Fast phase diversity wavefront sensing using object independent metrics

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ABSTRACT

Phase diversity methods allow the estimation of both the wavefront disturbance and the object that is being imaged and that is extended in space. In principle, phase diversity methods can also be used for wavefront sensing without the need of wasting part of the observed light to a dedicated wavefront sensor. However, the use of phase diversity in real-time applications is prevented by its high computational complexity determined by the number of parameters quantifying the wavefront and the object. Metrics that are independent on the object have been proposed to reduce the computational complexity which allow the exclusive estimation of the wavefront. Nevertheless, these still yield a nonlinear inverse problem. To further reduce the computational complexity of the wavefront estimation methods, linear approximations of these metrics are considered that enhance the estimate of the wavefront by solving a linear least squares problem. The estimation error is studied with respect to the presence of noise and metrics presented in literature are compared.

Keywords: Phase Diversity, Object Independent

1. INTRODUCTION

The phase of the wavefront is hard to retrieve, since an intensity measurement depends nonlinearly on the phase and is also the result of a convolution with an object. There are different methodologies to estimate the phase and one of these methods is Phase Diversity (PD). The idea of phase diversity is to split the optical path into multiple paths and introduce a known aberration in one of these paths, which can be seen in Fig. 1. The main advantages of this method compared to a wavefront sensor, such as a Shack-Hartmann sensor, are that the optical hardware needed for this technique is very simple and the object is taken into account. Parametric based phase retrieval has successfully been applied to applications outside of real-time adaptive optics such as the determinations of quasi-static aberrations for the Hubble telescope.

The main drawback of the current phase diversity methods for the use of wavefront sensing is that they are slow. This is due to the nonlinear dependence on the aberrations and the presence of an object. Two approaches are known to

![Figure 1: Principle of phase diversity: The object plane ( ); The propagation direction of the wavefront ( ); The unknown aberration caused by a structure with a spatially varying refractive index ( ); Known aberration caused by a structure with a spatially varying refractive index ( ); The image planes ( ).](image-url)
eliminate the object from the estimation problem: separable least squares\(^5\) and object independent metrics.\(^6\)

The main idea of separable least squares is that the estimation is separable and therefore an estimate of the object can be substituted back into the cost function. This cost function is therefore optimized for the aberrations. However, the cost function following from Maximum Likelihood (ML) estimation suffers from local optima. The idea of an object independent metric is to solve the inverse problem by the construction of a measure, such that its inverse is bijective and therefore solves the inverse problem independent of the object. An object independent metric is constructed by a combination of intensity measurements with different phase diversities, such that the object is eliminated from the metric by simple division in the Fourier domain.

First, in Section 2 a model for the incoherent image formation will be derived. In Section 3 this model will be used for the derivation of a set of metrics and their models. Subsequently, in Section 4 these models will be linearized using a Taylor approximation, such that an estimate can be obtained by inverting the obtained linear system. Finally, the metrics will be compared in Section 5 by simulation. This paper will end with a discussion of the obtained results.

2. PROBLEM FORMULATION

In this section, a model will be presented for the incoherent image formation. First, the transfer function from the object plane to the image plane will be derived for the optical setup given in Fig. 1, followed by the description of the noise process underlying the measurements. The image formation of an incoherent optical setup is given by the following linear convolution\(^7\)

\[
I_i(x, y) = \left(h_i \left(x', y' \right) \ast o \left(x', y' \right) \right)(x, y) + n_i(x, y),
\]

where \(I_i(x, y)\) denotes the \(i\)th acquired image, \(h_i\) denotes the space invariant Point Spread Function (PSF) of the \(i\)th optical path, \(o\) denotes the unknown and ideal object, \((h_i(x, y) \ast o(x, y))(x, y)\) denotes the convolution operation between the functions \(h_i\) and \(o\) over the coordinates \((x', y')\) expressed in the spatial coordinates \((x, y)\). The process \(n_i(x, y)\) following from the readout noise of the \(i\)th measurement is modeled as a spatially white Gaussian process, which has the property that the pixels are mutually uncorrelated. The object is normalized to unit surface to determine the Signal to Noise Ratio (SNR) in the image \((I_i)\), defined in Eq.( 1). Therefore,

\[
o(x, y) = \mu o_n(x, y),
\]

where \(o_n\) is the normalized object, \(o\) the observed object and \(\mu\) is the signal strength given by

\[
\mu = \int \int o(x, y) dx dy,
\]

where the differentials of \(x\) and \(y\) are denoted by \(dx\) and \(dy\). Furthermore, the space invariant PSF of the \(i\)th optical path is given by\(^7\)

\[
h_i(u, v) = \mathcal{F} \left( \Pi(x, y) e^{i\phi_i(x, y)} \right)(u, v) = \mathcal{F} \left( \Pi(x, y) e^{i\phi_i(x, y)} \right)(u, v),
\]

where \((u, v) = (\frac{2 \pi x}{f}, \frac{2 \pi y}{f})\), \(f\) is the focal length, \(\lambda\) is the wavelength in \([m]\), and \(\phi(x, y, \alpha)\) is the phase in unit \([rad]\) *, \(\mathcal{F} \) \((u, v)\) is the Fourier transform, and \(\Pi(x, y): \mathbb{R}^2 \rightarrow \mathbb{C}\) is known as the Generalized Pupil Function (GPF). The phase \(\phi_i(x, y)\), is expanded using a normalized Zernike basis

\[
\phi_1(x, y) = Z(x, y)^T \alpha,
\]

\[
\phi_2(x, y) = Z(x, y)^T (\alpha + \alpha_k),
\]

where \(\alpha \in \mathbb{R}^{l \times 1}\) are the \(l\) Zernike coefficients corresponding to the unknown aberration, \(\alpha_k \in \mathbb{R}^{l \times 1}\) are the \(l\) Zernike coefficients corresponding to the known aberration, and \(Z(x, y) \in \mathbb{R}^{l \times 1}\) is a vector of containing the \(l\) Zernike polynomials

\*The phase \(\phi(x, y, \alpha)\) is related to the Optical Path Difference (OPD) in \([m]\) given by \(\phi(x, y, \alpha) = \frac{2\pi}{\lambda} OPD\) in \([rad]\).

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evaluated in the spatial coordinate \((x, y)\). The normalized Zernike basis has the property that the variance of the wavefront is given by

\[
\sigma_w^2 = \sum_{i=1}^{l} \alpha_i^2 \tag{8}
\]

To obtain a PSF of unit surface, the pupil function \(\Pi(x, y)\) is chosen as follows

\[
\Pi(x, y) = \begin{cases} 
1 & x^2 + y^2 \leq r^2 \\
0 & x^2 + y^2 > r
\end{cases} \tag{9}
\]

where \(A\) is the physical surface of the pupil\(^1\). The convolution operation in Eq. (1) is eliminated by taking the Fourier transform, which gives

\[
Y_i(u, v) = W_i(u, v)F(u, v) + N_i(u, v), \tag{10}
\]

where \(Y_i(u, v) = \mathcal{F}(I_i(x, y))(u, v)\) is the amplitude spectrum of the image, \(W_i = \mathcal{F}(h_i(x', y'))\) \((u, v)\) is the optical transfer function, \(F(u, v) = \mathcal{F}(F(x, y))(u, v)\) is the amplitude spectrum of the object and \(N_i(u, v) = \mathcal{F}(n_i(x, y))(u, v)\) is the Fourier transform of the noise, which is also a white Gaussian noise process due to the fact that the Fourier transform is an orthogonal transformation.

### 2.1 Measurement Noise

Ideally, every photon that is incident to the camera is converted into exactly one electron. In this case, all electrons would be precisely counted and converted into a digital unit, telling exactly how much light was incident to each pixel. However, this is not possible since on-chip readout noise is present. Furthermore, since the photon count is sufficiently high, the count can be neglected and the measurements will be dominated by readout noise. This source of noise is Gaussian and the pixels are mutually uncorrelated. Therefore, the SNR of the camera measurement is given by

\[
SNR = \frac{\mu}{\sigma} \tag{11}
\]

where \(\mu\) is given by Eq. (3) and \(\sigma\) is the standard deviation of the Gaussian white noise process \(n_i(x, y)\) given in Eq. (1).

### 3. IMAGE METRICS

Here the metrics will be derived that form the basis of the estimation method presented in the next section. Now, let the measurement of the \(i\)th phase diversity metric in the Fourier domain be denoted by \(M_{i,meas}(u, v)\) and the metric model be defined by \(M_{i,mod}(u, v)\).

The first metric is the variance of the image\(^8,9\)

\[
M_{1,meas}(u, v) = \frac{Y_1(u, v)Y_1^*(u, v)}{Y_2(u, v)Y_2^*(u, v)} \tag{12}
\]

where \(Y_i(u, v)\) is the Fourier transform of the intensity measurement as defined in Eq. (10).

The second metric is the normalized difference of the variance between the two phase diversity images\(^6\)

\[
M_{2,meas}(u, v) = \frac{Y_1(u, v)Y_1^*(u, v) - Y_2(u, v)Y_2^*(u, v)}{Y_1(u, v)Y_1^*(u, v) + Y_2(u, v)Y_2^*(u, v) + \epsilon} \tag{13}
\]

where \(\epsilon << 1\) is a regularization parameter. This object independent metric is motivated by the fact that normalization eliminates the object.

\(^1\)This normalization is a direct consequence of Parseval’s Theorem.
Using a similar motivation as used for the second metric, the cross correlation between two diversities can be investigated. Therefore, for completeness, the third metric is the normalized cross correlation between two diversity images $Y_1$ and $Y_2$

$$M_{3,\text{meas}}(u, v) = \frac{Y_1(u, v)Y_2^*(u, v)}{\sqrt{Y_1^2(u, v) + \epsilon} \sqrt{Y_2^2(u, v) + \epsilon}},$$  

(14)

where $\epsilon \ll 1$ is again a regularization parameter.

To derive models for the metric measurements $M_{1,\text{meas}}$ to $M_{3,\text{meas}}$, $Y_i(u, v) = W_i(u, v)F(u, v) + N_i(u, v)$ is substituted

$$M_{1,\text{meas}}(u, v) = \left[\frac{W_1(u, v)W_1^*(u, v)F(u, v)F^*(u, v) + N_1'(u, v)}{W_2(u, v)W_2^*(u, v)F(u, v)F^*(u, v) + N_2'(u, v)}\right],$$  

(15)

$$M_{2,\text{meas}}(u, v) = \frac{W_1(u, v)W_1^*(u, v) - W_2(u, v)W_2^*(u, v)}{W_1(u, v)W_1^*(u, v) + W_2(u, v)W_2^*(u, v) + \epsilon(u, v)} + N'(u, v),$$  

(16)

$$M_{3,\text{meas}}(u, v) = \left[\frac{W_1(u, v)W_1^*(u, v) + \epsilon(u, v) + N_1'(u, v)}{W_2(u, v)W_2^*(u, v) + \epsilon(u, v) + N_2'(u, v)}\right],$$  

(17)

where $N'$ are unknown noise processes and $\epsilon'$ are regularization parameters depending on the object and the aberration present. From this can be concluded that $M_{1,\text{meas}}$ still includes the power spectrum of the object, and in spite of what is presented to be object independent metrics, $M_{2,\text{meas}}$ and $M_{3,\text{meas}}$ only become independent of the object when $N(u, v)$ and $\epsilon$ go to zero. To obtain models that do not contain information about the object, a uniform object ($F(u, v) = 1$) is assumed for $M_{1,\text{meas}}$ and a constant $\epsilon' \ll 1$ for $M_{2,\text{meas}}$ and $M_{3,\text{meas}}$, which leads to the following equations

$$M_{1,\text{mod}}(u, v) = \left[\frac{W_1(u, v)W_1^*(u, v)}{W_2(u, v)W_2^*(u, v)}\right],$$  

(18)

$$M_{2,\text{mod}}(u, v) = \frac{W_1(u, v)W_1^*(u, v) - W_2(u, v)W_2^*(u, v)}{W_1(u, v)W_1^*(u, v) + W_2(u, v)W_2^*(u, v) + \epsilon},$$  

(19)

$$M_{3,\text{mod}}(u, v) = \left[\frac{W_1(u, v)W_1^*(u, v) + \epsilon}{W_2(u, v)W_2^*(u, v) + \epsilon}\right].$$  

(20)

A forth metric following from separable least squares is equal to

$$M_4(u, v) = \frac{Y_1(u, v)W_2(u, v) - Y_2(u, v)W_1(u, v)}{W_1(u, v)W_1^*(u, v) + W_2(u, v)W_2^*(u, v) + \epsilon},$$  

(21)

where $\epsilon \ll 1$ is again a regularization parameter.

### 4. LINEAR APPROXIMATION AND WAVEFRONT ESTIMATION

The metrics defined in the previous section will be used to estimate the wavefront. First, the metrics will be linearized and the estimation will be done by inversion of this linearization. The metrics $M_1$ to $M_3$ can be linearized using a first order Taylor expansion around $\alpha_0$

$$M_{i,\text{mod}}(u, v) = M_{i,\text{mod}}(u, v)|_{\alpha_0} + \frac{\partial M_{i,\text{mod}}(u, v)}{\partial \alpha}|_{\alpha_0}(\alpha_0 - \alpha) + O(\alpha^2),$$  

(22)

where $\alpha = [\alpha_1 \ldots \alpha_l]^T \in \mathbb{R}^{l \times 1}$, $\frac{\partial M_{i,\text{mod}}(u, v)}{\partial \alpha} = \begin{bmatrix} \frac{\partial M_{1,\text{mod}}(u, v)}{\partial \alpha_1} & \ldots & \frac{\partial M_{l,\text{mod}}(u, v)}{\partial \alpha_l} \end{bmatrix} \in \mathbb{C}^{1 \times l}$, with $l$ the number of Zernike polynomials used. Omitting the second order error ($O(\alpha^2)$), a linear approximated model is obtained. Now, the
error between the measurement and the model can be solved analytically for different kinds of cost functions. Here, the two norm is chosen

\[ J_i(\alpha) = \sum_{j=1}^{m} \sum_{k=1}^{n} \left| M_{i, \text{meas}}(u_j, v_k) - M_{i, \text{mod}}(u_j, v_k) \right|_{\alpha_0} - \frac{\partial M_{i, \text{mod}}(u, v)}{\partial \alpha} \bigg|_{\alpha_0} (\alpha - \alpha_0)^2, \]  

(23)

and similarly for \( M_4 \)

\[ J_4(\alpha) = \sum_{j=1}^{m} \sum_{k=1}^{n} \left| M_4(u_j, v_k) \right|_{\alpha_0} - \frac{\partial M_4(u, v)}{\partial \alpha} \bigg|_{\alpha_0} (\alpha - \alpha_0)^2. \]  

(24)

These systems of equations are complex and therefore are solved by separating the complex and real equations and can be rewritten as

\[ J_i(\alpha) = \| y_i - X_i \alpha \|^2, \]  

(25)

\[ = \left\| \begin{bmatrix} \text{Im}(y_i) \\ \text{Re}(y_i) \end{bmatrix} - \begin{bmatrix} \text{Im}(X_i) \\ \text{Re}(X_i) \end{bmatrix} \alpha \right\|^2, \]  

(26)

where

\[ y_i \equiv \begin{bmatrix} M_{i, \text{meas}}(u_1, v_1) - M_{i, \text{mod}}(u_1, v_1) \bigg|_{\alpha_0} + \frac{\partial M_{i, \text{mod}}(u_1, v_1)}{\partial \alpha} \bigg|_{\alpha_0} (\alpha - \alpha_0) \\ \vdots \\ M_{i, \text{meas}}(u_m, v_n) - M_{i, \text{mod}}(u_m, v_n) \bigg|_{\alpha_0} + \frac{\partial M_{i, \text{mod}}(u_m, v_n)}{\partial \alpha} \bigg|_{\alpha_0} (\alpha - \alpha_0) \end{bmatrix}, \]  

(27)

\[ X_i \equiv \begin{bmatrix} \frac{\partial M_{i, \text{mod}}(u_1, v_1)}{\partial \alpha} \bigg|_{\alpha_0} \\ \vdots \\ \frac{\partial M_{i, \text{mod}}(u_m, v_n)}{\partial \alpha} \bigg|_{\alpha_0} \end{bmatrix}, \]  

(28)

\( \text{Re}(\cdot) \) denotes the real part and \( \text{Im}(\cdot) \) denotes the imaginary part. Equations similar to Eq. (27)- (28) can be derived for \( M_4 \). The system of equations is solved by the calculation of the pseudo inverse using the singular value decomposition.

### 5. EXPERIMENTAL STUDY

The accuracy and precision of the estimation methods based on the linearized metrics presented in the previous section have been investigated through executing simulations. The ideal object is chosen as \( \text{Erica} \), see Fig. 2. The unknown aberrations are caused by imperfections in the optical system, such as turbulence, and the true and unknown phase aberration \( \alpha_i \) is a linear combination of the first 6 Zernike polynomials neglecting piston, tip and tilt. To investigate the overall performance of the estimation method, the Zernike coefficients \( \alpha_i \), are drawn from \( U(-1, 1) \), which is a uniform distribution between \([-1, 1]\):

\[ \alpha_i \sim U(-1, 1), \quad i = 4, \ldots, 6, \]  

(29)

(30)

The magnitude squared of the vector \( \alpha_i \) is normalized to correspond to the variance of the initial wave front \( \sigma_w^2 \)

\[ \alpha_i = \frac{\alpha_i}{c}, \quad i = 4, \ldots, 6, \]  

(31)

(32)

where

\[ c = \sqrt{\frac{\sum_{i=4}^{6} \alpha_i^2}{\sigma_w^2}}, \]  

(33)
such that
\[ \sigma_w^2 = |\alpha_t|^2. \tag{34} \]

The expansion point of the linear approximation \( \alpha_0 \), which could be a static aberration following from design compromises or an initial estimate aberration originating from underlying temporal dynamics. Here, the known average aberration \( \alpha_0 \) is chosen to be equal to \( \sqrt{-\ln(0.8)} \approx 0.22 \text{ [rad]} \), which corresponds to diffraction limited Strehl ratio of 0.8. The simulated images are obtained by the convolution of the PSF and the ideal object, which is computed using FFTs. The simulated images are monochromatic and are sampled at the Nyquist rate and the obtained images are corrupted by additive Gaussian noise with variance \( \sigma^2 \). The SNR of the images is defined by Eq. (11).

<table>
<thead>
<tr>
<th>Metric</th>
<th>Defocus [rad]</th>
<th>Coma [rad]</th>
<th>Astigmatism [rad]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>4.5\pi</td>
<td>3\pi</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>-2.5\pi</td>
<td>-2\pi</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>-2.5\pi</td>
<td>1\pi</td>
</tr>
<tr>
<td>4</td>
<td>2\pi</td>
<td>-1\pi</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 1: Choice of phase diversity

### 5.1 Choice of Phase Diversity

A phase diversity has to be chosen that is inserted into one of the optical paths, to investigate the accuracy and precision of the estimation method based on the previously introduced metrics, as shown in Fig. 1. The optimal applied phase diversity is defined as the phase diversity that maximizes the precision of the estimation. This subject has been studied in the context of the so called Cramér-Rao Lower Bound (CRLB) and it is noted that an optimal applied phase diversity seems to exist. However, this diversity depends on the actual object and aberration present.\(^{10-12}\)

In the previous sections measures were defined that eliminate the object and the aberration is assumed to be close to the expansion point \( \alpha_0 \). However, since the noise processes underlying these metrics are unknown, the condition number is considered instead of the CRLB. Therefore, the phase diversity is chosen by minimizing the condition number of

\[
\begin{bmatrix}
\text{Im}(X_i) \\
\text{Re}(X_i)
\end{bmatrix},
\tag{35}
\]

which is the condition number of the least squares problem in Eq. (26). An appropriate optimization method for this problem is the interval analysis method,\(^{13}\) which assures to converge to a global minimum and can be used to execute the optimization unsupervised.\(^{7}\)

The optimization here is done in the absence of noise and the presence of a point source (supervised). Two higher order Zernike modes are selected from defocus, coma and astigmatism, that minimize the conditioning over the phase diversity magnitude range \([\frac{3}{4}\pi, 5\pi]\). The results for the different metrics are presented in Table 1.

![Figure 2: Object Erica.](image)

\(^{7}\)It is important to note that piston will never become measurable when introducing a phase diversity, which can be explained by investigating the influence of piston to the PSF.
5.2 Simulation Results

Simulations of the accuracy and precision of the estimation method based on the previously introduced metrics have been performed. The goal is to increase the initial Strehl ratio by reducing the variance of the wavefront ($\sigma_w^2$), such that the Strehl ratio is as close as possible to 1.\(^4\) The Strehl ratio is defined as

$$S(\sigma_w) = e^{-\sigma_w^2}$$

(36)

Note that it cannot be expected that the estimation will be done with the same accuracy and precision for varying Strehl ratios due to the region of validity of the approximation. The accuracy is here estimated by the sample mean of the bias

$$\mu_a = \frac{1}{N} \sum_{k=1}^{N} |\hat{\alpha}_{tk} - \alpha_{tk}|$$

(37)

and precision is estimated by the sample variance

$$\sigma_p^2 = \frac{1}{N-1} \sum_{k=1}^{N} (|\hat{\alpha}_{tk} - \alpha_{tk}| - \mu_a)^2$$

(38)

where $N = 64$ and the index $k$ denotes a sample vector (not an element). These estimations are transformed into a Strehl ratio ($S(\mu_a), 1 - S(\sigma_p)$) to make the visualization more insightful, see Fig. 3-4.

Furthermore, it is expected that as the variance increases, the Strehl ratio of the compensated wavefront will decrease. Therefore, the transformed accuracy $S(\mu_a)$ is compared against the variance of the initial wavefront $S(\{\alpha_{t}\})$. To visualize the Strehl ratio corresponding to the variance of the initial wavefront a dashed line is plotted $y = S(\{|\alpha_{t}|\})$, which is called the Compensation Threshold.

The performance of the estimation methods based on $M_1$ and $M_2$ with varying signal to noise ratios are plotted in Fig. 3 and for $M_3$ and $M_4$ in Fig. 4. The graph considering $M_1$ shows a variance over the whole domain of the Strehl ratio, therefore no usable estimate can be obtained using this metric based on two intensity measurements.

For $M_2$, a decrease in variance is observed as compared to $M_1$. A bias was witnessed at a 0 initial wavefront variance ($\sigma_w^2 = 0$), which is caused by the presence of an object and measurement noise. Furthermore, the variance of the wavefront does not decrease when the estimate is used for disturbance reduction, since the lines not significantly above the compensation threshold.

The behavior of $M_3$ is presented in Fig. 4, which shows again a decrease in variance and no bias at $\sigma_w^2 = 0$. However, since the variance of the wavefront is not reduced, the estimate cannot be used for disturbance reduction since the lines are approximately on the compensation threshold.

$M_4$, which is based on separable least squares, provides a significant improvement. The variance and bias are dependent on the signal to noise level. At least for a signal to noise level between 0 and 100 and an initial wavefront variance lower than $3\pi \ [rad^2]$, the estimate can be used to reduce the rms value of the initial wavefront.

6. CONCLUSIONS AND DISCUSSION

The heuristics based $M_1$ to $M_3$ do not decrease the variance of the initial wavefront. However, $M_4$ based on the separable least squares, does significantly decrease the variance of the wavefront. In practice more than 6 Zernike modes are desired, yet in this paper only the first 6 Zernike modes are used to prove the principle. The expansion point of $\alpha_0$ is the initial estimate of the aberration, which could be the static or average aberration of the optical system.

Since the computations only consist of vector matrix multiplications, a parallel GPU implementation using CUDA is possible.\(^{15}\) Therefore, computational speed of the proposed linear methods is dependent on the computer architecture of the system. It is difficult to define an object independent metric for the retrieval of aberrations, since the performance of the metrics investigated is influenced by the presence of an object, as visible in Fig. 3b.

Investigation of Eq. (24)-(26) shows that both $y_t$ and $X_t$ are corrupted by measurement noise and therefore should be solved by a Total Least Squares approach.\(^{16}\) Due to the underlying form of the covariance matrix the solution can only be obtained using a nonlinear optimization method, which is not practical. Therefore, row-wise and column-wise approximations\(^{17}\) to the covariance matrix were investigated but these methods do not result in an accuracy improvement.
Figure 3: The simulation results of $M_1$ and $M_2$ for different signal to noise levels. The Strehl ratio of the compensated wavefront ($[-]$) is plotted versus the variance of the initial wavefront ($[\text{rad}^2]$).

Figure 4: The simulation results of $M_3$ and $M_4$ for different signal to noise levels. The Strehl ratio of the compensated wavefront ($[-]$) is plotted versus the variance of the initial wavefront ($[\text{rad}^2]$).
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