DISC Course on Modeling and Control of Hybrid Systems 2017

Homework for Chapters 1 and 2

1. Select a system in your own field of research or interests that can be considered as a hybrid system (and that is not yet discussed in the lecture notes). Give a short description of the system, and describe the system (or a part of it) as a hybrid automaton.

2. The definition of hybrid automaton considered in the lecture notes is in fact that of an autonomous hybrid automaton, i.e., without inputs. Indicate how this definition can be extended to allow for inputs too. Be as general as possible.
   What restrictions or sets of restrictions should be imposed to make a hybrid automaton with inputs deterministic? Explain and motivate your answer.

3. Consider a cylindrical tank that can be filled with water (see Figure 1). The tank has an outlet valve V at the bottom (causing water to leave the tank) and at the top there is a feeding tap T (which adds water to the tank). The outlet valve V can be in two states: completely open or completely closed. We assume that the time needed for completely opening or closing the valve is negligible. The feeding tap T provides a variable inflow of water into the tank. The tank is open at the top, which implies that if the water level would exceed the height of the tank, the tank will overflow.
   Assume that the water level of the tank at time t is denoted by $h(t)$ (in m), and that the diameter of the tank is d (in m). Let $\alpha(t)$ be the inflow rate (in m^3/s) provided by tap T at time t, and let β be the constant[1] outflow rate (in m^3/s) due to valve V if the valve is open. Let H be the height of the tank. Finally, we assume that $0 < h(t_0) < H$.
   The valve V will be operated according to the sequence given in Table 1. During each phase the position of the tap T stays fixed. i.e., $\alpha(t)$ is a piecewise-constant function of the form $\alpha(t) = a(k)$ for $t \in [t_k, t_{k+1}]$.

   <u>Tasks</u>:

   - Derive a continuous-time PWA model that describes the evolution of the fluid level over time during a "outlet valve V open" phase $[t_{2\ell}, t_{2\ell+1}]$ for some integer ℓ.
     Also derive a continuous-time PWA model that describes the evolution of the fluid level over time during a "outlet valve V closed" phase $[t_{2\ell+1}, t_{2\ell+2}]$ for some integer ℓ.
   - Now assume that the phases have equal length T_{phase}. Construct a hybrid automaton that describes the evolution of the fluid level (hint: include the time t as a state variable).
   - Now we do no longer assume that the switching times are equidistant. Let $x(k)$ be the fluid level at the kth mode-switching time instant t_k (i.e., $x(k) = h(t_k)$). Now

[1] In general the outflow rate will also depend on the height of the water level, but for the sake of simplicity we assume it to be constant.

derive a discrete-time MMPS model that describes the evolution of $x(k)$ (Note that k is an event counter and not a sample step counter).

- Rewrite the MMPS model obtained in the previous step as an MLD model.

Upload your answers for this homework as a **pdf** file to to the DISC website.

Deadline: February 6, 2017

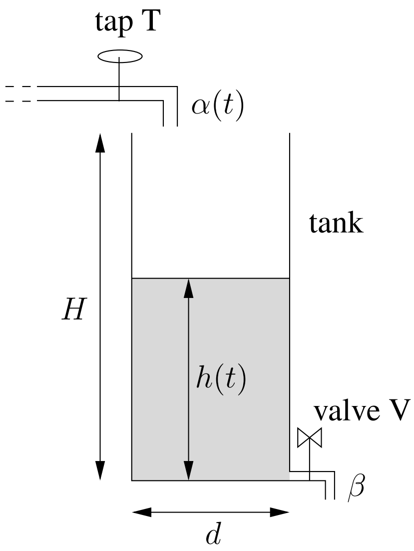

Figure 1: A water tank with an outlet valve and a feeding tap.

Period	State of valve V	Outflow rate for tap T (i.e. $\alpha(t)$)
t_0–t_1	open	$a(0)$
t_1–t_2	closed	$a(1)$
t_2–t_3	open	$a(2)$
t_3–t_4	closed	$a(3)$
⋮	⋮	⋮

Table 1: The operating scheme for the water tank.