2023-9-21

① A b

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0.1 & 0.2 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 100 \\ 14 \end{bmatrix}$$

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 + s_1 \\ x_4 + s_2 \end{pmatrix}$$

② ③

$$B = \begin{bmatrix} 1 & 1 \\ 0.2 & 0 \end{bmatrix}$$

① ④

$$N = \begin{bmatrix} 1 & 0 \\ 0.1 & 1 \end{bmatrix}$$

$$c = [-20 \quad -30 \quad 0 \quad 0]^T$$

$$x_B = \begin{bmatrix} x_2 \\ x_3 \end{bmatrix} \qquad x_N = \begin{bmatrix} x_1 \\ x_4 \end{bmatrix}$$

$$c_B = \begin{bmatrix} -30 \\ 0 \end{bmatrix} \qquad c_N = \begin{bmatrix} -20 \\ 0 \end{bmatrix}$$

$$Ax = b \qquad Bx_B + Nx_N = b$$

$$c^T x = c_B^T x_B + c_N^T x_N$$

$$x = x_0 + \lambda^T x_N$$

$$x = 10$$

$$\lambda^T = [0 \quad -20 \quad \textcircled{-20} \quad 0 \quad 50 \quad 6]$$

$$\arg\min_i \{\lambda_i \mid \lambda_i < 0\}$$

(2)

(1) $B x_B = b$

$B y = N_i$

$B(x_B - \varepsilon y) + N_i \cdot \varepsilon = b$

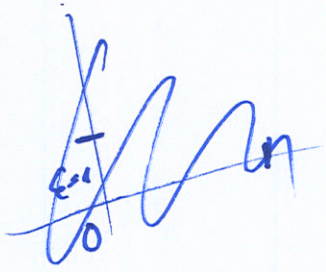
new $(x_B)_j$ — new $(x_N)_i > 0$

(1) one of them $= 0$

(2) rest ≥ 0

$B x_B - \varepsilon B y + N_i \varepsilon = b$ (indeed correct!)

(1)



$(x_B)_j - \varepsilon y_j = 0 \rightarrow \varepsilon = \frac{(x_B)_j}{y_j}$

$\varepsilon_1 = 2 \rightarrow x_1 = 0 \rightarrow x_3 \ll 0$

$\varepsilon_2 = 4 \rightarrow x_2 = 0$

$\boxed{\varepsilon_3 = 1} \rightarrow x_3 = 0$

> 0

$$\arg\min_j \left\{ \frac{(x_B)_j}{y_j} \;\middle|\; y_j > 0 \right\}$$

(3)

$$A = \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0.1 & 0.2 & 0 & 1 \end{array}\right] \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$N = \begin{bmatrix} 1 & 1 \\ 0.1 & 0.2 \end{bmatrix}$$

$$b = \begin{bmatrix} 100 \\ 14 \end{bmatrix}$$

$$x_B = \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} \qquad x_N = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

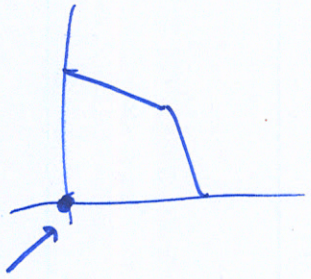
$$= B^{-1} \cdot b = \begin{bmatrix} 100 \\ 14 \end{bmatrix}$$

$$X = (x_1\ x_2)^T = [0\ 0)^T$$

La faisible

$$\lambda^T = [-20 \quad -30]$$

N_2



$$By = N_2 \rightarrow y = \begin{bmatrix} 1 \\ 0.2 \end{bmatrix}$$

$$\frac{(x_B)_1}{y_1} = \frac{100}{1} = 100; \qquad \frac{(x_B)_2}{y_2} = \frac{14}{0.2} = 70$$

ε_2

$$B_{new} = \begin{bmatrix} 1 & 1 \\ 0 & 0.2 \end{bmatrix} \qquad N_{new} = \begin{bmatrix} 1 & 0 \\ 0.1 & 1 \end{bmatrix}$$

(4) $|x_1 + x_2 - 2x_3 + x_4 + 9| \leq 2$

$-2 \leq x_1 + x_2 - 2x_3 + x_4 + 9 \leq 2$

linear inequality

↓

LP

$|x_1| + |x_2| + |x_3|$

min $x_1 x_2 x_3$

$t_1 + t_2 + t_3$

min $t_1 t_2 t_3$
$x_1 t_2 t_3$

in optimum: $t_i^* = |x_i^*|$

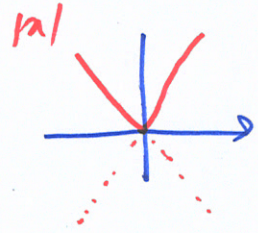
$t_1 + t_2 + t_3$

min $x_1 t$

$x_1^* = 5$
$t_1^* = 6 - t_1^* = 5$

$t_1 \geq |x_1|$
$t_2 \geq |x_2|$
$t_3 \geq |x_3|$

$t_1 \geq x_1 \quad t_1 \geq -x_1$
$t_2 \geq x_2 \quad t_2 \geq -x_2$
$t_3 \geq x_3 \quad t_2 \geq -x_3$



$|x| = \max(x, -x)$

LP!