Online take-home exam

Modeling and Control of Hybrid Systems (SC42075)

June 22, 2020

- This exam is an online open-book take-home exam using the Brightspace Assignments function.
- Please recall that the number of questions is larger than what the majority of students will be able to answer within the allocated time span. So there is ***no need at all to worry if you cannot answer all questions***. Just start with the questions that you feel most familiar with and try to answer as many questions correctly as possible.
- The exam consists of 4 questions; the maximal score for each question is marked in red next to the given question.
- Make sure to clearly motivate your answers; so make sure to provide both the final results as well as important intermediate steps and the procedure followed to reach the results. Just listing the final answer without any explanation is not sufficient.
- It is also important to note that similarities in replies among different students will be penalized, and in the worst case reported to the Board of Examiners.
- Matlab and other computation tools can be used for verification purposes only; they cannot be used to replace calculations by hand.
- I wish all of you good luck with the exam!

Questions

1. Give a *new* example (i.e., one that has not been discussed in the lecture notes, the slides, the lectures, the assignment of this and previous years) and that illustrates the following concept/definition and use the example to explain the given concept/definition in your own words:

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   (a) Zeno behavior

   (b) region graph

   (c) globally uniform asymptotic stability for switched systems

   (d) timed Petri net

   (e) reset map

   (f) right accumulation point of an event sequence

2. Let α be the last non-zero digit of your student ID, and let β be the one but last non-zero digit of your student ID[1]. List the values of α and β in your reply, and use their *numerical* values in the questions below.

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   For each of the following systems with a specific property, give a *new* example (i.e., one that has not been discussed in the lecture notes, the slides, the lectures, the assignment of this and previous years) of a system that satisfies the given property, if possible. Motivate your answer.

   (a) a nondeterministic timed automaton

   (b) a nondeterministic unconstrained MMPS system

   (c) a completely well-posed MLD system

   (d) a non-convex function without any affine parts and with a generalized gradient that is equal to $[0, \max(\alpha, \beta)]$

3. Let α be the last non-zero digit of your student ID, and let β be the one but last non-zero digit of your student ID[1]. List the values of α and β in your reply, and use their *numerical* values in the questions below.

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   Consider the following PWA system:

$$x(k+1) = \begin{cases} \alpha x(k) - \beta u(k) & \text{if } x(k) + u(k) \geqslant \beta \\ \beta x(k) + u(k) & \text{if } x(k) + u(k) < \beta \end{cases}$$
$$y(k) = \alpha x(k)$$

   Tasks:

   (a) Transform the given system into an LC system.
   If you need to impose additional constraints to allow you to make the transformation, then you are allowed to do so, provided that you add a *numerical* constraint (possibly with rather arbitrary values; so purely symbolic constraints are not allowed) and that motivate why such a constraint is needed. Also make sure not to include more constraints than what is really needed.
   Motivate your answer, as well as the intermediate steps.

   (b) Is the resulting system 100 % mathematically equivalent to the original PWA system, taking the extra constraints you may have added into account? Why (not)? Motivate your answer.

[1] So if your student ID is 12340090, then $\alpha = 9$ and $\beta = 4$.

4. This question involves nondeterminism of hybrid automata.

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(a) Briefly explain in your own words what nondeterminism means.

(b) Discuss when and how nondeterminism can be useful.

(c) What is according to you the main disadvantage of a nondeterministic model. Explain and motivate your answer.

(d) What conditions should be imposed on a hybrid automaton to make it fully deterministic? Try to keep the number of conditions and their restrictiveness as small as possible.

End of the exam