Short solutions for Exam of November 2016 “Optimization in Systems and Control” (SC42055)

This document concisely lists the solutions for the exam of November 2016. Note that other solutions might also be correct to some degree, and that **you should extensively motivate your answers in the actual exam!** (cf. the worked solutions for Sample Exams 1 and 2 and for the exams of October 2013 and October 2014).

Short answers for Question 1

P1. MILP (8 times — not that the non-convex constraint $1 \leq 0.7|x_1| + 0.2|x_2| + 0.5|x_3|$ can be split in the union of $2^3 = 8$ linear constraints)
M12: Branch-and-bound method for mixed-integer linear programming

P2. NCC: Nonconvex constrained (nonconvex due to last constraint)
multi-start M10: Sequential quadratic programming

P3. NCC: Nonconvex constrained (nonconvex due to, e.g., the term x_4x_5)
multi-start M10: Sequential quadratic programming

P4. QP
M3: Ellipsoid algorithm (as there is no dedicated QP algorithm in the list, except maybe a single iteration of M10), or M10: Sequential quadratic programming (single iteration only)

P5. NCU: Nonconvex unconstrained (after elimination of x_1)
multi-start M5: Levenberg-Marquardt

P6. NCC: Nonconvex constrained (nonconvex as x belongs to a discrete set)
multi-start M11: Simulated annealing

P7. NCU: Nonconvex unconstrained
multi-start M11: Simulated annealing (gradient and Hessian not analytically computable, and objective function requires time-consuming numerical evaluation)

P8. CP: Convex
M3: Ellipsoid algorithm

P9. NCC: Nonconvex constrained
multi-start M8: Lagrange method + DFP quasi-Newton algorithm

P10. CP: Convex
M3: Ellipsoid algorithm

The answers below are short answers only; **you should extensively motivate your answers in the actual exam!** (cf. the worked solutions for Sample Exams 1 and 2 and for the exams of October 2013 and October 2014).

Short answers for Question 2

1. See lecture notes (explain tree search; branching, i.e., splitting into LP subproblems; bounding, i.e., different ways in which subtrees can be marked as not requiring further exploration; stopping criterion)
   Assume that the branch-and-bound algorithm is not ended prematurely (e.g., due to maximum number of iterations being reached, maximum CPU time being reached, or maximum memory being used). Then it will always yield a globally optimal solution when no more subtrees should be explored. If the feasible set is bounded, then clearly the algorithm will terminate in a finite amount of time. This also holds if the feasible set is unbouded and the optimal solution is finite (as after some iterations the remaining feasible set will be bounded), or if the feasible set is unbounded and the optimal solution is not finite (as then eventually one of the subproblems will yield $-\infty$ as the optimal objective function value). So as a consequence, the branch-and-bound algorithm will terminate in a finite amount of time and yield the globally optimal solution.

2 a) The objective function is convex since it can be written as a sum of squares and an affine term.
b) $(1,2)$ is a global minimum

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Short answers for Question 3

a. We have

$$M = \frac{1}{1+PK}\begin{bmatrix} P & PK & -PK \\ -P & 1 & -1 \end{bmatrix} = \begin{bmatrix} P(1-PQ) & PQ & -PQ \\ -P(1-PQ) & 1-PQ & -(1-PQ) \end{bmatrix}$$

b. For stability we need Q to be stable. This is always true for finite θ_i. We have $M_{r\to y} = PQ$. Moreover, we need that $P(1)Q(1) = 1$ due to the final value property. We have $P(1) = 2$ and $Q(1) = \theta_0 + \theta_1 + \theta_2$. Hence, we obtain the condition

$$\theta_0 + \theta_1 + \theta_2 = [P(1)]^{-1} = 0.5 \ .$$

c. We know $e(k) = \dfrac{-1}{1+P(q)K(q)} n(k) = -\Big(1 - P(q)Q(q)\Big) n(k)$. The induced $(2,2)$-norm is equal to the system infinity norm. This means we have to solve

$$\min_{\theta} \|1 - P(q)Q(q)\|_\infty \ .$$

Next, fill out P and Q.

d. The minimum value of $\|1-P(q)Q(q)\|_\infty$ is reached for $1 - P(q)Q(q) = 0$. So $Q(q) = P^{-1}(q) = 1 - 0.5q^{-1}$ and therefore

$$\theta_0 = 1, \ \ \theta_1 = -0.5, \ \ \theta_2 = 0 \ .$$

e. Let the input and output signals of the model error Δ be given by $\varepsilon(\cdot)$ and $\delta(\cdot)$, respectively. The closed-loop transfer from $\delta(k)$ to $\varepsilon(k)$ is given by

$$\frac{WP}{1+PK} = WP(1-PQ) \ .$$

For robust stability we need

$$\|WP(1-PQ)\|_\infty \leq 1 \ .$$