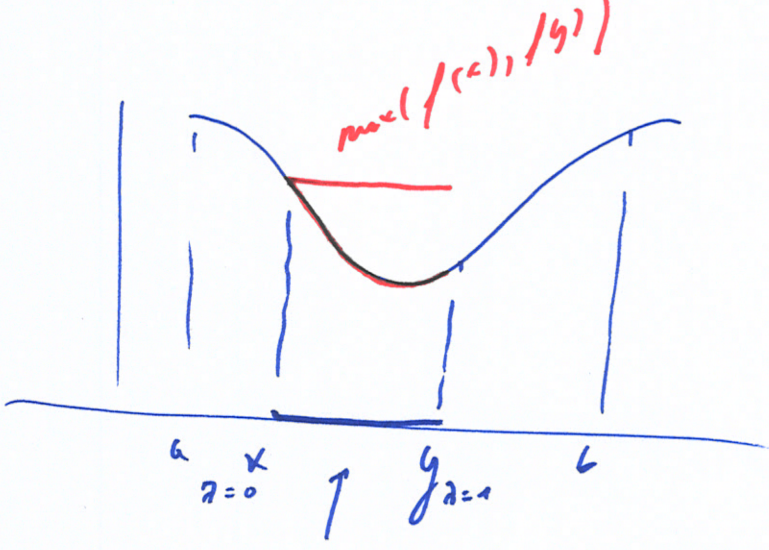
1

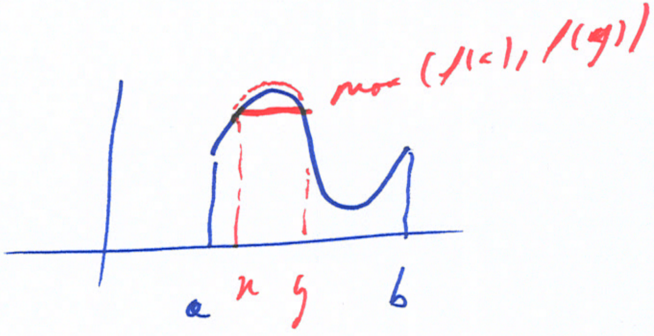
2023-9-19

quasi convex

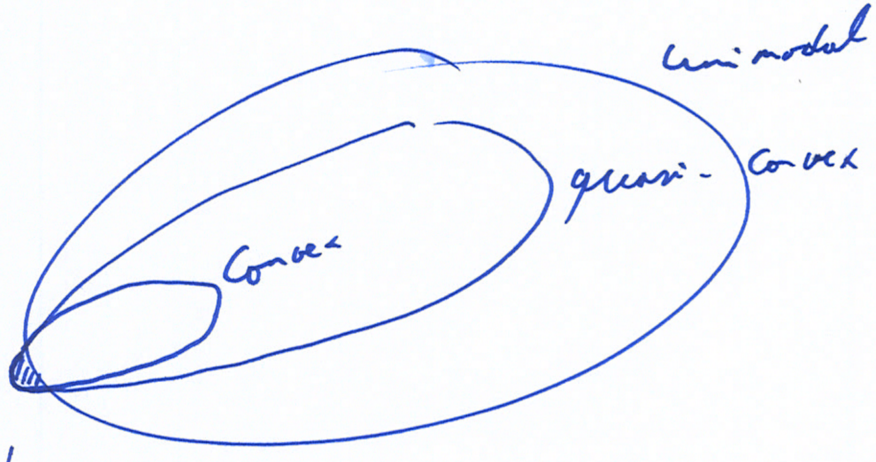


$(1-\lambda)x+\lambda y \quad \lambda \in [0,1]$

[a,b] convex

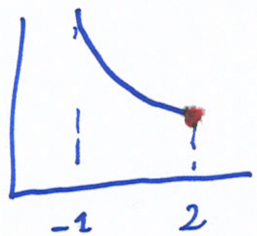


not quasi convex.



. not unimodal
. quasiconvex
. convex

$2 \in dom(f) = [-1, 2)$

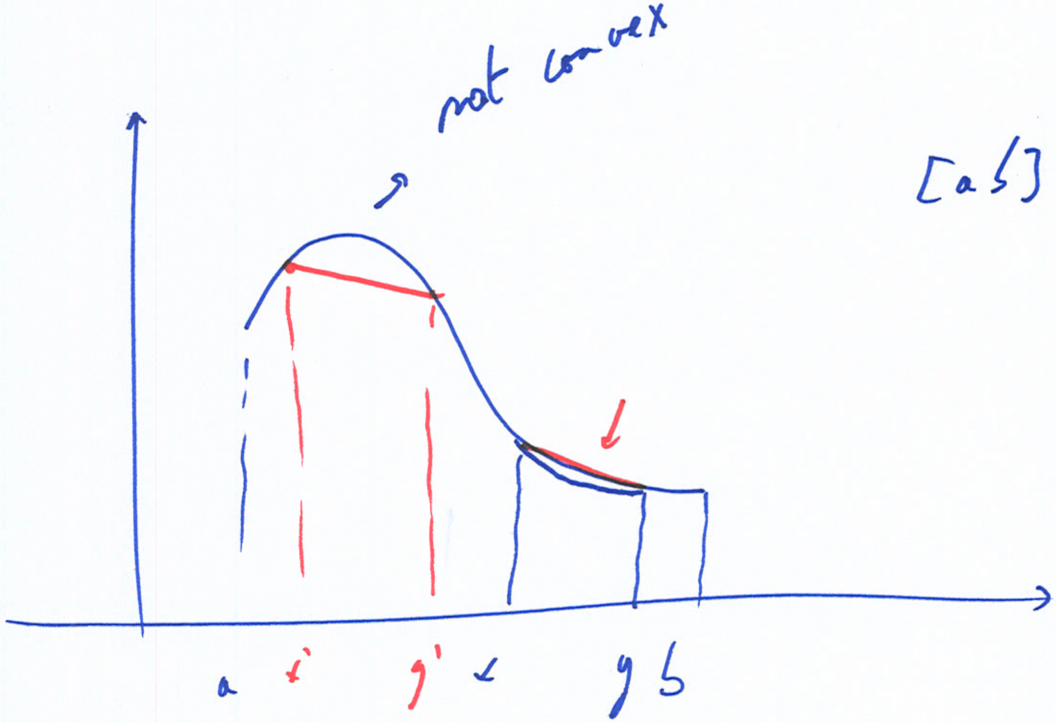


$\exists\, x^* \in dom(f)$
$f(x^*) \le f(x) \quad \forall x \in dom(f)$

2023-9-19

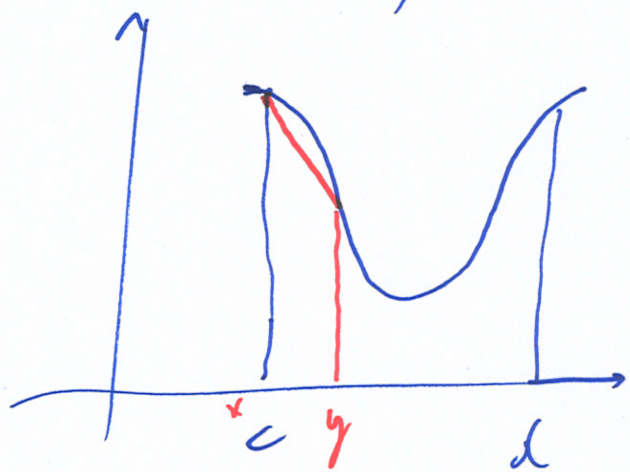
(2)

1D case: unimodal = quasiconvex (except special case with $x^* \notin \mathrm{dom}(f)$)



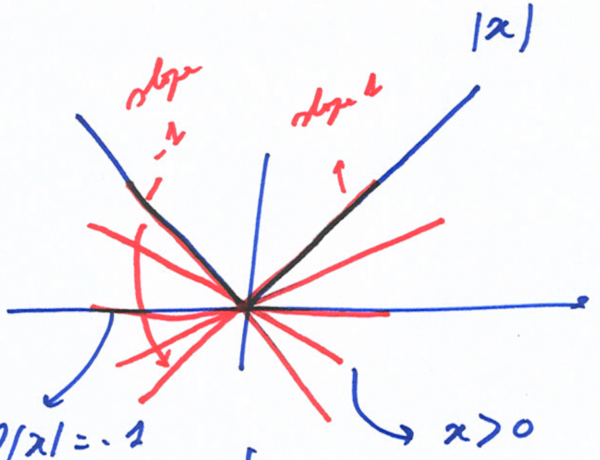
[a b] convex.

unimodal
quasiconvex
~~not~~ ~~convex~~



(3) $|x|$

Convex



$x < 0$ $D|x| = -1$

$x > 0$ $D|x| = 1$

$x = 0$

$D|x| \in [-1, 1]$ subgradient

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positive definite matrix

$$A = \begin{bmatrix} \boxed{4} & 1 & 0 \\ 1 & 6 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$A_{\{1\}\{1\}} = 4 \xrightarrow{\det} 4 > 0$

$A_{\{1,2\}\{1,2\}} = \begin{bmatrix} 4 & 1 \\ 1 & 6 \end{bmatrix} \longrightarrow 4 \cdot 6 - 1 \cdot 1 = 23 > 0$

$A_{\{1,2,3\}\{1,2,3\}} = \begin{bmatrix} 4 & 1 & 0 \\ 1 & 6 & 0 \\ 0 & 0 & 3 \end{bmatrix} \longrightarrow 4 \cdot 6 \cdot 3 - 1 \cdot 1 \cdot 3 = 69 > 0$

$\downarrow$

A positive definite

$A > 0$

A_{JJ}

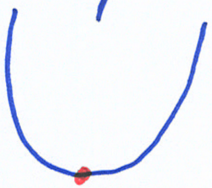
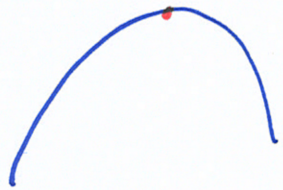
$J = \{1\}$

$= \{1, 2\}$

$= \{1, 2, 3\} \ldots$

(4) scalar function $\quad f'=0 \rightarrow$ horizontal tangent line
slope 0

$f''>0$ $\qquad$ $f''<0$ $\qquad$ $f''=0$

saddle point

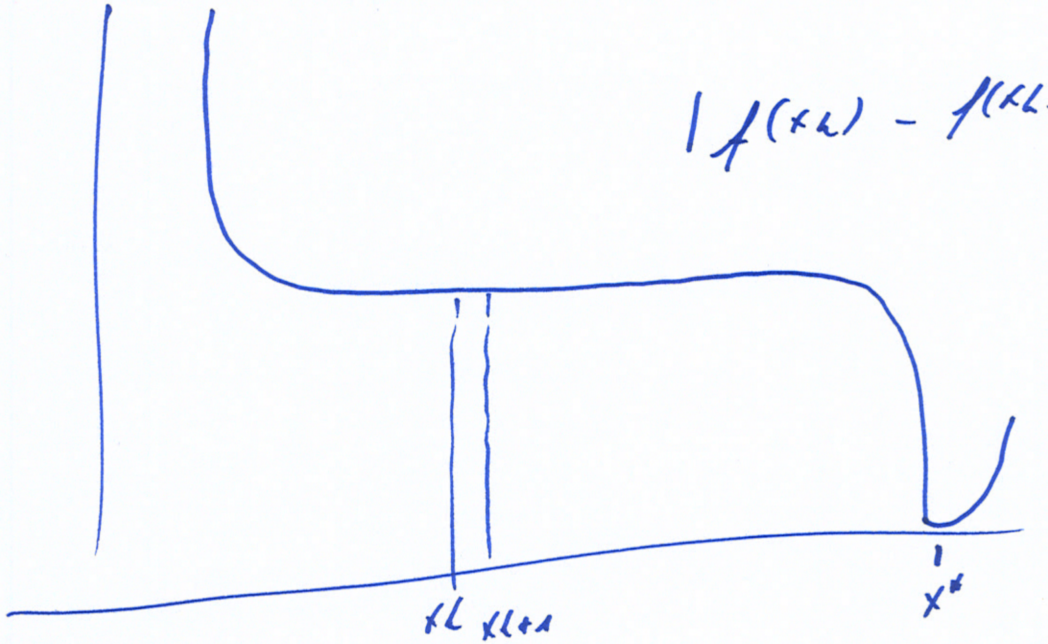
minimum $\qquad$ maximum

e.g. $\quad x^2 \qquad -x^2 \qquad -x^3$

necessary $\qquad$ minimum $\Rightarrow$ $f'=0$

necessary & sufficient $\qquad$ minimum $\Leftarrow$ $f'=0$, $f''>0$

g



$|f(x_k) - f(x_{k+1})| \leq \varepsilon$

but not yet at minimum