Exam Modeling and Control of Hybrid Systems (SC4160)

April 18, 2012

This exam has 5 questions, all of which will be weighted equally in the final mark.

Questions:

1. Give the definition of a general max-min-plus-scaling (MMPS) system. Under which conditions can a general MMPS system be transformed into a piecewise affine (PWA) system? Explain how the transformation can be done (the exact formulas for the equivalences used should not be given, but the nature of the formulas/equivalences should be indicated).

2. Explain the following concepts and give an illustrative example for each of them:
   - timed automaton
   - Petri net
   - bisimulation
   - generalized gradient

3. Discuss the relations and differences between stochasticity and non-determinism in hybrid systems (i.e., briefly explain each of the concepts, discuss why and when they are needed, and how they are related or how they differ).

4. This question involves global uniform asymptotic stability of switched systems \( \dot{x}(t) = f_\sigma(x(t)) \) with switching signal \( \sigma \).
   a) Explain the concept of global uniform asymptotic stability of a switched system.
   b) Suppose there are \( N \) subsystems (i.e., \( \sigma \) always belongs to the set \( \{1, \ldots, N\} \)). Is it correct to claim that the switched system is globally uniformly asymptotically stable if and only if each of the subsystems \( \dot{x}(t) = f_i(x(t)) \) for each fixed \( i \in \{1, \ldots, N\} \) is stable? Explain.
   c) What are the main strengths and weaknesses of a Lyapunov function approach to stability of switched systems?
5. Consider a switched system with dynamics

\[
\dot{x}(t) = \begin{cases} 
  f_-(x(t)) & \text{if } x(t) \in C_- \\
  f_+(x(t)) & \text{if } x(t) \in C_+ 
\end{cases}
\]

Let \( x_0 \) be a point on the boundary of \( C_- \) and \( C_+ \) as indicated in the figures below.

Next indicate how the system will evolve if it starts from the point \( x_0 \) (if there are multiple solutions possible, give all of them) in each of the cases a), b), and c)

1) if Filippov solutions are not allowed
2) if Filippov solutions are allowed

Explain and motivate your answer.