Numerical computation of the gradient

2018-09-25 ①

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{bmatrix}$$

→ analytical gradient

→ numerical gradient

$$\frac{\partial f}{\partial x_i}(\hat{x}) \approx \frac{f\left(\hat{x} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} \delta\right) - f(\hat{x})}{\delta} \qquad \delta \text{ small}$$

($\leftarrow$ ith component)

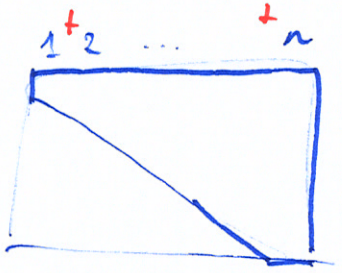
$$= \lim_{\delta \to 0} (\quad " \quad)$$

" $\frac{n(n+1)}{2}$

n function evaluations for ∇f

$O(n^2)$ " " for H

$O(n^2) = \frac{n(n+1)}{2}$



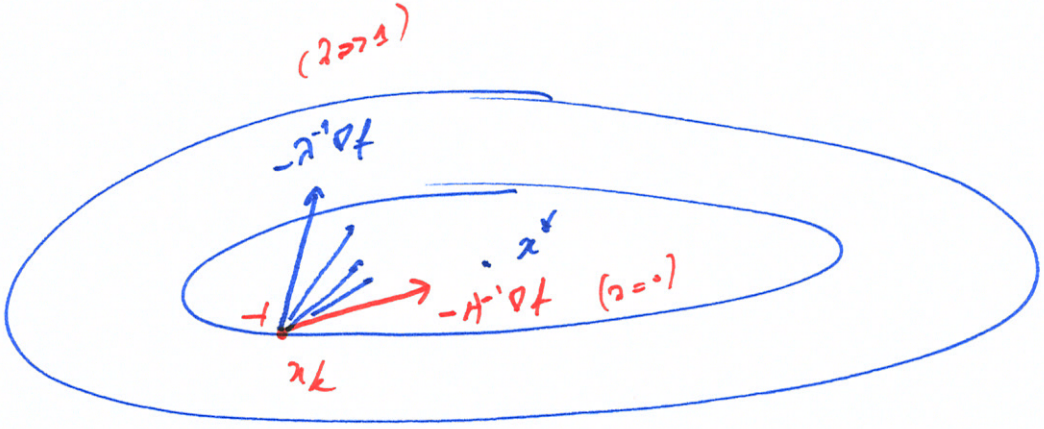
Levenberg - Marquardt

$\hat{H} \rightarrow \lambda I + H$

$$x_{opt} = x_0 - H^{-1} \cdot \nabla f$$

$\lambda = 0$	$\rightarrow$	$-H^{-1} \cdot \nabla f$	$\rightarrow$ Newton method.
$\lambda \gg 1$	$\rightarrow$	$-\lambda^{-1} \nabla f$	$\rightarrow$ steepest descent.

$-\nabla f$: steepest descent

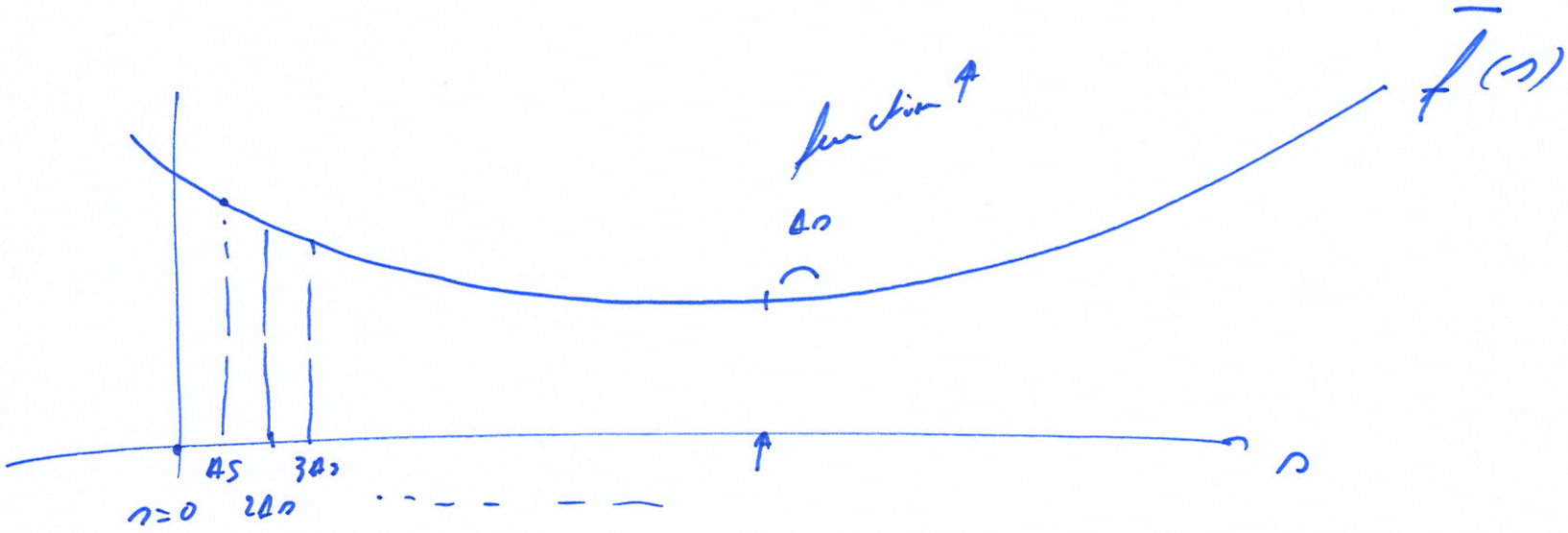


Least squares

$$e = \begin{bmatrix} e_1 \\ \vdots \\ e_N \end{bmatrix}$$

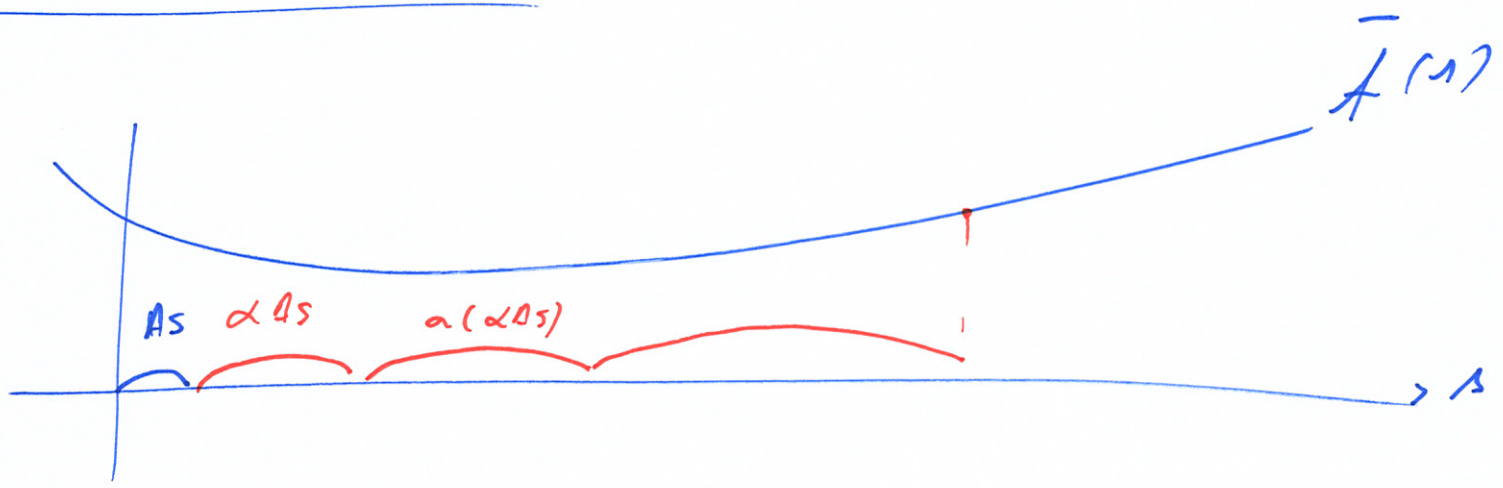
$$J = \sum_{i=1}^{N} e_i^2 = e^T \cdot e$$

Fixed step size

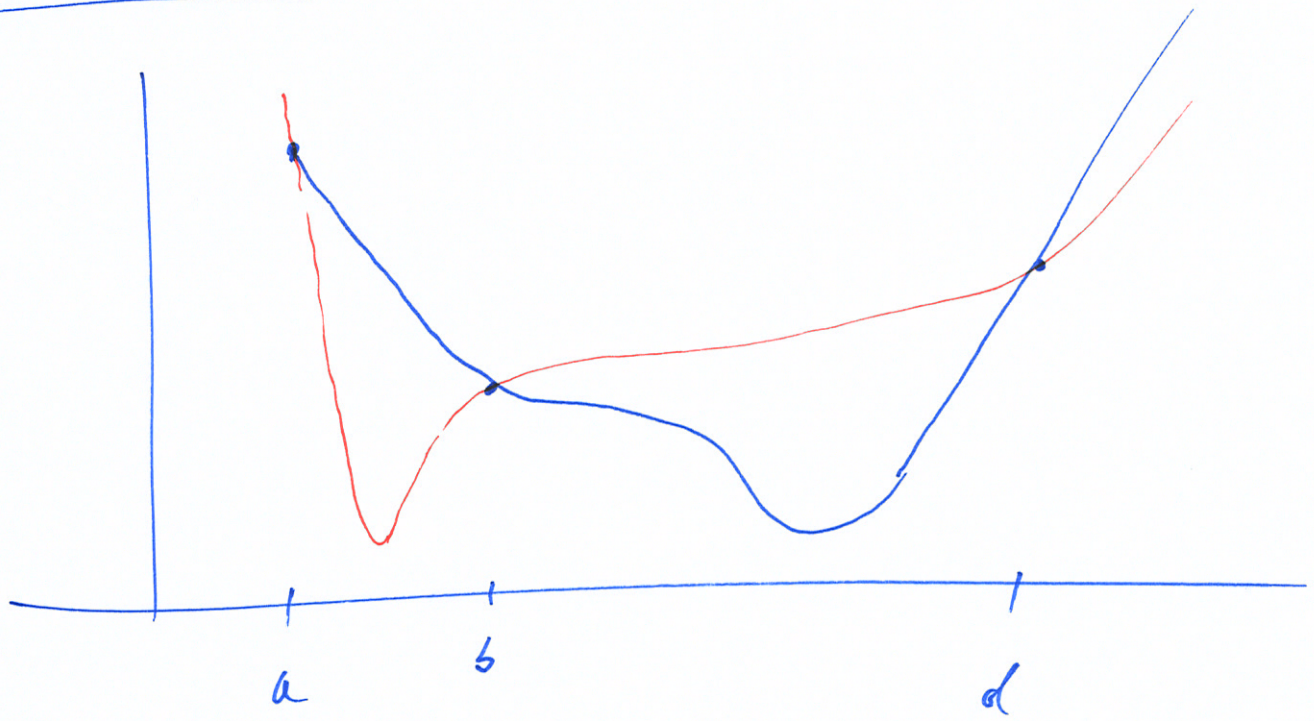


$$s_{l+1} = s_l + \Delta s = l \cdot \Delta s$$

Variable step size

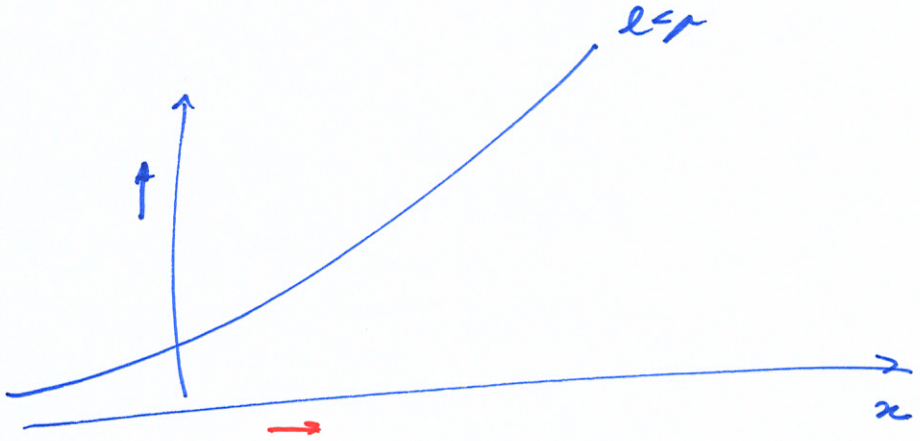


Golden section



1 extra point
↓
not enough
to reduce
interval.
⇓
we need 2
points

Test



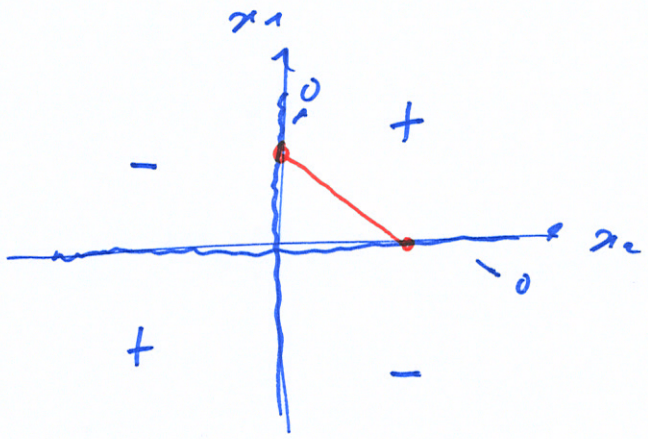
$\max \quad x_1 - x_2$

$x \in \mathbb{R}^3$

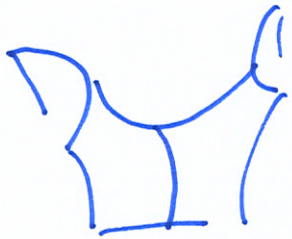
$\angle P!$

$$-1 \leq 2x_1 + 3x_2 - 5x_3 \leq 1$$

x_1 x_2



→ not convex!



a saddle function
see slide 29/33
of Introduction lecture

$\Rightarrow$ min $x_1 x_2 + x_2 x_3$ is not a convex problem.
$x \in \mathbb{R}$

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min $x_1^2 + 2x_1 x_2 + 3x_2^2 + 4x_3^2$
$x \in \mathbb{R}^3$

s.t. $x_1^2 + x_2^2 + x_3^2 \leq 1$

$(x_1 + x_2)^2 + 2x_2^2 + 4x_3^2$
≥ 0 ≥ 0 ≥ 0

sum of squares

→ NLP

- positive definite
- convex.

$$(x_1 + x_2)^2 + 2x^2 + 4x_3^2$$ → convex. quadratische

mit

$$-5 \leq x_1 + 4x_2 - 8x_3 \leq 20$$ → linear

↓

QP