

# sc4026

## Exercise Session 3

Alessandro Abate  
a.abate@tudelft.nl

Solomon Zegeye  
s.k.zegeye@tudelft.nl

Delft Center for Systems and Control, TU Delft

September 24, 2009

## Lyapunov Stability Check

Consider the continuous-time system  $\dot{x} = 0.8x$  and the test function  $V(x) = 2x$ . Here the state is a scalar. Which one of the following statements is true?

1.  $V(x)$  is a Lyapunov function for this system and therefore the system is asymptotically stable;
2.  $V(x)$  is not a Lyapunov function for this system and therefore the system is not stable;
3.  $V(x)$  is not a Lyapunov function for this system, but given this information, we cannot conclude anything about the stability of the system.

## Lyapunov Stability

Study the stability of the system:

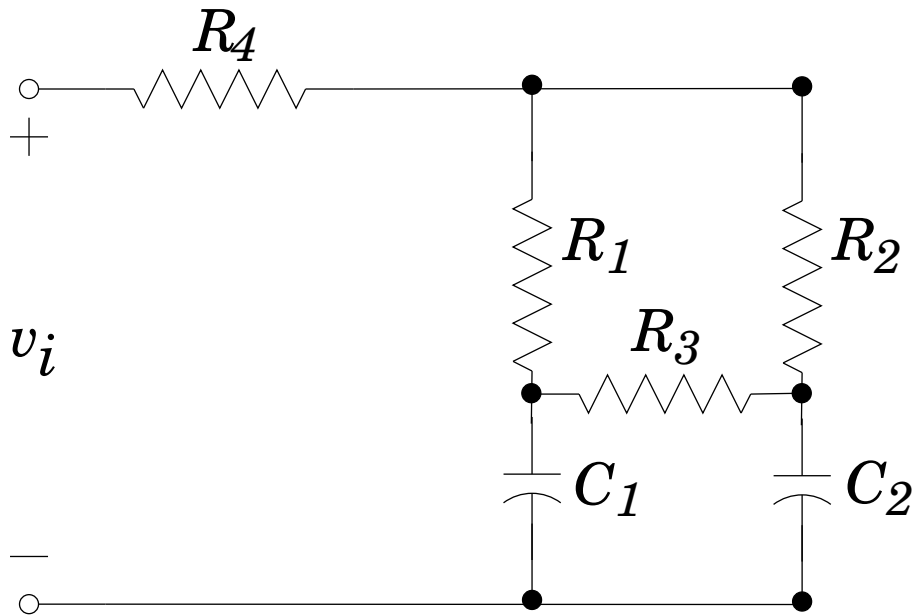
$$\ddot{y} + y - \left( \frac{\dot{y}^3}{3} - \dot{y} \right) = 0.$$

## Controllability

Show that the property of controllability for a linear system is preserved under similarity transformation.

## Electrical System with Symmetry

Defining two state variables as the voltages across the two capacitors  $C_1, C_2$ , is the system shown in the Figure controllable from  $v_i$ ? Give an explanation (why or why not?) in terms of the circuit behavior. (HINT: Consider both the case in which the circuit is balanced ( $C_1 R_1 = C_2 R_2$ ), and the case in which the circuit is not balanced ( $C_1 R_1 \neq C_2 R_2$ )).



## Stability Implications of the Integration of an Ordinary Differential Equation

Consider the simple two-dimensional model  $\dot{x} = Ax, x \in \mathbb{R}^2$ , where

$$\begin{bmatrix} -0.1 & 1 \\ -1 & -0.1 \end{bmatrix}.$$

Integrate the model in time, using an automatic procedure of your choice in MATLAB. Then, integrate the model “manually,” by employing a first-order Euler scheme, with increasing step size. Show that, for a large enough step size, the asymptotic stability of the original ordinary differential equation is corrupted.