sc4026
Bonus Point Exercise 4

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Separation Principle

In class, we showed the validity of the separation principle by directly looking at the state-space matrix of the overall closed-loop system. See Lecture 6, page 4 and 5, for more details.

Prove the same thing, starting again form the matrix on page 4, using the *Schur complement*, which is a neat result in linear algebra that is summarized as follows.

Let $S \in \mathbb{R}^{(n+m)\times(n+m)}$ be a matrix partitioned as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix},$$

with $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{m\times n}$ and $D \in \mathbb{R}^{m\times m}$. If A is invertible then

$$\begin{bmatrix} I_n & 0 \\ -CA^{-1} & I_m \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} I_n & -A^{-1}B \\ 0 & I_m \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & D - CA^{-1}B \end{bmatrix},$$

where $S_A = D - CA^{-1}B$ is called the *Schur complement* of matrix A.