sc4026
Bonus Point Exercise 1

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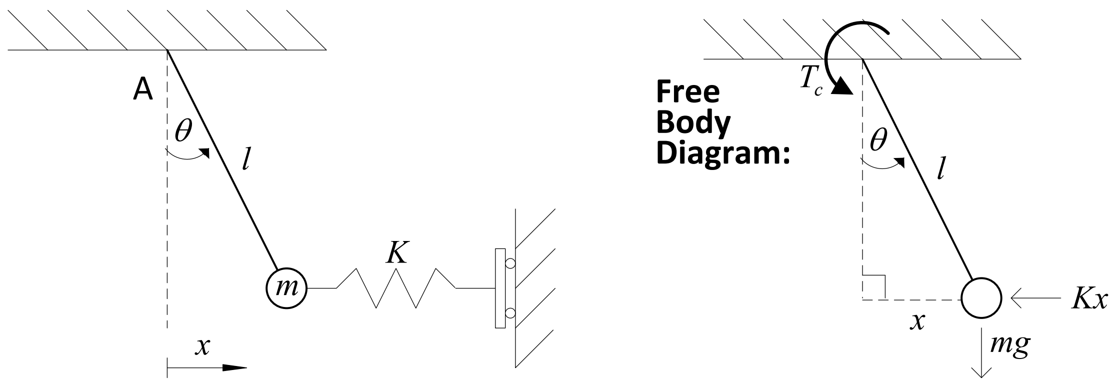
Linearization Example

In the following diagram, we have a ball of mass m swinging on a mass-less, perfectly rigid rod of length l. The ball is attached by a spring to a mass-less and friction-less cart, whose only purpose is to keep the spring horizontal with the ball. There is a torque input applied at the base of the rod (point A).



Variables and parameters for the model:

- m, mass of ball
- l, length of pendulum
- g, gravitational constant
- K, spring constant
- T_c, moment input
- θ, angle of pendulum

- x, horizontal position of ball (zero is directly beneath pivot)
- I, moment of inertia (given to be ml^2)

Solve the following points:

1. Build up a state-space representation of the controlled system described above;
2. Linearize the model around the spatial-input operating point $(\theta, T_c) = (0, 0)$.

Hints for the solution

Sum of the torques around the pivoting point A is equal to the moment of inertia of the ball for the pivot point A multiplied with angular acceleration:

$$\sum \tau = I\alpha = (ml^2)\ddot{\theta}$$

From this equation you will obtain the nonlinear equation of motion. Define the state space vector $x = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}$, and obtain the state-space description of the system. State space description will have the form

$$\dot{x} = \begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ f(\theta, T_c) \end{bmatrix}$$

Linearize function $f(\theta, T_c)$:

$$f(\theta, T_c) \approx f(0,0) + \left.\frac{\partial f}{\partial \theta}\right|_{(0,0)} (\theta - 0) + \left.\frac{\partial f}{\partial T_c}\right|_{(0,0)} (T_c - 0).$$

Substitute in this linearized equation in the nonlinear model and you have linearized system!