sc4026
Homework Set 2

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A magnetically suspended steel ball, linearized: part 2

The simplied dynamics of a magnetically suspended steel ball are given by:

$$m\ddot{y} = mg - c\frac{u^2}{y^2},$$

where the input u represents the current supplied to the electromagnet, y is the vertical position of the ball, which may be measured by a position sensor, g is gravitational acceleration, m is the mass of the ball, and c is a positive constant such that the force on the ball due to the electromagnet is cu^2/y^2 . Assume a normalization such that $m = g = c = 1$.

In the homework set 1, you answered to the following three questions:

- Using the states $x_1 = y$ and $x_2 = \dot{y}$ write down a nonlinear state space description of this system;
- What equilibrium control input u_e must be applied to suspend the ball at $y = 1$ m?
- Write the linearized state space equations for state and input variables representing perturbations away from the equilibrium obtained in the previous part.

Now, answer to the following set of new questions:

1. Is the linearized model stable? Can you find a Lyapunov function for the linearized model? What can you conclude about the stability of the nonlinear system close to the equilibrium point x_e ?
2. Is the linearized model controllable? Observable?
3. Design a state feedback controller for the linearized system, to place the closed loop poles at points $-1, -1$;
4. Design an observer, so that the state estimate error dynamics has eigenvalues at $-5, -5$;
5. (to be solved after attending Lecture 6) Combine your answers above to find an output feedback controller $K(\cdot)$ for the linearized system that places the closed loop system poles at $-1, -1, -5, -5$;
6. (to be solved after attending Lecture 6) Now, suppose that you applied this controller to the original nonlinear system. Discuss how you would expect the system to behave. How would the behavior change if you had chosen controller poles at $-5, -5$, and observer poles at $-20, -20$?

Stability, controllability, and observability of a 3-d model

Given the LTI state-space model characterized by the following matrices

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -3 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}.$$

1. Find the characteristic polynomial of A. Is the dynamical part of the model ($\dot{x} = Ax$) stable? If so, show that you can find a Lyapunov function for the system (this can be done either by hand with the definition of Lyapunov function, or with MATLAB solving the associated Lyapunov equation).
2. Is the model (A, B) controllable? If so, find its controllable canonical form, and derive the similarity transformation (matrix T) that obtains such a form.
3. Find the set of eigenvalues of the closed loop system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \quad x(0) = x_0 \\ y(t) &= Cx(t) \\ u(t) &= -Kx(t) + v(t), \end{aligned}$$

that can be obtained for any possible real-valued $K \in \mathbb{R}^3$.

4. Is the model (A, C) observable? Why?
5. In the case of absence of observability, use the Hautus test to find the unobservable eigenvalues.
6. Does there exist a $K \in \mathbb{R}^3$ such that $(A - BK, C)$ is observable? Motivate your answer.

Lateral Dynamics of an Aircraft

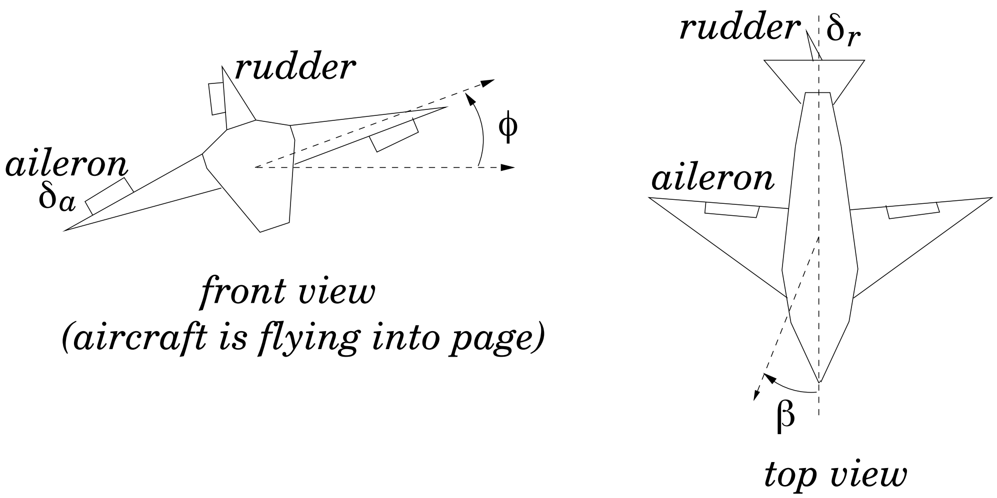

Figure 1: Lateral axes of an aircraft.

An approximate linear model of the lateral dynamics of an aircraft, for a particular set of flight conditions, has the linearized state and control vectors:

$$x = \begin{bmatrix} p \\ r \\ \beta \\ \phi \end{bmatrix}, \quad u = \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix},$$

where p and r are incremental roll and yaw rates, β is an incremental sideslip angle, and ϕ is an incremental roll angle. The control inputs are the incremental changes in the aileron angle δ_a and in the rudder angle δ_r, respectively. These variables are shown in the Figure above. The state-space equation for this model is $\dot{x} = Ax + Bu$, where

$$A = \begin{bmatrix} -10 & 0 & -10 & 0 \\ 0 & -0.7 & 9 & 0 \\ 0 & -1 & -0.7 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 20 & 2.8 \\ 0 & -3.13 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

1. Suppose a malfunction prevents manipulation of the input δ_r. Is it possible to completely control the aircraft using only δ_a?
2. If you had your choice of <u>only one</u> of the following sensors, which would you choose? Explain.
   - A rate gyro which measures the roll rate p;
   - A bank indicator which measures ϕ.