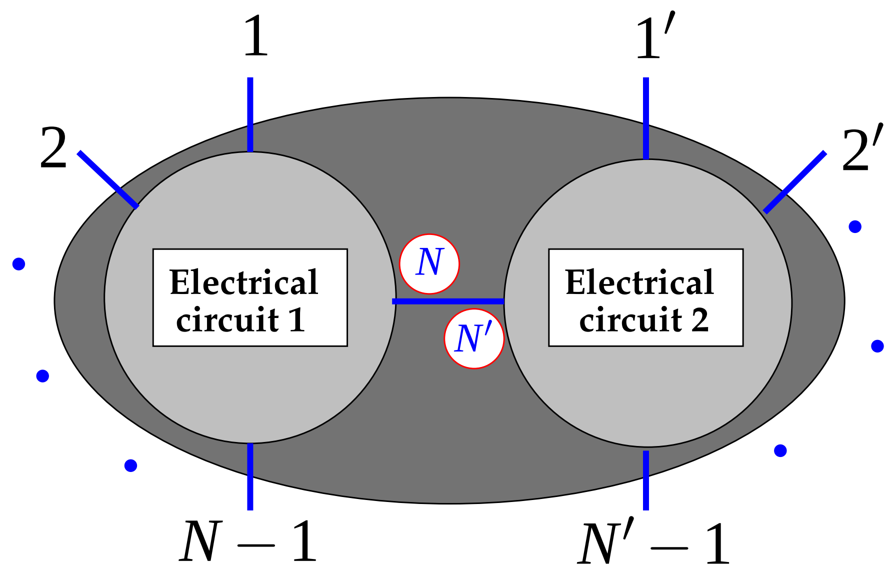
INTERCONNECTION

Interconnection of circuits



$$V_N = V_{N'} \quad \textbf{and} \quad I_N + I_{N'} = 0.$$

Interconnection of circuits



$$V_N = V_{N'} \quad \textbf{and} \quad I_N + I_{N'} = 0.$$

Behavior after interconnection:

$$\mathscr{B}_1 \sqcap \mathscr{B}_2$$

$$:= \{(V_1, \ldots, V_{N-1}, V_{1'}, \ldots, V_{N'-1}, I_1, \ldots, I_{N-1}, I_{1'}, \ldots, I_{N'-1}) \mid$$

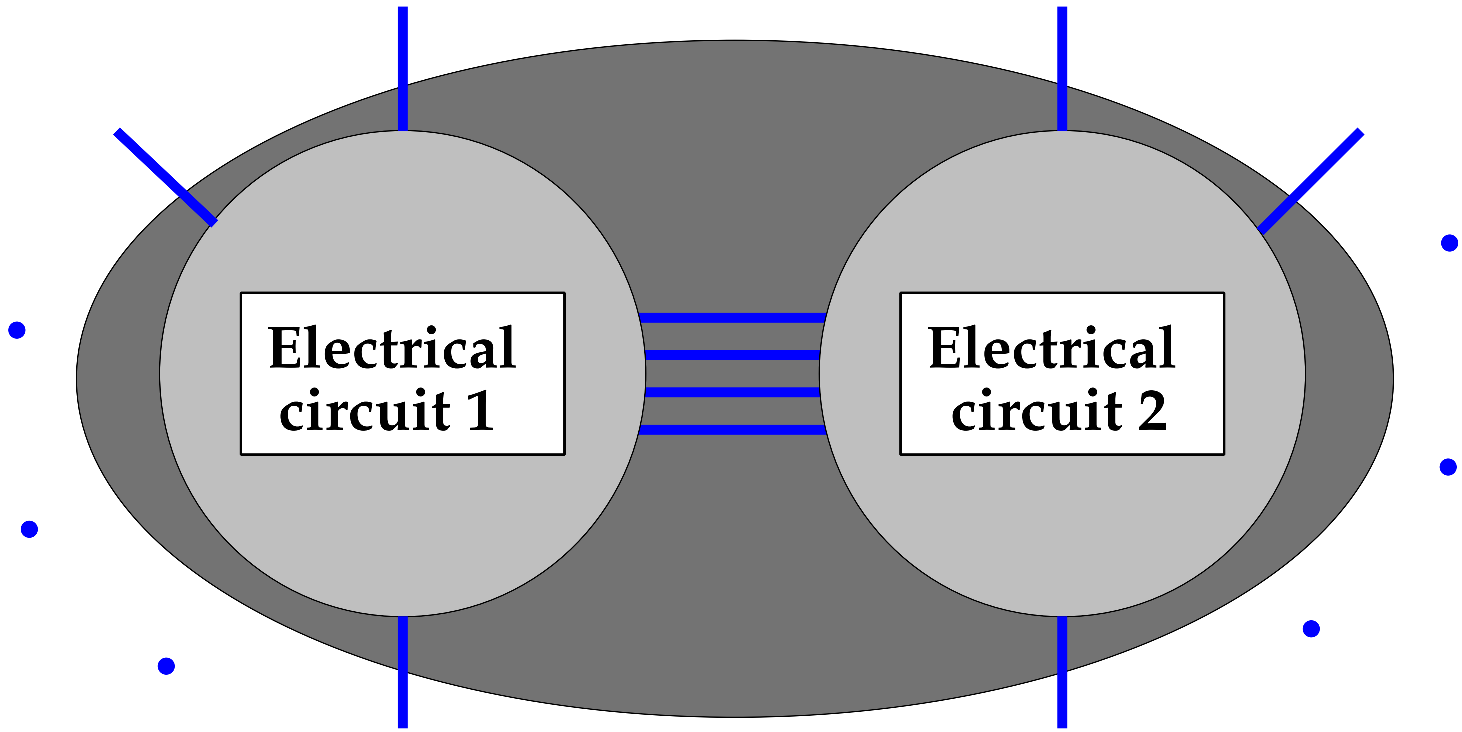
$$\exists V, I \textbf{ such that}$$

$$(V_1, \ldots, V_{N-1}, V, I_1, \ldots, I_{N-1}, I) \in \mathscr{B}_1 \textbf{ and}$$

$$(V_{1'}, \ldots, V_{N'-1}, V, I_{1'}, \ldots, I_{N'-1}, -I) \in \mathscr{B}_2\}.$$

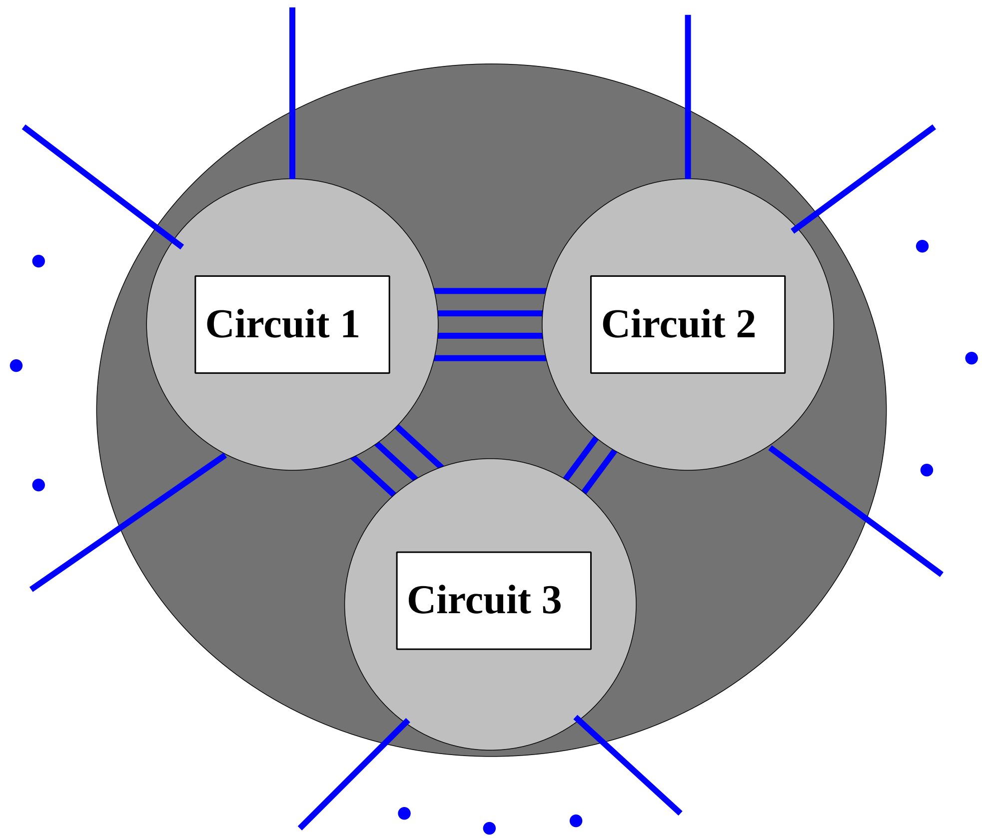
Interconnection of circuits

⇝ **more terminals connected**

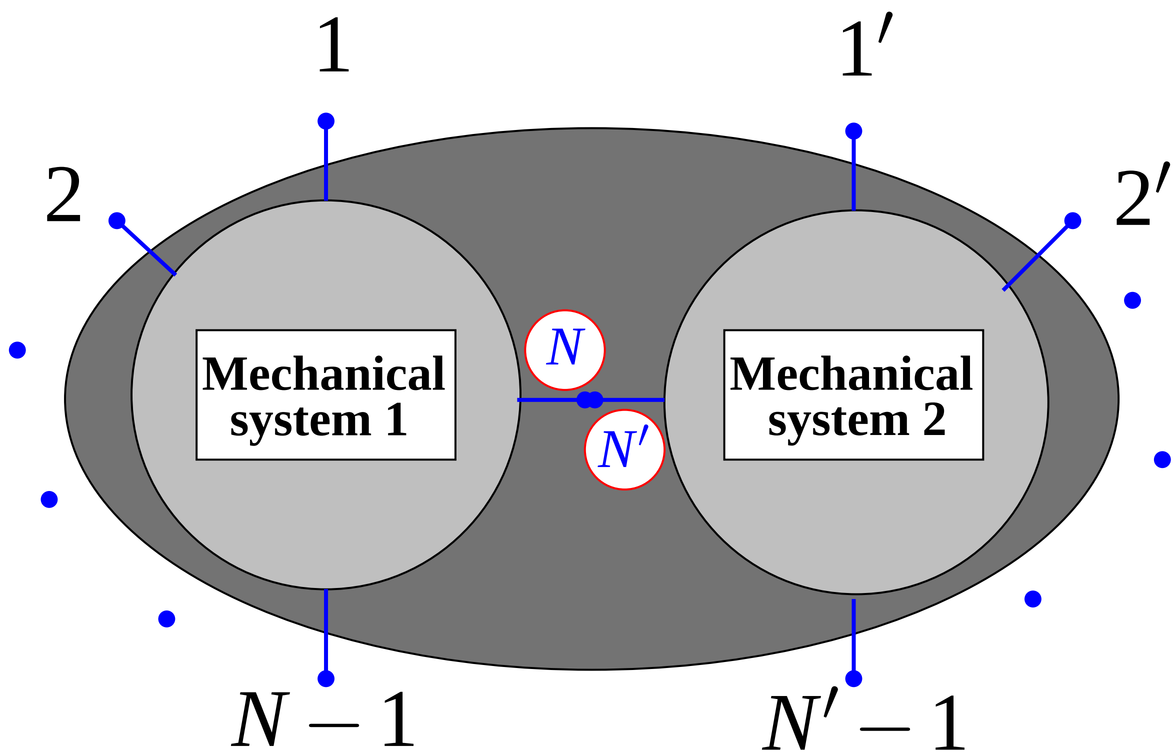


Interconnection of circuits

⇝ **more circuits connected**



Interconnection of mechanical systems



$$q_N = q_{N'} \quad \textbf{and} \quad F_N + F_{N'} = 0.$$

Other systems

- **Thermal systems:**

  At each terminal: a temperature and a heat flow.

$$T_N = T_{N'} \quad \textbf{and} \quad Q_N + Q_{N'} = 0.$$

- **Hydraulic systems:**

  At each terminal: a pressure and a mass flow.

$$p_N = p_{N'} \quad \textbf{and} \quad f_N + f_{N'} = 0.$$

- **...**

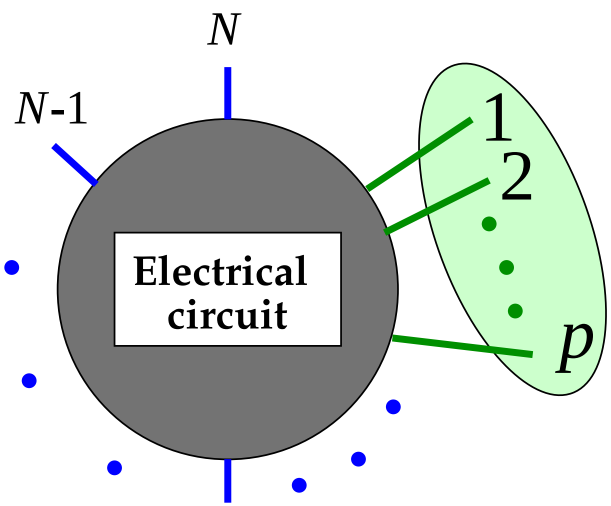
Sharing variables

$$
\begin{aligned}
V_N = \ & V_{N'} \quad \textbf{and} \quad & I_N + I_{N'} = \ & 0, \\
q_N = \ & q_{N'} \quad \textbf{and} \quad & F_N + F_{N'} = \ & 0, \\
T_N = \ & T_{N'} \quad \textbf{and} \quad & Q_N + Q_{N'} = \ & 0, \\
p_N = \ & p_{N'} \quad \textbf{and} \quad & f_N + f_{N'} = \ & 0, \\
& \vdots & &
\end{aligned}
$$

Interconnection means variable sharing.

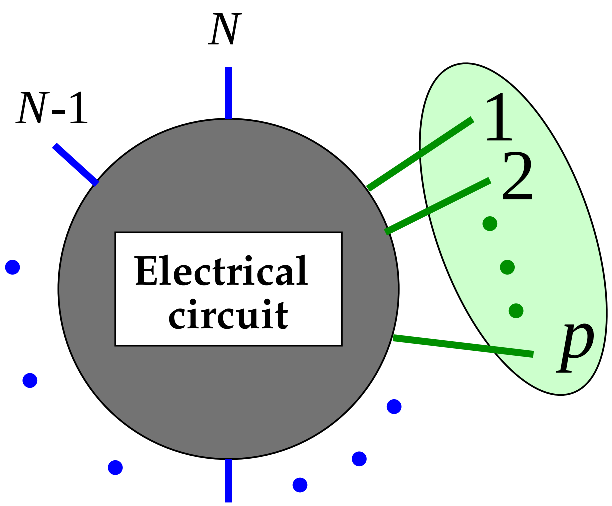
ENERGY TRANSFER

Ports



Terminals $\{1, 2, \ldots, p\}$ **form a port** $:\Leftrightarrow$

Ports



Terminals $\{1, 2, \ldots, p\}$ **form a port** $:\Leftrightarrow$

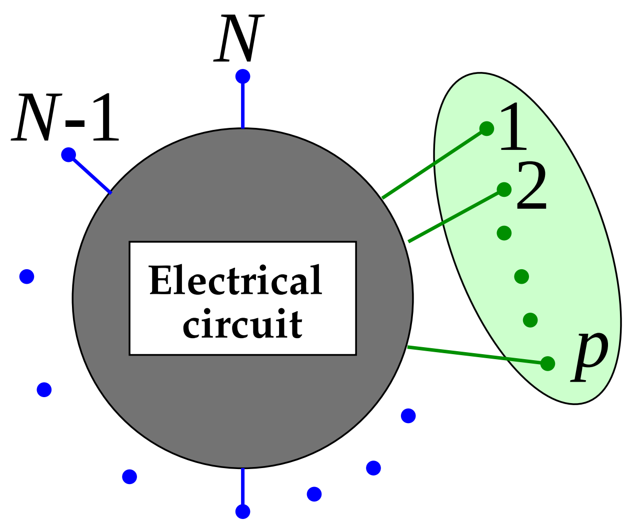
$(V_1, \ldots, V_p, V_{p+1}, \ldots, V_N, I_1, \ldots, I_p, I_{p+1}, \ldots, I_N) \in \mathscr{B}, \alpha : \mathbb{R} \to \mathbb{R}$

$\Rightarrow$ $(V_1 + \alpha, \ldots, V_p + \alpha, V_{p+1}, \ldots, V_N, I_1, \ldots, I_p, I_{p+1}, \ldots, I_N) \in \mathscr{B}$

and $I_1 + \cdots + I_p = 0.$

'port KVL' **and** ***'port KCL'*****.**

Ports



If terminals $\{1, 2, \ldots, p\}$ **form a port, then**

power in along these terminals = $V_1(t)I_1(t) + \cdots + V_p(t)I_p(t)$,

energy in = $$\int_{t_1}^{t_2} (V_1(t)I_1(t) + \cdots + V_p(t)I_p(t))\, dt.$$

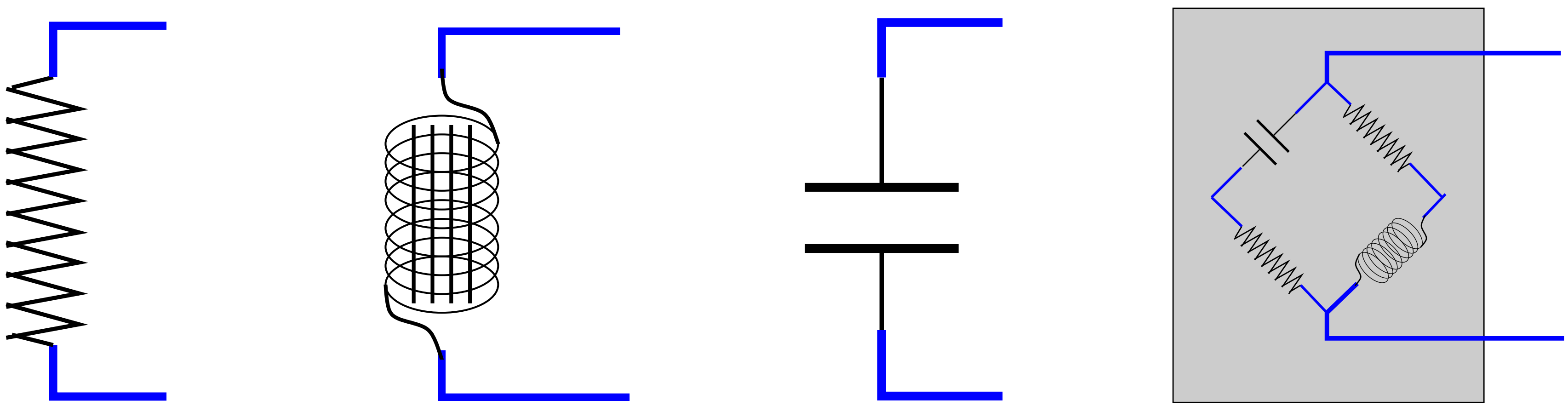
This interpretation in terms of power and energy is not valid

unless these terminals form a port !

Examples

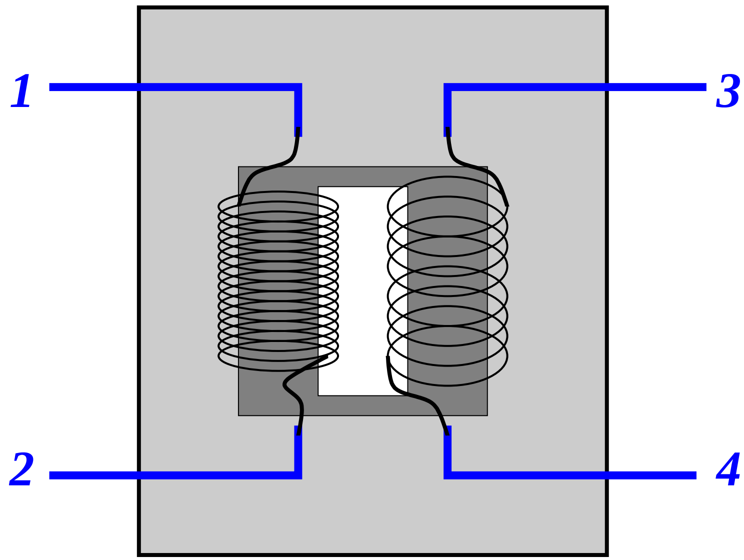
Examples of 1-port 2-terminal devices:

resistors, capacitors, inductors, memristors,
any 2-terminal circuit composed of these.



Examples

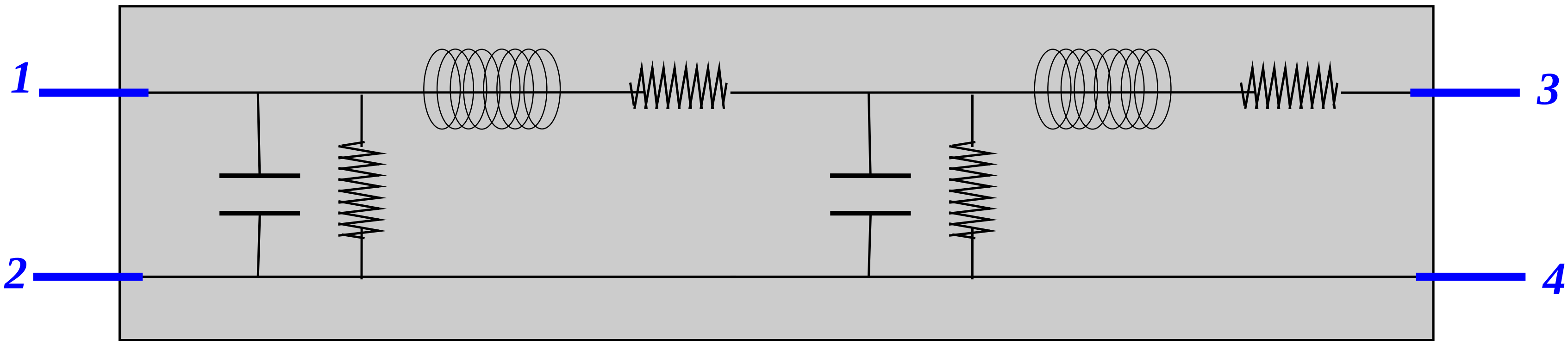
Example of 2-port 4-terminal device:



$$V_1 - V_2 = n\left(V_3 - V_4\right), -nI_1 = I_3 \qquad I_1 + I_2 = 0, I_3 + I_4 = 0$$

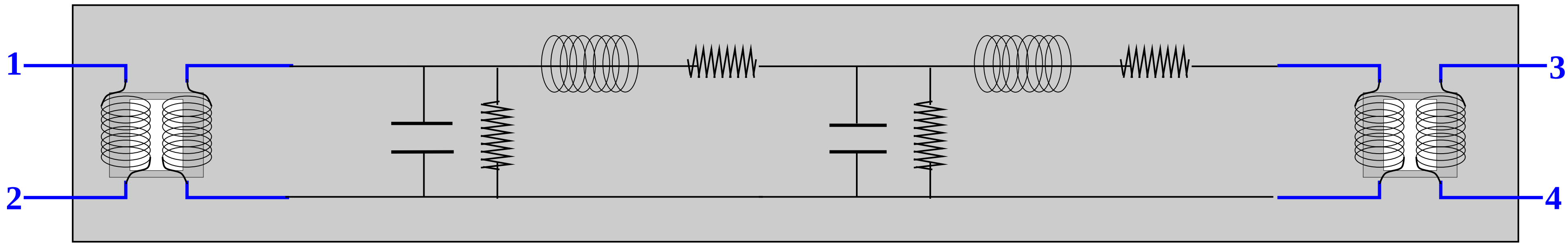
Terminals $\{1,2\}$ **and** $\{3,4\}$ **(and** $\{1,2,3,4\}$**) form ports.**

Examples

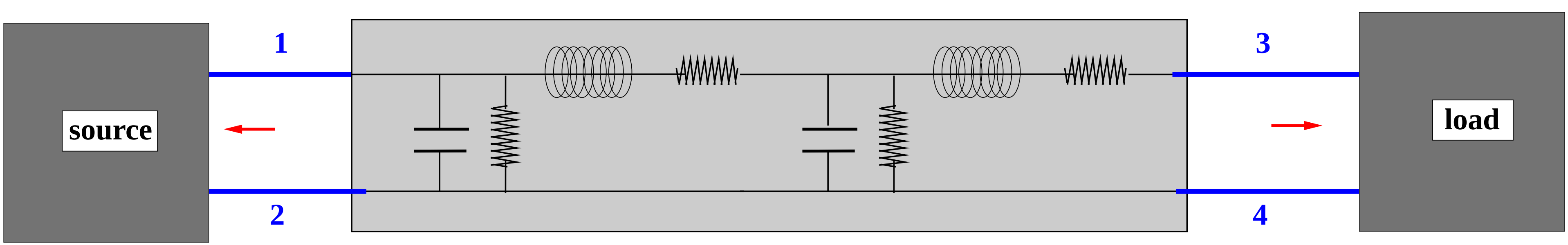


Terminals $\{1, 2, 3, 4\}$ form a port. But $\{1, 2\}$ and $\{3, 4\}$ do not.

Examples

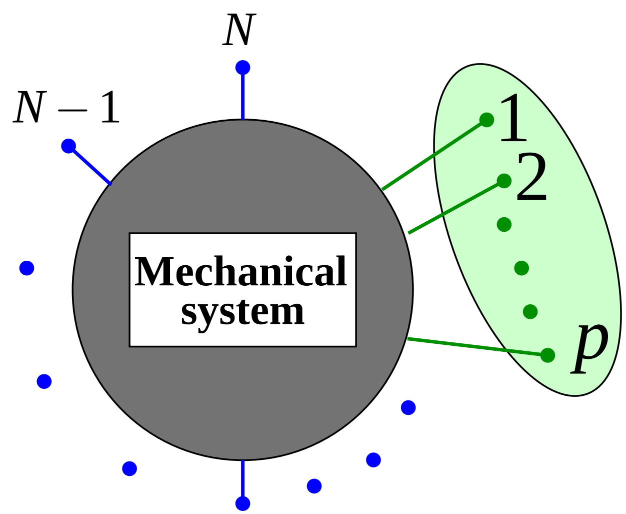


Terminals $\{1,2\}$ and $\{3,4\}$ form a port.



Terminals $\{1,2\}$ and $\{3,4\}$ form 'internal' ports.

Mechanical ports



Terminals $\{1,2,\ldots,p\}$ **form a (mechanical) port** $:\Leftrightarrow$

$$(q_1,\ldots,q_p,q_{p+1},\ldots,q_N,F_1,\ldots,F_p,F_{p+1},\ldots,F_N)\in\mathscr{B},$$

and $v : t\in\mathbb{R}\mapsto(a+bt)\in\mathbb{R}^\bullet$, **with** $a,b\in\mathbb{R}^\bullet$

$$\Rightarrow\ (q_1+v,\ldots,q_p+v,q_{p+1},\ldots,q_N,F_1,\ldots,F_p,F_{p+1},\ldots,F_N)\in\mathscr{B}$$

and $F_1+F_2+\cdots+F_p=0.$

'invariance under uniform motion' **and** ***'KFL'*.**

Power and energy

If terminals $\{1, 2, \ldots, p\}$ form a port, then

$$\textbf{power in} = F_1(t)^\top \tfrac{d}{dt} q_1(t) + \cdots + F_p(t)^\top \tfrac{d}{dt} q_p(t),$$

and

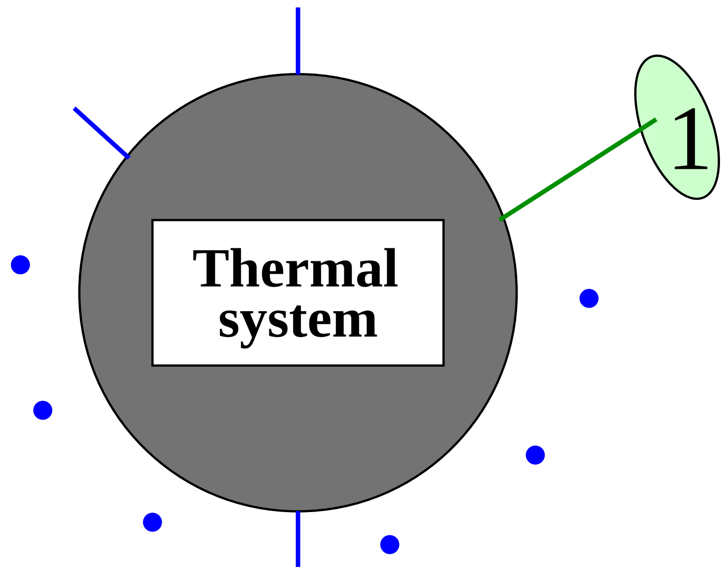
$$\textbf{energy in} = \int_{t_1}^{t_2} \left(F_1(t)^\top \frac{d}{dt} q_1(t) + \cdots + F_p(t)^\top \frac{d}{dt} q_p(t) \right) dt.$$

This interpretation in terms of power and energy is not valid

unless these terminals form a port !

Other domains

temperature T, heat flow Q



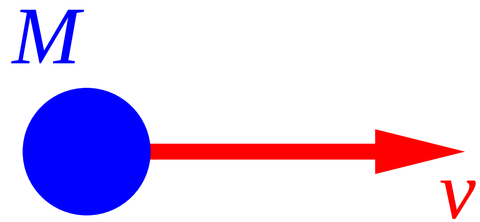
- **Thermal systems: Every terminal forms a port.**

$$\textbf{power in} = Q_1(t).$$

- $\ldots$

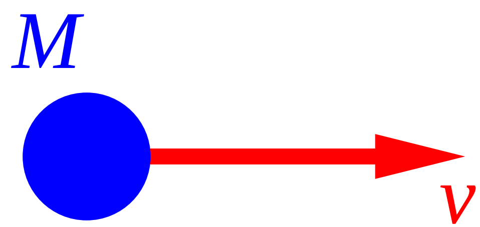
KINETIC ENERGY

Kinetic energy and invariance under uniform motions



What is the kinetic energy?

Kinetic energy and invariance under uniform motions



What is the kinetic energy?

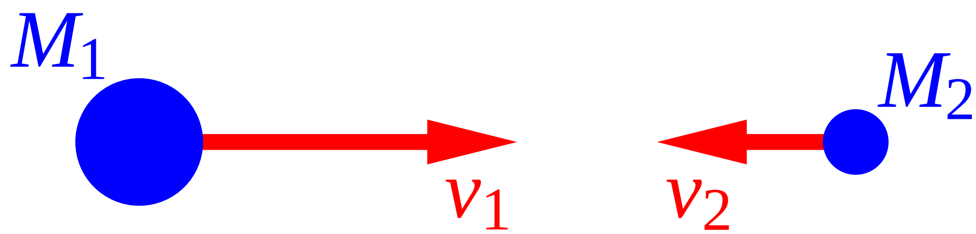
$$\mathscr{E}_{\textbf{kinetic}} = \frac{1}{2} M \, ||v||^2$$

Willem ’s Gravesande
1688–1742

Émilie du Châtelet
1706–1749

This formula is not invariant under uniform motion.

Kinetic energy and invariance under uniform motions



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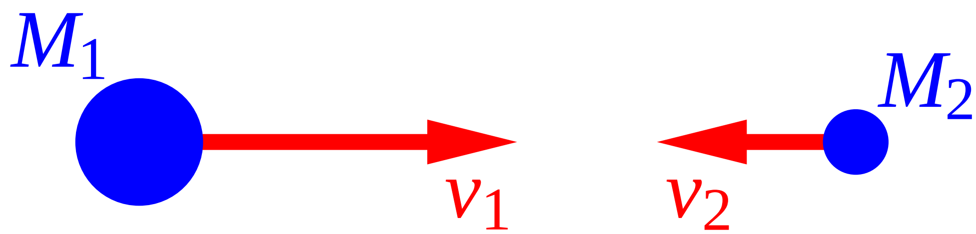
Kinetic energy and invariance under uniform motions



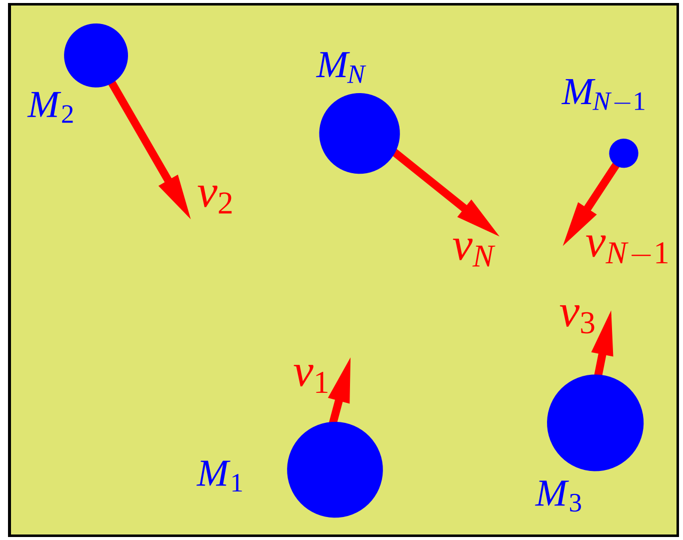
What is the kinetic energy?

$$\mathscr{E}_{\textbf{kinetic}} = \frac{1}{2} \frac{M_1 M_2}{M_1 + M_2} ||v_1 - v_2||^2$$

Invariant under uniform motion.

Can be justified by mounting a damper or a spring between the masses.

Kinetic energy

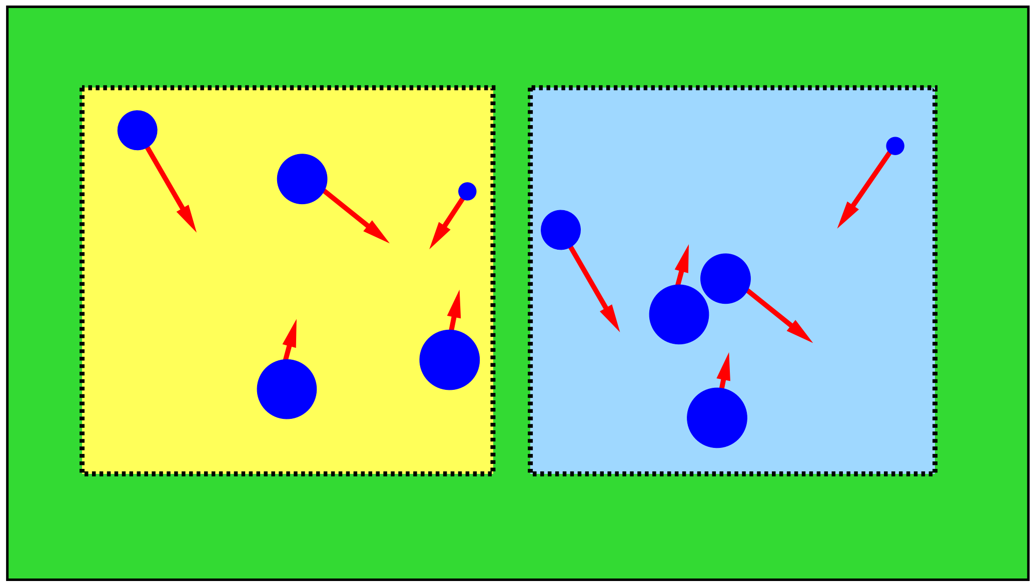


$$\mathscr{E}_{\textbf{kinetic}} = \boxed{\frac{1}{4} \sum_{i,j \in \{1,2,\ldots,N\}} \frac{M_i\, M_j}{M_1 + M_2 + \cdots + M_N} ||v_i - v_j||^2.}$$

$$\textbf{KFL} \quad \Rightarrow \qquad \frac{d}{dt} \mathscr{E}_{\textbf{kinetic}} = \sum_{i \in \{1,2,\ldots,N\}} F_i^\top v_i.$$

Kinetic energy

Kinetic energy is not additive.



Total kinetic energy $\neq$ sum of the parts.

Kinetic energy

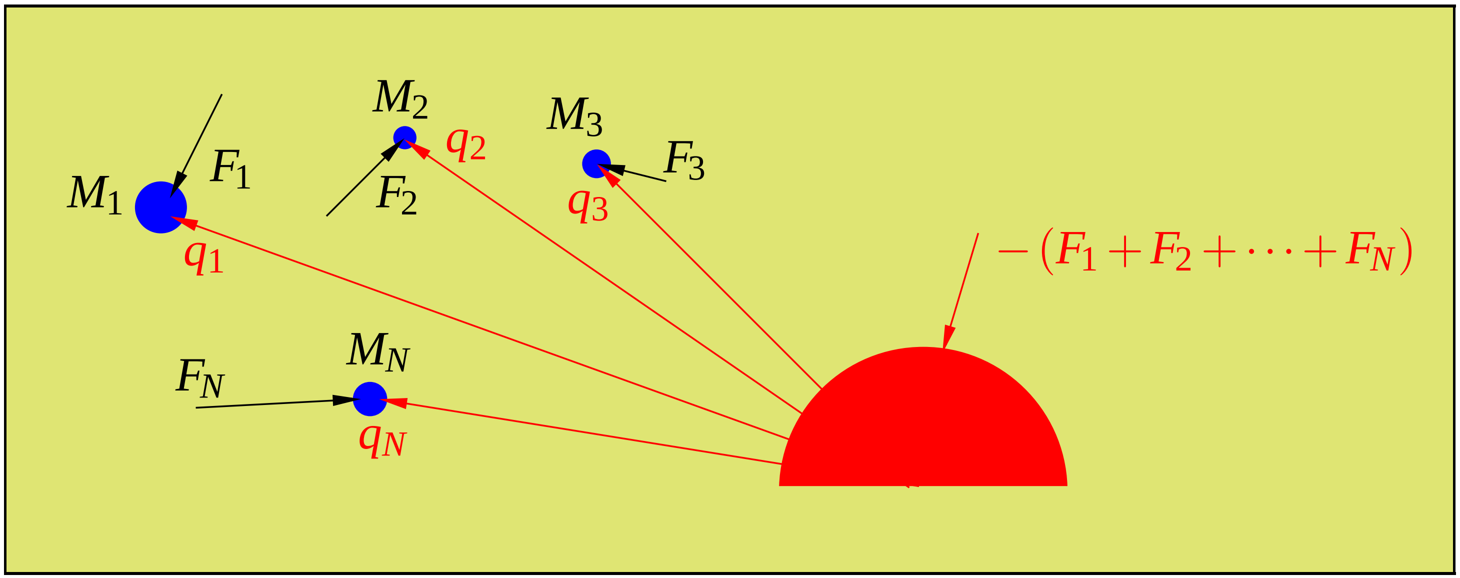
$$\mathscr{E}_{\textbf{kinetic}} = \frac{1}{4} \sum_{i,j \in \{1,2,\ldots,N\}} \frac{M_i \, M_j}{M_1 + M_2 + \cdots + M_N} ||v_i - v_j||^2 .$$

Distinct from the classical expression of the kinetic energy,

$$\mathscr{E}_{\textbf{classical}} = \frac{1}{2} \sum_{i \in \{1,2,\ldots,N\}} M_i \, ||v_i||^2 .$$

Kinetic energy

Reconciliation: $M_{N+1} = \infty, F_{N+1} = -(F_1 + F_2 + \cdots + F_N)$,



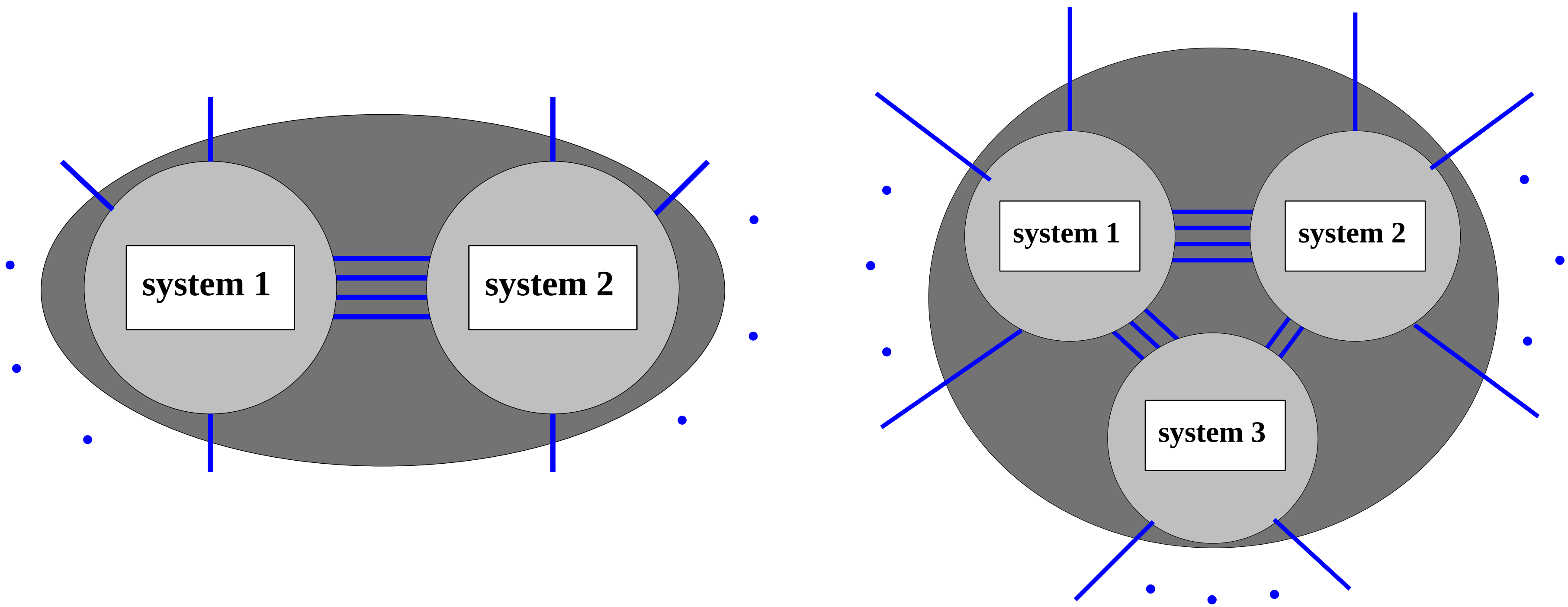
measure velocities w.r.t. this infinite mass, then

$$\frac{1}{4} \sum_{i,j \in \{1,2,\ldots,N,N+1\}} \frac{M_i\, M_j}{M_1 + M_2 + \cdots + M_N + M_{N+1}} ||v_i - v_j||^2$$

$$\longrightarrow \quad \frac{1}{2} \sum_{i \in \{1,2,\ldots,N\}} M_i\, ||v_i||^2.$$

PORTS and TERMINALS

Energy transfer

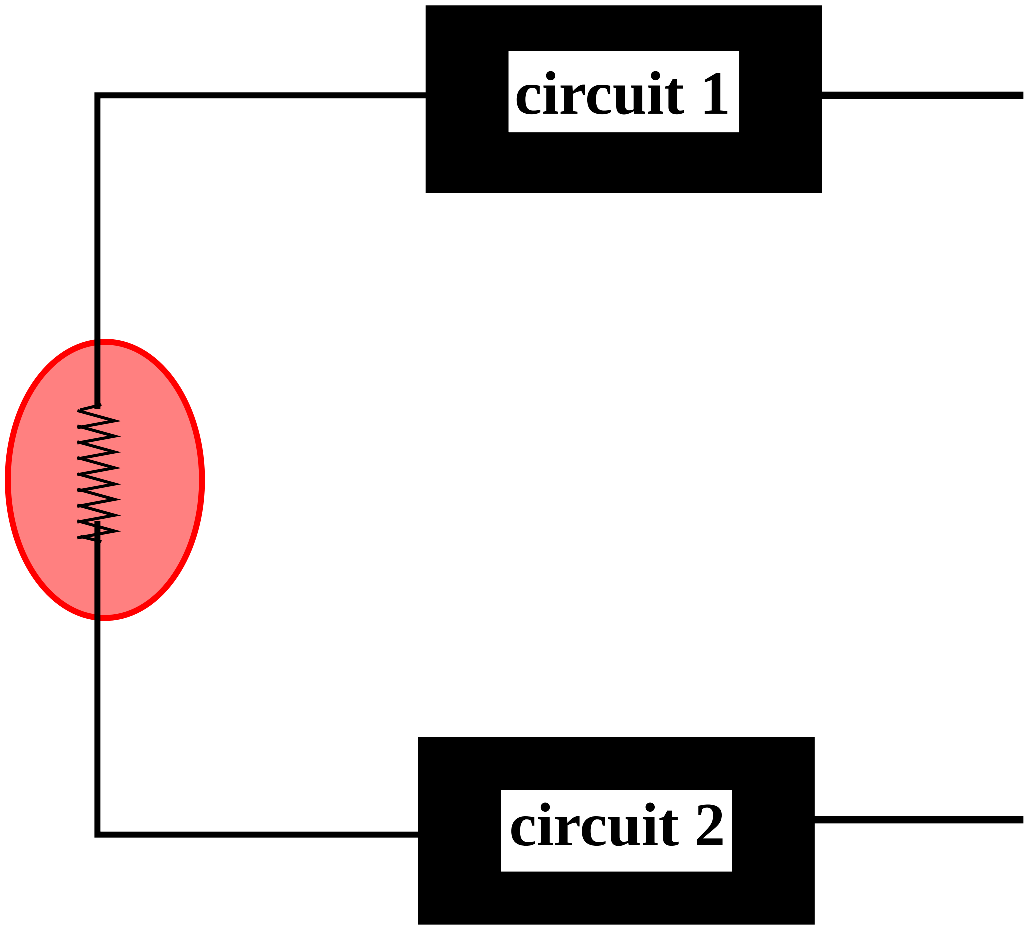


One cannot speak about

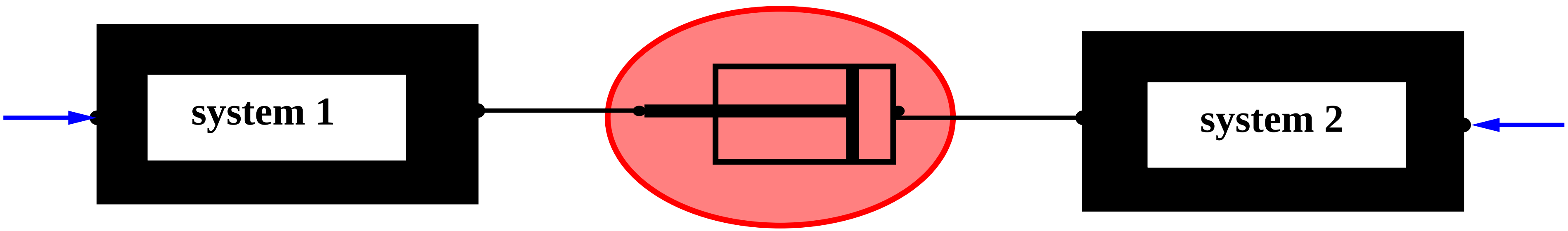
“ the energy transferred from system 1 to system 2 ”
or ***“ from the environment to system 1 ”*****,**

unless the relevant terminals form a port.

Energy transfer



Energy transfer



Terminals and ports

Terminals are for interconnection, ports are for energy transfer.

Interconnection is ‘local’,
power and energy transfer involve ‘action at a distance’.

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The basis of bond-graph (and related) modeling methodologies that

‘In physical systems, the interaction between subsystems is always related to an exchange of energy’

is flawed.

CONCLUSION

Interconnection

- **Physical systems are not input/output devices. Interconnection of physical systems should not be viewed in terms of output-to-input assignment.**

A physical system is not a signal processor.

Interconnection

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A physical system is not a signal processor.

- **Interconnection of physical devices is distinctly different from equating power and energy flow.**

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Interconnection

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  A physical system is not a signal processor.

- **Interconnection of physical devices is distinctly different from equating power and energy flow.**

  Interconnection is 'local',

  power and energy involve 'action at a distance'.

- **Interconnection of physical systems involves**

  variable sharing.

Geniet van jouw emeritaat !!!

The behavioral approach to open and interconnected systems, *Control Systems Magazine*, volume 27, pages 46-99, 2007.

Copies of the lecture frames will be available from/at

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Thank you

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