sc4026
Homework Set 3

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A. Abate

Linear Quadratic Regulator, via Simulations

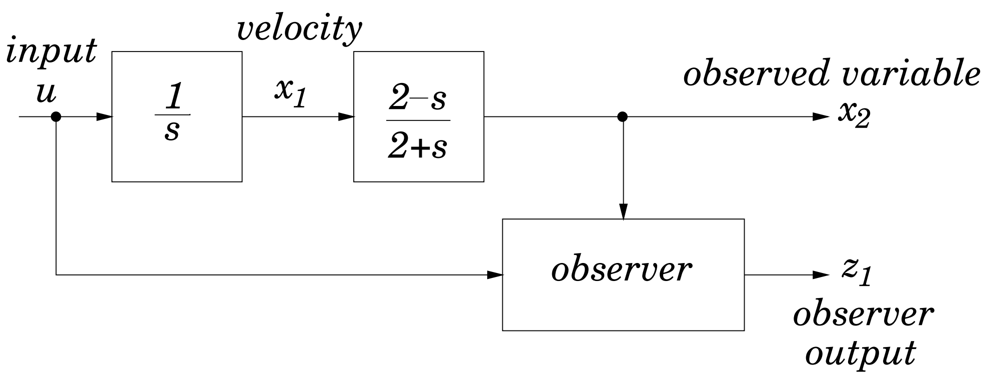
Consider an object of mass $m = 1$ moving along the x-axis in response to a force input $u(t)$. The objects dynamics can be described simply as $\ddot{x}(t) = u(t)$. This model is also known as *the double integrator*.

- Is the system controllable?

Suppose you would like to design an input $u(t)$ that will move the object from any initial position and velocity, to come to rest at the position $x_e = 0$.

- Using the linear quadratic regulator (LQR) discussed in class, formulate an appropriate quadratic cost functional, and solve the problem in MATLAB, showing simulations of your results for different weightings on the state and input. More precisely, you can either employ the weighting matrices with their specific parameters as used in class (see Exercise Session 5), or choose identity matrices that are uniformly weighted respectively by parameters α, β. Then, select 5 configurations for the pair (α, β) (same magnitude, $\alpha > \beta, \alpha >> \beta, \alpha < \beta, \alpha << \beta$) and perform simulations.

Observer Design Problem



The Figure above shows a velocity observation system, where x_1 is the velocity to be observed. An observer is to be constructed to track x_1, using the control u and the output x_2 as its inputs. The variable x_2 is obtained from x_1 through a sensor having the following transfer function

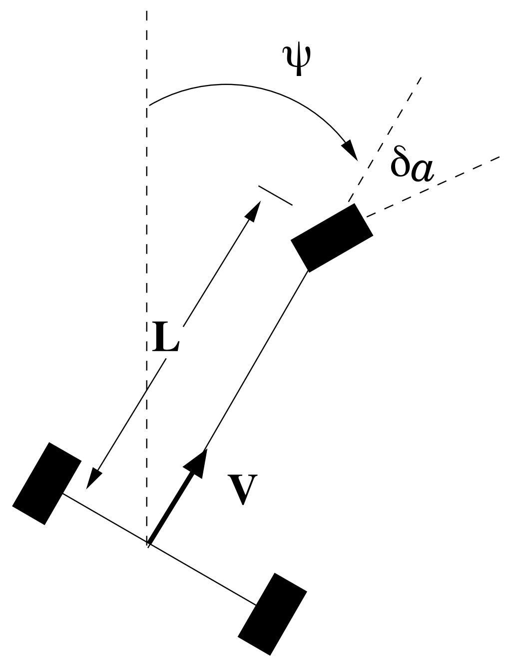
$$G(s) = \frac{2-s}{2+s}.$$

Answer to the following points:

1. Derive a set of state-space equations for the system, with state variables x_1 and x_2 , input u and output x_2. Later, check your result by calculating the transfer function of the obtained model.
   Plot the poles and the zeros of the transfer function in MATLAB. Also, represent its Bode plot with MATLAB.

2. Design an observer with states z_1 and z_2 to track respectively x_1 and x_2. Choose both observer eigenvalues to be at -4. Write out the state space equations for the observer.
3. Derive the combined state equation for the system plus the observer. Take as state variables $x_1, x_2, e_1 = x_1 - z_1, e_2 = x_2 - z_2$. Consider u as the input and z_1 as the output. Is this system controllable and/or observable? If meaningful, try to give physical reasons for any states being uncontrollable or unobservable.
4. What is the transfer function relating u to z_1? (Explain your result.)

Heading Controller for Aircraft Ground Control



The above figure displays a top view of a tricycle landing gear of an aircraft. Consider the case in which the vehicle is moving with constant forward velocity V (achieved by an input motor thrust, which is not shown here). The only input we would like to consider here is the heading actuator δ_a, which affects the heading ψ. The dynamics between heading actuator and heading, for small angle changes, can be modeled as

$$\Psi(s) = \frac{V}{L}\frac{1}{s(\tau s + 1)}\Delta_a(s),$$

where in the formula we have denoted with capitals letters the respective Laplace transforms, and where $V, L, \tau > 0$.

1. Write a block diagram representation of the feedback loop that presents a controller (call it $C(s)$) and the system block with input δ_a and output ψ. The block diagram should also encompass a reference signal. (If necessary, you can get inspiration from the first Section of Chapter 9 in the A.M.)
2. Based on this representation, design a controller (the simplest possible) for this system, so that a given step reference heading $\psi_{ref} = 1$ is achieved with no steady state error. (In other words, apply a step input at the reference – recall that its Laplace transform is $\frac{1}{s}$ – and make sure to get zero steady state error.)
3. Furthermore, design a controller for this system (again, the simplest possible), so that a given step reference heading rate $\omega_{ref} = \dot{\psi}_{ref} = 1$ (corresponding to a change in the heading) is achieved with no steady state error.