sc4026
Exercise Session 4

Alessandro Abate
a.abate@tudelft.nl

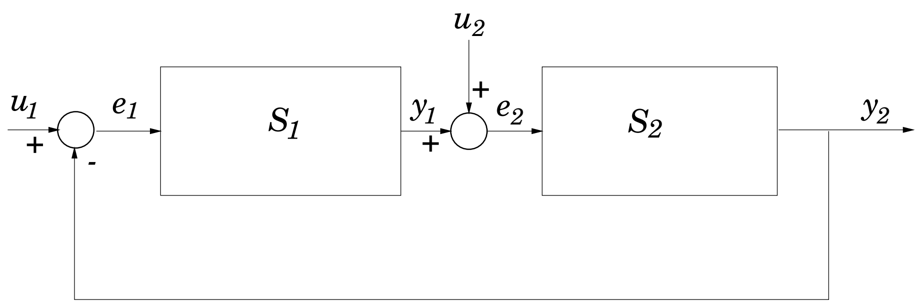
Solomon Zegeye
s.k.zegeye@tudelft.nl

Delft Center for Systems and Control, TU Delft

October 1, 2009

A. Abate, S.K. Zegeye

Controllability and Observability of Feedback Interconnection



Consider the two systems shown in the Figure above, in which S_1 represents the system

$$\dot{x}_1 = A_1 x_1 + B_1 e_1$$
$$y_1 = C_1 x_1,$$

where $A_1 \in \mathbb{R}^{n_1 \times n_1}$, $B_1 \in \mathbb{R}^{n_{i1}}$, $C_1 \in \mathbb{R}^{n_{o1}}$; and S_2 represents

$$\dot{x}_2 = A_2 x_2 + B_2 e_2$$
$$y_2 = C_2 x_2,$$

where $A_2 \in \mathbb{R}^{n_2 \times n_2}$, $B_1 \in \mathbb{R}^{n_{i2}}$, $C_1 \in \mathbb{R}^{n_{o2}}$.
Assume that both S_1 and S_2 are controllable and observable.

1. Write a state-space representation of the overall system that is interconnected as shown in Figure. The input of the

interconnection is $[u_1 \;\; u_2]^T$, the output $[y_1 \;\; y_2]^T$, and the state vector $[x_1 \; x_2]^T$;

2. Show that the interconnection is both controllable and observable.

Pole placement

Consider the following dynamical system:

$$\frac{d^4\theta}{dt^4} + \alpha_1 \frac{d^3\theta}{dt^3} + \alpha_2 \frac{d^2\theta}{dt^2} + \alpha_3 \frac{d\theta}{dt} + \alpha_4 \theta = u,$$

where θ is the state, u represents a control input, whereas α_i are real scalars. Assuming that $\frac{d^3\theta}{dt^3}, \frac{d^2\theta}{dt^2}, \frac{d\theta}{dt}$ and θ can all be measured, design a state feedback control scheme (use a single input) which places the closed-loop eigenvalues at $s_1 = -1, s_2 = -1, s_3 = -1 + j, s_4 = -1 - j$.

Observability and Observer Design

Consider a process in the state-space form with

$$A = \begin{pmatrix} 0.5 & 1 \\ 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0.1 \\ 0.5 \end{pmatrix},$$
$$C = \begin{pmatrix} 1 & 0 \end{pmatrix}, \quad D = 0.$$

Address the following items:

1. Check whether the system is observable. Motivate your answer;
2. Compute the observer gain K such that the observer poles are $s_1 = -0.3$ and $s_2 = -0.7$. Draw a block diagram of the observer;
3. Verify the above computations in MATLAB.

Model formulation, its controllability and observability

Let an LTI system be defined by the differential equations:

$$\frac{d^2}{dt^2}y_1(t) - y_2(t) + \frac{d}{dt}u_1(t) = 0$$

$$-\frac{d}{dt}y_2(t) + y_1(t) + u_2(t) = 0,$$

where $y_1(t), y_2(t)$ are outputs, $u_1(t), u_2(t)$ are inputs.

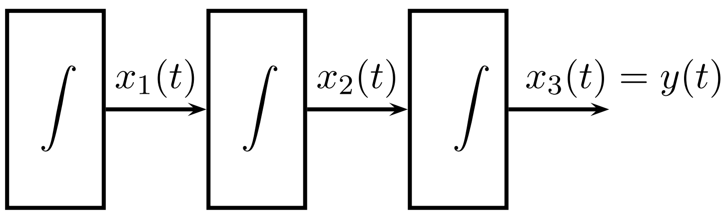
1. Make a block diagram of the system;
2. Formulate the system through its state-space model;
3. Analyze the model to study its controllability and observability.

Observability and Observer Design

Consider an LTI system

$$\dot{x}(t) = Ax(t) + Bu(t), \quad x(0) = x_0 \in \mathbb{R}^3,$$
$$y(t) = Cx(t),$$

defined by the following block diagram



and having $x(0) = [1\,0\,0]^T$.

1. Show that (A, C) is observable;
2. Synthesize an observer that estimates $x(t)$, and which is driven by $y(t)$;
3. Let $\hat{x}(t)$ be the states of the designed observer. Compute $\hat{x}_1(t)$, for $t \longrightarrow \infty$.