Short solutions for Exam of November 2018 "Optimization in Systems and Control" (SC42055)

This document concisely lists the solutions for the exam of November 2018. Note that other solutions might also be correct to some degree, and that **you should extensively motivate your answers in the actual exam!** (cf. the worked solutions for Sample Exams 1 and 2 and for the exams of October 2013 and October 2014).

Short answers for Question 1

6 P1. NCC: Nonconvex constrained (nonconvex due to $\mathbb{Z}^4$)
multi-run M11 (simulated annealing)

6 P2. NCU: Nonconvex unconstrained (nonconvex due to nonlinear plant)
multi-run M11 (simulated annealing — gradient and Hessian not analytically computable, and objective function requires time-consuming numerical evaluation)

6 P3. $2 \times$ CP: Convex (the 3rd constraint is actually nonconvex, but it can be split as a union of two convex constraints[1], so *two* convex problems result)
M9 (penalty function) or M10 (barrier function)

6 P4. NCC: Nonconvex constrained (due to the term $-6x_2^2$ in simplified version of second constraint)
multi-start M9 (penalty function), multi-start M10 (barrier function), or multi-run M11 (simulated annealing)

6 P5. CP: Convex
M5 (Levenberg-Marquardt)

6 P6. NCC: Nonconvex constrained (as the simplified objective function is quadratic, but not convex)
multi-start M3 (gradient projection)

6 P7. $8 \times$ LP: Linear (as the term inside the square in the objective function is positive, the square can be dropped, and as the 3rd constraint is actually nonconvex, but it can be written as a union of 8 affine constraints[2]; so *8* linear programming problems result)
M1 (simplex)

[1] Note that $\min(L_1, L_2) \leq 42$ with L_1 and L_2 linear expressions, is equivalent to $L_1 \leq 42$ **or** $L_2 \leq 42$.

[2] In particular, $\|x\|_\infty \geqslant 3$ is equivalent to $\max_{i\in\{1,2,3,4\}}(|x_i|) \geq 3$ or $\max_{i\in\{1,2,3,4\}}(x_i, -x_i) \geq 3$ or $x_1 \geq 3$ **or** $-x_1 \geq 3$ **or** $x_2 \geq 3$ **or** $-x_2 \geq 3$ **or** ... **or** $x_4 \geq 3$ **or** $-x_4 \geq 3$, i.e., the union of $2 \cdot 4 = 8$ affine constraints.

6 P8. 2 $\times$ QP: Quadratic (the second constraint is nonconvex but is can be written as: $2x_1 + 3x_2 - 4x_3 + 8x_4 \geq \cosh^{-1}(3)$ **or** $2x_1 + 3x_2 - 4x_3 + 8x_4 \leq -\cosh^{-1}(3)$, so *2* quadratic programming problems result)

6 P9. NCC: Nonconvex constrained (nonconvex due to the equality constraint (as it is not affine))
multi-start M7 (Lagrange + DFP)

6 P10. NCC: Nonconvex constrained (as log is a nonconvex function)
multi-start M9 (penalty function), multi-start M10 (barrier function), or multi-run M11 (simulated annealing)

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Short answers for Question 2

11 1a. See lecture notes (explain/stress tree search, branching, bounding (both of infeasible branches and branches that would not lead to a lower objective function, stop criterion) — best add an illustrative drawing

2 1b. A mixed-integer linear programming problem is convex if there are no integer variables (i.e., the set of integer variables is empty) or if there are integer variables but their values are fixed (i.e., the set of integer variables is a singleton).

2 1c. No, since a vertex does not necessarily have integer values for all components that should be integer.

2 2a. Since the objective function and the constraints are linear, the problem is convex.

6 2b. The optimal solution can be found in a graphical way: $x^* = [0 \;\; 2]^{\mathrm{T}}$.

2 2c. Since the problem is convex, any local optimum is also a global optimum.

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Short answers for Question 3

4 a. We have

$$M = \frac{1}{1+PK}\begin{bmatrix} P & PK \\ -PK & K \end{bmatrix}$$

6 b. After substitution we obtain

$$M = \begin{bmatrix} P(1-PQ) & PQ \\ -PQ & Q \end{bmatrix}$$

As the entries of M are sums and products of P (which is stable) and Q, and since the set of stable transfer functions is closed under addition and multiplication, we can conclude that all entries of M are stable transfer functions if Q is stable. Note that Q is always stable as its poles ($q = 0$) are inside the unit disc for any $(\theta_1, \theta_2, \theta_3)$. This implies that $\hat{S} = \mathbb{R}^3$. Hence, $\hat{S}$ is a convex set.

5 c. After substitution we obtain

$$M = \frac{1}{\kappa q+2}\begin{bmatrix} \frac{\kappa q+1}{q} & 1 \\ -1 & q \end{bmatrix}$$

There are 2 different poles: $q = 0$ (which is a stable pole) and $q = -\frac{2}{\kappa}$. The latter pole is stable (i.e., inside the unit disc) if $\left|-\frac{2}{\kappa}\right| < 1$ or — equivalently — if $\kappa < -2$ or $\kappa > 2$. Thus $\tilde{S} = (-\infty, 2) \cup (2, \infty)$, which is a nonconvex set.