sc4026
Exercise Session 5

Alessandro Abate
a.abate@tudelft.nl

Aleksandar Haber
a.haber@tudelft.nl

Delft Center for Systems and Control, TU Delft

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A. Abate, A. Haber

State vs Output Feedback

Consider the model

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 5 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 0 \\ -2 \end{bmatrix} u,$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} x.$$

If legitimate, design a state-feedback controller so that the closed-loop dynamics have eigenvalues $-1, -2, -1 \pm j$. Furthermore, if it makes sense, design an observer that has dynamics depending on the following eigenvalues: $-3, -2, -3 \pm j2$, then exploiting the separation principle close the overall loop by designing an observer-based feedback that again places the poles in $-1, -2, -1 \pm j$.

Linear Quadratic Regulator, in Theory

Consider an object of mass $m = 1$ moving along the x-axis in response to a force input $u(t)$. The objects dynamics can be described simply as $\ddot{x}(t) = u(t)$. This model is also known as *the double integrator*.

- Is the system controllable?

Suppose you would like to design an input $u(t)$ which will move the object from any initial position and velocity, to come to rest at the position $x_e = 0$.

- Using the linear quadratic regulator (LQR) discussed in class, formulate an appropriate quadratic cost functional, and solve the problem by hand, showing simulations of your results for different weightings on the state and input.

Stabilization by state-feedback: its dependence on the underlying controllability and its fragility

Consider the model $\dot{x} = Ax + Bu$, where

$$A = \begin{bmatrix} -\alpha & \epsilon \\ 0 & -\beta \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

and where $\alpha, \beta \geq 0, \epsilon \approx 0$.

- Design a general state feedback, deriving the eigenvalues of the obtained closed-loop system;
- Reason on the effect of small perturbations on ϵ, with particular focus on the case $\epsilon \downarrow 0$;
- Reason on the effect of perturbations on the parameters α, β (for simplicity, you can fix one parameter, then move the second – and viceversa).

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Eigenvalue Assignment via Controllability Canonical Form

Argue that it is extremely convenient to assign eigenvalues via state-feedback through the controllability canonical form. Draw the same conclusion for the observer design case.