

# sc4026

## Exercise Session 6

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## Integral Control

Consider the first-order system described by

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{s + 3},$$

that is, such that  $A = -3$ ,  $B = 1$ ,  $C = 1$ .

Design a system with integral control and two poles at  $s = -5$ .

□

## Observable Canonical Form from polynomials of Transfer Function

Consider the following transfer function:

$$\frac{Y(s)}{U(s)} = \frac{(s + 10)(s^2 + s + 25)}{s^2(s + 3)(s^2 + s + 36)}$$

Give a state-space description of the system in its canonical controllable form.

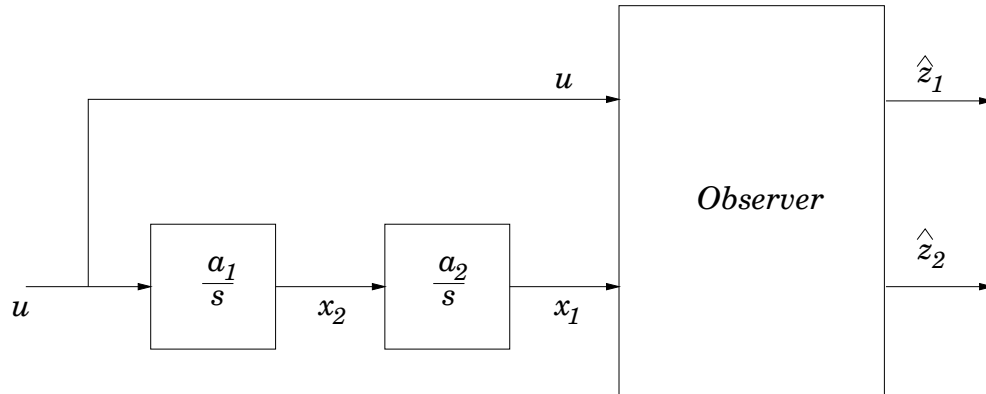
## Direct Derivation of Observable Canonical Form from Transfer Function

Given

$$\frac{d^3y}{dt^3} + 2\frac{d^2y}{dt^2} + \frac{dy}{dt} + y = 3\frac{d^3u}{dt^3} + 5\frac{d^2u}{dt^2} + \frac{du}{dt} + 2u,$$

where  $y(0) = 1, \dot{y}(0) = 0, \ddot{y}(0) = 0$ . Find the observable canonical form.

## Observer Design for DC Servo



The Figure above shows a block diagram representation of a simple model of a DC servo system. The variable  $x_1$  is a voltage signal that corresponds to the output, whereas  $x_2$  is the angular velocity.

1. Design an observer, with observer gain matrix  $L = [l_1 \ l_2]^T$ , for the variables  $x_1, x_2$ , and such that the characteristic polynomial associated with the error dynamics is given by:

$$\Delta(s) = s^2 + 2\zeta\omega s + \omega^2.$$

2. The observer is a system with inputs  $u$  and  $x_1$ , and with outputs  $\hat{z}_1$  and  $\hat{z}_2$ . Thus, there are four possible transfer functions between inputs and outputs these may be included as elements in a  $2 \times 2$  matrix. Evaluate the following matrix

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of transfer functions  $M(s)$  between the inputs to the observer  $u$  and  $x_1$ , and its outputs  $\hat{z}_1$  and  $\hat{z}_2$

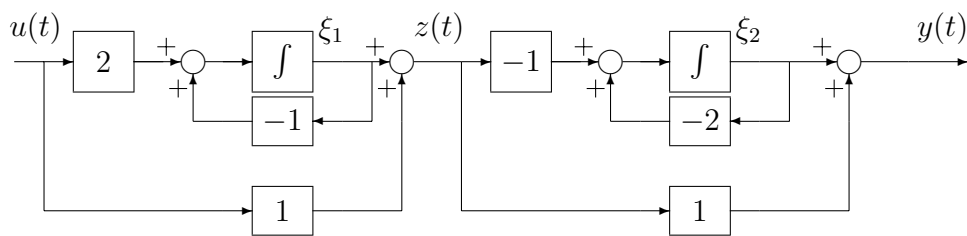
$$M(s) = \begin{bmatrix} \frac{Z_1(s)}{U(s)} & \frac{Z_1(s)}{X_1(s)} \\ \frac{Z_2(s)}{U(s)} & \frac{Z_2(s)}{X_2(s)} \end{bmatrix},$$

as a function of gains  $l_1$  and  $l_2$ , as well as of the system parameters  $a_1, a_2$ .

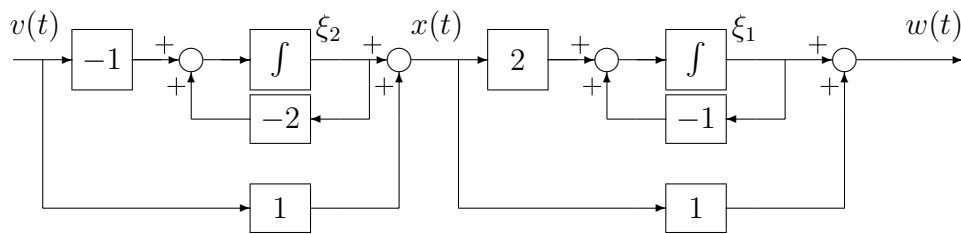
3. Determine  $M(s)$  as  $l_2 \rightarrow \infty$ . Discuss the meaning of the result.

## Loss of Observability and/or Controllability

Recall a topic developed in Lecture 4. Consider system in series:



Show that it is controllable, though not observable. Show that this corresponds to zero/pole cancellation in the Transfer Functions. Now consider system in series:



Show that it is observable, though not controllable. Show that this corresponds to zero/pole cancellation in the Transfer Functions.