Short solutions for Exam of October 2019 "Optimization in Systems and Control" (SC42055)

This document concisely lists the solutions for the exam of October 2018. Note that other solutions might also be correct to some degree, and that **you should extensively motivate your answers in the actual exam!** (cf. the worked solutions for Sample Exams 1 and 2 and for the exams of October 2013 and October 2014).

Short answers for Question 1

7.5 P1. NCU: Nonconvex unconstrained (eliminate x_1 from constraint)
multi-start M6 (Levenberg-Marquardt)

7.5 P2. MILP: Mixed integer linear
M12 (branch-and-bound)

7.5 P3. NCC: Nonconvex constrained
multi-start M10 (penalty + Powell — gradient and Hessian not analytically computable, and objective function requires time-consuming numerical evaluation)

7.5 P4. LP: Linear
M1 (simplex)

7.5 P5. $2 \times$ CP: Convex (the 3rd constraint is actually nonconvex, but it can be split in two sets of convex constraints[1], so *two* convex problems result)
M11 (barrier function + steepest descent — note that multi-start is not needed provided that a convex barrier function is selected)

7.5 P6. NCC: Nonconvex constrained (due to the term -3 in the argument of cosh the objective function is not convex)
multi-start M11 (barrier function + steepest descent), or multi-start M10 (penalty + Powell — if one argues that gradient is not defined everywhere for the absolute value function (but note that we could just take the subgradient then))

7.5 P7. NCC: Nonconvex constrained (the objective function can be recast as a quadratic but nonconvex function, the constraints can be rewritten as linear constraints)
multi-start M3 (gradient projection), multi-start M11 (barrier function + steepest descent)

7.5 P8. CP: Convex (note that the last term of the max function is actually convex due to the first constraint of the given optimization problem)
M11 (barrier function + steepest descent — note that multi-start is not needed provided that a convex barrier function is selected)

[1] Note that $\max(L_1, L_2) \geq 25$ with L_1 and L_2 linear expressions, is equivalent to $L_1 \geq 25$ **or** $L_2 \geq 25$.

The answers below are short answers only; **you should extensively motivate your answers in the actual exam!** (cf. the worked solutions for Sample Exams 1 and 2 and for the exams of October 2013 and October 2014).

Short answers for Question 2

13 1. See lecture notes (give, explain, and motivate update formula with $\hat{H} = H + \lambda I$, advantages w.r.t. Newton (in case of singular H) and quasi-Newton (which works with an approximation of the real Hessian), effect of small/large λ, and add an illustrative drawing)

2 2a. After transforming the max problem into a min problem, the objective function becomes concave while the constraints are linear. So the problem is not convex.

10 2b. In the point $(1,0)$ the *negative* gradient[2] points towards the infeasible region; so we have to project and move on the x-axis, the line minimum[3] $(2,0)$ is found on the right-most boundary of the feasible set; in $(2,0)$ the *negative* gradient points towards the infeasible region — its projection is $(0,0)$ — so $(2,0)$ is a local optimum

[2] After writing the optimization problem in the standard form (with minimization instead of maximization), the gradient of the objective function is given by $\nabla f(x,y) = \begin{bmatrix}-2x+y & -4y+x+4\end{bmatrix}^{\mathrm{T}}$. So $\nabla f(1,0) = \begin{bmatrix}-2 & 5\end{bmatrix}^{\mathrm{T}}$ and thus $-\nabla f(1,0) = \begin{bmatrix}2 & -5\end{bmatrix}^{\mathrm{T}}$.

[3] Just optimizing the step size yield the line maximum in $(0,0)$.

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<u>Short answers for Question 3</u>

6 a. We have

$$M = \frac{1}{1+P_1K_1+P_1P_2K_2}\begin{bmatrix} K_2 & -(K_1+P_2K_2) \\ P_1P_2K_2 & P_2 \end{bmatrix}$$

9 b. After substitution we obtain

$$M = \begin{bmatrix} Q_2 & -(Q_1+P_2Q_2) \\ P_1P_2Q_2 & P_2(1-P_1Q_1-P_1Q_1Q_2) \end{bmatrix}$$

As the entries of M are sums and products of P_1, P_2, Q_1, Q_2 (which are all stable), and since the set of stable transfer functions is closed under addition and multiplication, we can conclude that all entries of M are stable transfer functions. So the system is then closed-loop stable.