sc4026
Exercise Session 2

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Basic review of Eigenvalues and Eigenvectors Computation

Find the eigenvalues and eigenvectors (by hand) of the following matrix:

$$A = \begin{bmatrix} 5 & 2 & -2 \\ 1 & 7 & -1 \\ -3 & 0 & 6 \end{bmatrix}$$

Do similarly with the aid of MATLAB.

Matrix Exponential

Compute e^{At}, through its diagonalization, where:

$$A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 1 & -0.5 \end{bmatrix}.$$

The diagonalization version should include the explicit use of the similarity transformation.

Solution to state space models

Find the output response to a step input and to an impulse, where the system is given by:

$$X = AX + BU$$
$$Y = CX + DU,$$

where

$$X(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix}, D = 0,$$

and where A is

$$\begin{bmatrix} 1 & 3 \\ 0 & 4 \end{bmatrix}, \begin{bmatrix} \alpha & -\beta \\ \beta & \alpha \end{bmatrix}, \text{ and} \begin{bmatrix} -1 & 3 \\ 0 & -4 \end{bmatrix}.$$

In the first of the three cases above, do the computation by hand and then quickly check your result in MATLAB. The second case can be handled, if necessary, with the MATLAB symbolic toolbox. The third case should be verified in MATLAB with the commands `step` and `impulse`. Compute for this last case the steady-state gain.

Coordinate Scaling

Perform, if possible, a coordinate scaling of the following model (undamped spring-mass system):

$$m\ddot{q} + kq = u.$$

Direct Stability Analysis

For a given linear state-space description of the system:

$$\dot{\mathbf{x}} = A\mathbf{x} + \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \mathbf{u}$$

$$\mathbf{y} = \begin{bmatrix} 1 & 3 & 2 \end{bmatrix} \mathbf{x} \tag{1}$$

where the eigenvalues of the system are $s_1 = 2i$, $s_2 = -2i$ and $s_3 = -2$, analyze the stability of the zero equilibrium point <u>using the definition of stability</u> (ϵ-δ reasoning). Give δ as function of ϵ!

Equilibria and Phase Portrait

Determine the equilibrium point and sketch the phase portraits[1] of the systems given in the following polar coordinates:

$$\dot{\rho} = \rho(\rho^2 - 1) \ , \ \dot{\theta} = -1 \tag{2}$$

$$\dot{\rho} = -\rho(\rho^2 - 1) \ , \ \dot{\theta} = -1 \tag{3}$$

$$\dot{\rho} = \rho(\rho^2 - 1)^2 \ , \ \dot{\theta} = -1 \tag{4}$$

$$\dot{\rho} = \rho(\rho^2 - 1)(\rho^2 - 4) \ , \ \dot{\theta} = -1 \tag{5}$$

Here ρ is the radius of the point in the plane and θ is the angle between the radius line and the positive direction of the x axis (θ is positive in the counter-clockwise direction). In other words: $x = \rho\cos(\theta)$ and $y = \rho\sin(\theta)$.

[1] Sketch the phase portrait by hand.