Quadratic Programming Assignment

SC42055 Optimization in Systems and Control

D_1, D_2 and D_3 are parameters changing from 0 to 9 for each student according to the last three numbers of his/her Student ID (with D_3 being the right-most digit).

A university wants to create a model to predict the number of students enrolled at the university during an academic year k. The students are divided in three groups according to the level of education: B.Sc. students ($x_1(k)$), M.Sc. students ($x_2(k)$) and Ph.D. students ($x_3(k)$).

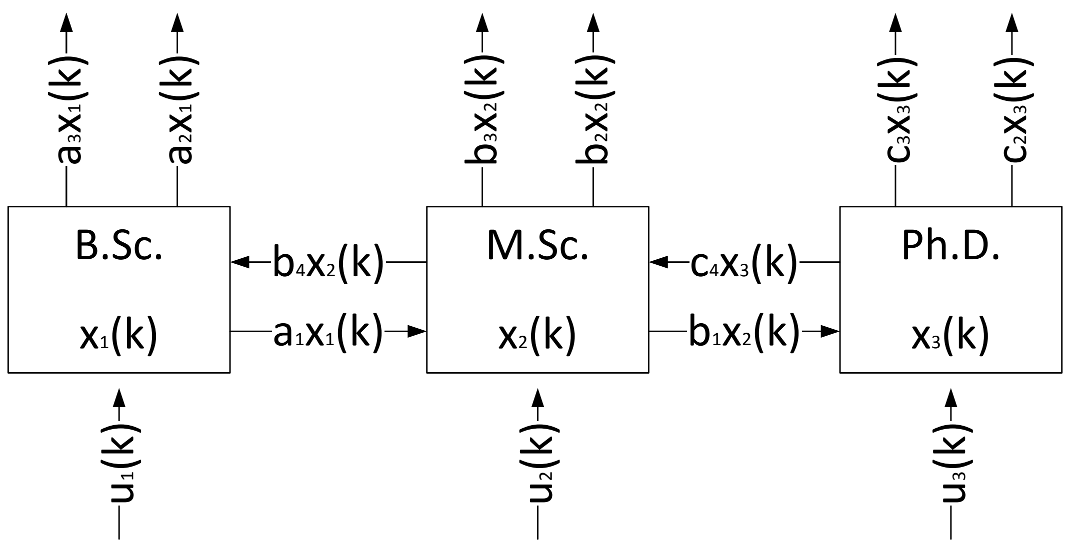

Figure 1: Schematic representation of the considered model

For year $k+1$, the number of students enrolled in the B.Sc. program is increased/decreased by:

- $u_1(k)$: The number of new students, coming from other institutions, starting their college education in year $k+1$.
- A fraction b_4 of the M.Sc. students of year k starting a new B.Sc. degree on year $k+1$.
- A fraction a_1 of the B.Sc. students graduating in year k and starting an M.Sc. degree in year $k+1$.
- A fraction a_2 of the B.Sc. students graduating in year k and going to the labor market.
- A fraction a_3 of the B.Sc. students quitting without finishing their studies.

Similarly, the number of the students in the M.Sc. program is increased/decreased by:

- $u_2(k)$: The number of students, coming from other universities, starting an M.Sc. program in year $k + 1$.
- A fraction c_4 of the Ph.D. students of year k starting a new M.Sc. degree in year $k + 1$.
- A fraction b_1 of the M.Sc. students graduating in year k and starting a Ph.D. degree in year $k + 1$.
- A fraction b_2 of the M.Sc. students graduating in year k and going to the labor market.
- A fraction b_3 of the M.Sc. students quitting without finishing their studies.
- A fraction b_4 of the M.Sc. students graduating and starting a new B.Sc. degree in year $k + 1$.

Finally, the number of the Ph.D. students is increased/decreased by:

- $u_3(k)$ students, coming from other universities, starting a Ph.D. program in year $k + 1$.
- A fraction c_2 of the students finishing their Ph.D. in year k and going to the labor market.
- A fraction c_3 of the Ph.D. students quitting without finishing their studies.
- A fraction c_4 of the Ph.D. student finishing their Ph.D. in year k and starting a new M.Sc. degree in year $k + 1$.

Parameters a_3, b_3 and c_3 are assumed to be known: $a_3 = 0.05 + D_1/200$, $b_3 = 0.05 - D_2/200$ and $c_3 = 0.1 + D_3/200$.

1. Formulate the discrete-time linear state-space model of the system that predicts the number of students enrolled at each level for the following year $k+1$ based on the data of year k :

   $$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ y(k) = Cx(k) + Du(k) \end{cases}$$

   The model considers three outputs ($y_1(k)$, $y_2(k)$ and $y_3(k)$) including the number of students graduated every year at each level (i.e. the students that start a new degree plus the students going to the labor market after graduation for each educational level):

   $$\begin{cases} y_1(k) = a_1x_1(k) + a_2x_1(k) \\ y_2(k) = b_1x_2(k) + b_2x_2(k) + b_4x_2(k) \\ y_3(k) = c_2x_3(k) + c_4x_3(k) \end{cases}$$

2. The university keeps a record of the number of students enrolled, the number of new students coming from other universities and the number of students graduated for the last 10 years. This data can be seen on Table 1. Based on the data, and using QP and MATLAB, identify the optimal value of the parameters of the model ($a_1, a_2, b_1, b_2, b_4, c_2$, and c_4).

Table 1: Record of the number of current, new and graduated students per level and year

Year	B.Sc.			M.Sc.			Ph.D.		
	Current	New	Graduated	Current	New	Graduated	Current	New	Graduated
2009	2800	1273	840	900	100	432	220	42	24
2010	2971	1142	891	776	101	372	257	41	28
2011	2940	1106	882	729	102	350	275	39	30
2012	2885	1110	866	704	100	338	283	38	31
2013	2855	1125	857	686	94	329	287	38	32
2014	2852	1138	856	668	91	321	289	41	32
2015	2863	1156	859	656	101	315	292	41	32
2016	2887	1200	866	661	101	317	294	40	32
2017	2945	1125	884	665	100	319	294	42	32
2018	2905	-	872	671	-	322	297	-	33

3. Simulate the identified model using, for the inputs, the actual numbers of new students shown in Table I for every year and the initial point given by the data of 2009. Analyze the obtained predictions and compare them with the actual number of students observed every year.

4. Assuming that the university will be able to fix the number of students entering the university at each level ($u_1(k)$, $u_2(k)$, $u_3(k)$, $u_1(k+1)$, $u_2(k+1)$, and $u_3(k+1)$) during the next two years (2018 and 2019), formulate the QP problem that minimizes the number of new students entering the university while ensuring that there are, at least, 2500 B.Sc. students, 600 M.Sc. students and 200 Ph.D. students enrolled in the university during 2019 and 2020. Solve this problem using MATLAB and comment on the obtained solution.

The written report on the practical exercise, including the MATLAB code used, should be emailed to José Ramón Domínguez Frejo (j.r.dominguezfrejo@tudelft.nl) before Monday, October 9, 2017 at 17.00 p.m. as one pdf file. Please note that you will lose 0.5 point from your grade on the report for each (started) day of delay in case you exceed the deadline.