sc4026
Exercise Session 1

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1. Properties of the matrix exponential

Show, by using a few terms of the infinite series expansion, the following equalities:

- $e^{A(t+s)} = e^{At}e^{As}$
- $e^{(A+B)t} = e^{At}e^{Bt}$ if $AB = BA$ (i.e., A and B commute)
- $\frac{d}{dt}e^{At} = Ae^{At} = e^{At}A$
- $\left(e^{At}\right)^{-1} = e^{-At}$

2. Elaboration of the Predator-Prey Model

This model, studied in Lecture 1 (see class notes), is also known under the known appellative of “Lotka Volterra” model.
The state variables are the time-dependent population level for lynxes ($l(t), t \geq 0$) and that of hares ($h(t), t \geq 0$).
Assume that the control Input $b(u)$ (hare birth rate), a function of food, is kept constant: $b(u) = b > 0$. The mortality rate is the constant parameter $d > 0$. The interaction rates are the constant parameters a, c.

The model has the following dynamics:

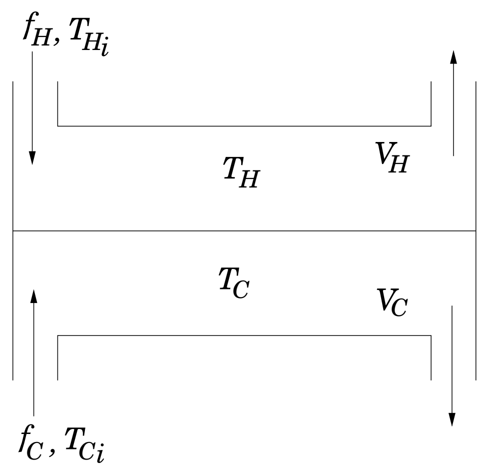
$$\begin{cases} \dot{h}(t) & = bh(t) - a\,l(t)h(t) \\ \dot{l}(t) & = c\,l(t)h(t) - d\,l(t) \end{cases}$$

Questions:

1. Determine all equilibria of this system;
2. Linearize the system about each equilibrium that you found in the first part, and write the results in the form of a first order differential equation;
3. Program your model into Simuling as well as into MATLAB, choosing representative values of a, b, c, and d. Show, using simulation, how the predator and the prey populations evolve for the following three initial conditions:

- no preys, “a few” predators
- “a few” preys, no predators
- d/c preys, b/a predators

3. Heat Exchanger



Consider the heat exchanger in the figure, where f_C and f_H are the flows (assumed to be constant) of cold and hot water, T_H and T_C represent the temperatures in the hot and cold compartments, respectively, T_{Hi} and T_{Ci} denote the temperature of the hot and cold inflow, respectively, and V_H and V_C are the volumes of hot and cold water. The temperature in each compartment evolves according to:

$$V_C \frac{dT_C}{dt} = f_C(T_{Ci} - T_C) + \beta(T_H - T_C)$$

$$V_H \frac{dT_H}{dt} = f_H(T_{Hi} - T_H) - \beta(T_H - T_C)$$

Let the inputs to this system be $u_1 = T_{Ci}, u_2 = T_{Hi}$, let the outputs be $y_1 = T_C$ and $y_2 = T_H$, and assume that $f_C = f_H =$

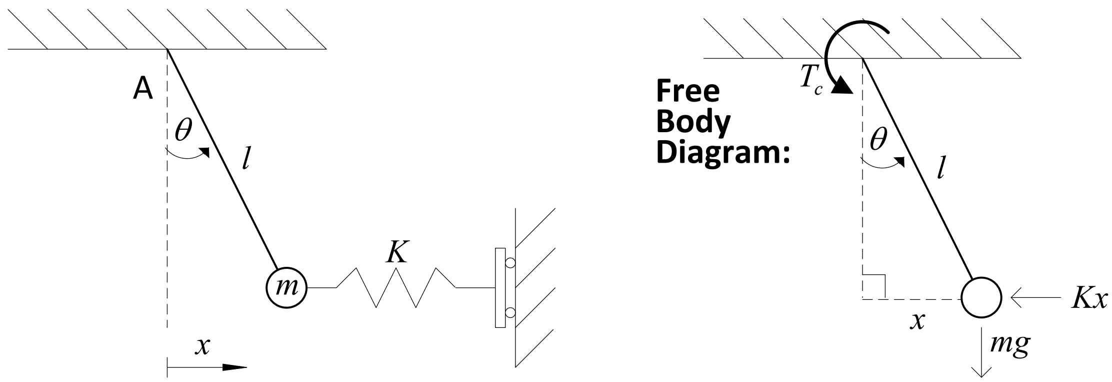
$0.1(m^3/\min)$, $\beta = 0.2(m^3/\min)$ and $V_H = V_C = 1(m^3)$. Assume that both compartments start at a temperature of 1 *units of temperature*.

Solve the following:

1. Write state space and output equations for this system;
2. In the absence of any input, determine $y_1(t)$ and $y_2(t)$;
3. Now, suppose $T_{Hi} = 10$ and $T_{Ci} = -10$ *units of temperature*. Determine $y_1(t)$ and $y_2(t)$.

4. (or extra points) Linearization Example

In the following diagram, we have a ball of mass m swinging on a mass-less, perfectly rigid rod of length l. The ball is attached by a spring to a mass-less and friction-less cart, whose only purpose is to keep the spring horizontal with the ball. There is a torque input applied at the base of the rod (point A).



Variables and parameters for the model:

- m, mass of ball
- l, length of pendulum
- g, gravitational constant
- K, spring constant
- T_c, moment input
- θ, angle of pendulum

- x, horizontal position of ball (zero is directly beneath pivot)
- I, moment of inertia (given to be ml^2)

Solve the following points:

1. Build up a state-space representation of the controlled system described above;
2. Linearize the model around the spatial-input operating point $(\theta, T_c) = (0, 0)$.