Online take-home exam

Modeling and Control of Hybrid Systems (SC42075)

July 1, 2021

- This exam is an online open-book take-home exam using the Brightspace Assignments function.
- Please recall that the number of questions is larger than what the majority of students will be able to answer within the allocated time span. So there is ***no need at all to worry if you cannot answer all questions***. Just start with the questions that you feel most familiar with and try to answer as many questions correctly as possible.
- The exam consists of 4 questions; the maximal score for each question is marked in red next to the given question.
- Make sure to clearly motivate your answers; so make sure to provide both the final results as well as important intermediate steps and the procedure followed to reach the results. Just listing the final answer without any explanation is not sufficient.
- It is also important to note that similarities in replies among different students will be penalized, and in the worst case reported to the Board of Examiners.
- Matlab and other computation tools can be used for verification purposes only; they cannot be used to replace calculations by hand.
- I wish all of you good luck with the exam!

Questions

For all the questions below, define

- α as the last **non-zero** digit of your student ID, and
- β as the one but last **non-zero and non-α** digit of your student ID.

So if your student ID is 12490090, then $\alpha = 9$ and $\beta = 4$. Moreover, we always have $\alpha \neq \beta$.

When applicable in the questions below (i.e., when α or β are mentioned), list the values of α and β in your replies, and use their *numerical* values – so do not work with symbolic expressions.

1. Give a *new* example (i.e., one that has not been discussed in the lecture notes, the slides, the lectures, the assignment of this and previous years) that illustrates the following concept/definition and use the example to **explain** the given concept/definition in your own words:

8

   (a) Filippov solution
   (b) nondeterministic (discrete-event) automaton
   (c) generalized gradient
   (d) bisimulation

2. First list the numerical values of α and β as defined above. Verify once more and confirm explicitly whether α and β indeed satisfy all the requirements.

10

   Next, for each of the following systems with a specific property, give a *new* example (i.e., one that has not been discussed in the lecture notes, the slides, the lectures, the assignment of this and previous years) of a system that satisfies the given property. **Motivate** your answer.
   Note that there always exists an example.

   (a) a timed automaton with live-lock behavior, where one of the invariant-set conditions contains 3α
   (b) a nondeterministic PWA system, where one of the coefficients inside the model equations is equal to β
   (c) a completely well-posed MLD system, where one of the coefficients inside the model equations is equal to $2|\alpha - \beta|$
   (d) a non-convex function without any affine parts and with a generalized gradient that is equal to $[-1, \alpha + \beta]$
   (e) a deterministic switched system with 3 modes that does not require Filippov solutions and that has a state vector with 2 components
   Hint: represent the system by the vector field of its dynamics

3. First list the numerical values of α and β as defined above. Verify once more and confirm explicitly whether α and β indeed satisfy all the requirements.

7

   Next, consider the following PWA system:

$$x(k+1) = \begin{cases} x(k) + 2\beta u(k) & \text{if } x(k) - \alpha u(k) \geqslant \beta \\ 2x(k) + (-\alpha + 2\beta)u(k) - \beta & \text{if } x(k) - \beta u(k) \leqslant \alpha \end{cases}$$
$$y(k) = 2\alpha x(k) + \beta$$

   with $x(k) \in [-50, 75]$ and $u(k) \in [-60, 60]$.

   (a) Is this PWA system well-posed? Motivate your answer.

../..

(b) Transform the given system into an LC system. Motivate your answer, as well as the intermediate steps.

(c) Is the resulting system 100 % mathematically equivalent to the original PWA system? Why (not)? Motivate your answer.

4. (a) What are the main advantages and disadvantages of a timed automaton model?

6 (b) Under which conditions can a PWA model be transformed into a *mathematically* fully equivalent MMPS model? Try to keep the number of conditions and their restrictiveness as small as possible. Motivate your answer.

(c) What are the main strengths and weaknesses of a Lyapunov function approach to stability of switched systems?

End of the exam