Non-Linear Optimization Assignment

SC42055 Optimization in Systems and Control

E_1, E_2 and E_3 are parameters changing from 0 to 9 for each group according to the mean of the last three numbers of their Student IDs:

$E_1 = \frac{D_{a,1}+D_{b,1}}{2}$, $E_2 = \frac{D_{a,2}+D_{b,2}}{2}$, $E_3 = \frac{D_{a,3}+D_{b,3}}{2}$,

where $D_{a,3}$ is the right-most digit of one student and $D_{b,3}$ is the right-most digit of the other student.

Traffic congestion on freeways is a critical problem due to its negative impact on the environment and many other important consequences (higher delays, waste of fuel, a higher accident risk probability, etc.). Since the construction of new freeways is not always a viable option or is too costly, other solutions have to be found. In many cases, the use of dynamic control signals such as ramp metering, variable speed limits, reversible lanes, and route guidance may be a cost-efficient and effective solution. In general, dynamic traffic control uses measurements of the traffic conditions over time and computes dynamic control signals to influence the behavior of the drivers and to generate a response in such a way that the performance of the network is improved, by reducing delays, emissions, fuel consumption etc.

Figure 1: Ramp Metering on the A13 in Delft

The most used control input in freeway traffic control is ramp metering, a device (usually a basic traffic light) located at an on-ramp that regulates the flow of traffic entering the freeway according to current traffic conditions. Ramp metering systems have proved to be successful in decreasing traffic congestion and improving driver safety.

The goal of this assignment is to compute the values of the ramp metering rates (i.e. the percentage of ramp flow that is allowed to enter) that minimize the congestion that appears on a freeway stretch.

Let us assume that we want to model and control the behavior of the freeway stretch shown in Figure 2. The network considered has 4 segments with a length of 1 kilometer (each one) and 3 lanes. A ramp metering is located on the on-ramp on segment 4.

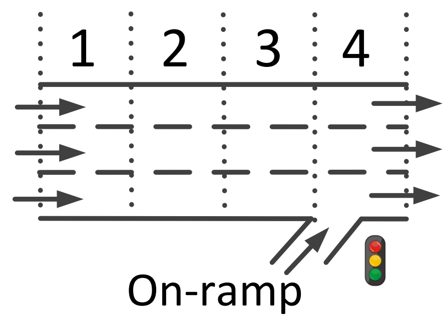

Figure 2: Schematic representation of the considered freeway stretch

The macroscopic model METANET will be used to model the behavior of the freeway. METANET represents the traffic network as a graph where the segments are indexed by index i and have a length of L meters with λ lanes. The freeway is dynamically characterized by the traffic density of each segment ($\rho_i(k)$), the mean speed of each segment ($v_i(k)$) and the queue of vehicles waiting on the on-ramp ($w_\mathrm{r}(k)$). The index k is the time step corresponding to instant $t = kT$, where T is the simulation time step (in this case, $T = 10$ s). The traffic density is defined as the number of vehicles per unit length of the roadway and is expressed in the units vehicles/(km lane). The mean speed and the queue are expressed in km/h and number of vehicles, respectively.
Two main equations describe the system dynamics of METANET model. The first one expresses the conservation of vehicles:

$$\rho_i(k+1) = \rho_i(k) + \frac{T}{\lambda_i L_i}(q_{i-1}(k) - q_i(k) + q_{\mathrm{r},i}(k)) \tag{1}$$

where $q_{\mathrm{r},i}(k)$ is the traffic flow that enters the freeway link i from the connected on-ramp (if any). The traffic flow in each link $q_i(k)$ can be computed for each time step using $q_i(k) = \lambda\rho_i(k)v_i(k)$.
The second equation expresses the mean speed as a sum of the previous mean speed, a relaxation term, a convection term, and an anticipation term:

$$v_i(k+1) = v_i(k) + \frac{T}{\tau}(V(k) - v_i(k)) + \frac{T}{L}v_i(k)(v_{i-1}(k) - v_i(k)) - \frac{\mu T}{\tau L}\frac{\rho_{i+1}(k) - \rho_i(k)}{\rho_i(k) + K} \tag{2}$$

where τ, μ, and K are model parameters that are assumed to be constant for all segments, and $V(k)$ is the desired speed for the drivers that is modeled by the following equation:

$$V(k) = v_\mathrm{f} e^{-\frac{1}{a}\left(\frac{\rho_i(k)}{\rho_\mathrm{c}}\right)^a} \tag{3}$$

where a is a model parameter, v_f is the free flow speed that the cars reach in steady state, and ρ_c is the critical density (i.e. the density corresponding to the maximum flow).

In order to complete the model, the following equation defines the flow that enters from the controlled on-ramp on segment 4:

$$q_\mathrm{r}(k) = \min\left(r(k)C_\mathrm{r}, D_\mathrm{r}(k) + \frac{w_\mathrm{r}(k)}{T}, C_\mathrm{r}\frac{\rho_\mathrm{m} - \rho_4(k)}{\rho_\mathrm{m} - \rho_\mathrm{c}}\right) \tag{4}$$

where $r(k)$ is the ramp metering rate applied at time step k, ρ_m and C_r are model parameters, $D_\mathrm{r}(k)$ is the demand of the on-ramp, and $w_\mathrm{r}(k)$ is the queue length on a ramp on segment 4, the dynamics of which are defined by:

$$w_\mathrm{r}(k+1) = w_\mathrm{r}(k) + T(D_\mathrm{r}(k) - q_\mathrm{r}(k)) \tag{5}$$

It has to be noted that the ramp metering rate $r(k)$ is defined between 0 and 1 since it corresponds to the percentage of ramp flow that is allowed to enter and that the initial ramp queue is equal to 0 ($w_\mathrm{r}(1) = 0$).
Finally, some boundary conditions are defined:

- The downstream density of the final segment is considered to be equal to the density of the last segment: $\rho_5(k) = \rho_4(k)$.
- The speed upstream the first segment is considered to be equal to the speed of the first segment: $v_0(k) = v_1(k)$.
- The flow entering the first segment ($q_0(k)$) and the on-ramp demand ($D_\mathrm{r}(k)$) are inputs of the system.

The Total Time Spent (TTS) by the drivers during each interval $[kT, (k+1)T]$ is taken as the output of the system:

$$y(k) = T\sum_{i\in \mathrm{O}} w_i(k) + TL\lambda \sum_{i\in I} \rho_i(k) \tag{6}$$

where O is the set of all the segments with an on-ramp and I is the set of all the segments. For the freeway considered, the length of each segment (L) is one kilometer, the simulation time step (T) is 10 s and the METANET model parameters have the values shown in the following table:

Table 1: METANET parameters

τ	μ	C_r	ρ_m
10 s	80 $\frac{\mathrm{km}^2}{\mathrm{h}}$	2000 $\frac{\mathrm{veh}}{\mathrm{h}}$	120 $\frac{\mathrm{veh}}{\mathrm{km\ lane}}$
K	a	v_f	ρ_c
10	2	110 $\frac{\mathrm{km}}{\mathrm{h}}$	28 $\frac{\mathrm{veh}}{\mathrm{km\ lane}}$

Moreover, assume that, at $k = 0$, the densities of all the segments are equal to 20 veh/km lane and the speeds are equal to 90 km/h, that the ramp demand is constant and equal to $D_r(k) = 1500$ veh/h and that the flow entering the mainline is defined as follow:

$$q_0(k) = \begin{cases} 7000 + 100E_1 \text{ veh/h} & \text{if } k < 12 \\ 2000 + 100E_2 \text{ veh/h} & \text{if } k \geq 12 \end{cases}$$

Tasks:

1. Formulate the discrete-time state-space model that predicts the density and the speed of each segment, and the number of vehicles in the on-ramp queue for the following simulation time step $k+1$:

$$\begin{cases} x(k+1) = f(x(k), u(k)) \\ y(k) = g(x(k), u(k)) \end{cases} \tag{7}$$

Note: The state x(k) includes 9 variables: 4 densities (one for each segment), 4 speeds (one for each segment), and the on-ramp queue $w_{\mathrm{r}}(k)$. They output $y(k)$ includes one variable: The TTS for time step k. All the parameters have to be expressed with the same units: h, km, veh...

2. Formulate the optimization problem in order to find the values of the ramp metering rates that minimize the Total Time Spent (TTS) by the drivers in the network for the following 10 minutes ($[kT, (k+60)T]$).

3. Select an appropriate optimization algorithm (explaining your choice) and write the corresponding MATLAB code.

4. Run the optimization algorithm using two different starting points: $r(k) = 0.99\ \forall\ k$ and $r(k) = 0\ \forall\ k$. Is there a substantial difference between the obtained solutions? Why?

5. Can you prove that the solution obtained is the global optimum? How can the solution be improved (if possible)?

6. Find the ramp metering rates $r(k)$ that minimize the Total Time Spent (TTS) by the drivers in the freeway for the following 10 minutes ($[kT, (k+60)T]$) while limiting the maximum number of vehicles waiting on the queue to $20 - E_3$ vehicles.

7. Plot the queues and the ramp metering rates obtained for each case and compare them with the ones obtained for the no control case ($r(k) = 1\ \forall\ k$) . Moreover, find the TTS of the different cases and compare them. Explain the results obtained .

The written report on the practical exercise, including the MATLAB code used, should be emailed to José Ramón Domínguez Frejo (j.r.dominguezfrejo@tudelft.nl) before Monday, October 30, 2017 at 17.00 p.m. as one pdf file. Please note that you will lose 0.5 point from your grade on the report for each (started) day of delay in case you exceed the deadline.