sc4026
Homework Set 1

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handed out: September 14
due in class: September 24, 2009

A. Abate

A magnetically suspended steel ball, linearized

The simplied dynamics of a magnetically suspended steel ball are given by:

$$m\ddot{y} = mg - c\frac{u^2}{y^2},$$

where the input u represents the current supplied to the electromagnet, y is the vertical position of the ball, which may be measured by a position sensor, g is gravitational acceleration, m is the mass of the ball, and c is a positive constant such that the force on the ball due to the electromagnet is cu^2/y^2 . Assume a normalization such that $m = g = c = 1$.

Questions:

1. Using the states $x_1 = y$ and $x_2 = \dot{y}$ write down a nonlinear state space description of this system;
2. What equilibrium control input u_e must be applied to suspend the ball at $y = 1$ m?
3. Write the linearized state space equations for state and input variables representing perturbations away from the equilibrium obtained in the previous part.

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Step and Impulse Response

Consider the following system (it actually represents the heading dynamics of a tricycle landing gear on the ground, with heading actuator on the front wheel as input):

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}$$

Assume that $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. What is the steady state error in the output to a step input $u(t)$? What is the impulse response?
Use MATLAB to verify your calculations.

Local vs Global validity of Linearization procedure

The purpose of this exercise is to illustrate how linearized model can sometimes have a totally different (in qualitative sense) global behavior than that of the original nonlinear system and, as such, can lead to incorrect conclusions about the global properties of the equilibrium point.
Consider the mathematical models of the following two nonlinear systems:

$$\dot{x}_1 = -x_2 + x_1(x_1^2 + x_2^2)$$
$$\dot{x}_2 = x_1 + x_2(x_1^2 + x_2^2),$$

and

$$\dot{x}_1 = -x_2 - x_1(x_1^2 + x_2^2)$$
$$\dot{x}_2 = x_1 - x_2(x_1^2 + x_2^2).$$

For each of the two systems, do the following:

1. Determine its equilibrium point;
2. Linearize the nonlinear model around the equilibrium point;
3. Sketch the phase portrait of the linearized model[1]. Compare

[1]The phase portrait can be sketched 1. by hand, or 2. in MATLAB, or 3. via the "ODE software for MATLAB" by Prof. Polking, available at `http://math.rice.edu/~dfield/` .

this phase portrait with the phase portrait of the linearized model of the other nonlinear system. What do you observe?

4. Using polar coordinates transform the original nonlinear systems and sketch the phase portraits of the transformed system (do not linearize it!). Compare this phase portraits with those of the linearized models from the previous task. What do you observe?
5. Qualitatively answer the following question: does linearized model preserve the "true nature" of the nonlinear system in this example? Summarize your conclusions in the form of a short paragraph.

Your answers should include all the mathematical transformations that you have performed. For example, when analyzing the equilibrium point, you will have to solve mathematically correctly the algebraic nonlinear system of equations, or when using the polar coordinates to transform the system you will have to include the every step of transformation.

Asymptotic Stability Analysis

Consider the system

$$\dot{x} = A\,x, \qquad x_0 \in \mathbb{R}^n.$$

Formally prove that the above system is (globally) asymptotically stable if and only if the real parts of the eigenvalues of A are all less than zero.

Now, consider the model (here again $x \in \mathbb{R}^n$)

$$\begin{aligned} \ddot{x} + d\dot{x} + k\,x &= u, \\ y &= x. \end{aligned}$$

1. Write the system equations in state space form;
2. For what values of the parameters k and d is the system asymptotically stable?