

⑤

QP2023-9-21

$$x_1^2 + 4x_1x_2 + 9x_2^2$$

$$H = \begin{bmatrix} 2 & 4 \\ 4 & 18 \end{bmatrix}$$

Diagram showing the mapping of the quadratic form coefficients to the Hessian matrix H. The coefficient 2 is linked to the top-left element, 4 to the top-right element, and 4 to the bottom-left element. The coefficient 9 is linked to the bottom-right element. Red arrows indicate the flow from the quadratic form to the matrix elements.

$$\frac{1}{2} x^T H x$$

$$\begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} \alpha x_1 + \beta x_2 \\ \gamma x_1 + \delta x_2 \end{bmatrix}$$

H symmetric

$$\gamma = \beta$$

$$\frac{1}{2} (\alpha \underline{x_1}^2 + \beta \underline{x_1 x_2} + \gamma \underline{x_1 x_2} + \delta \underline{x_2}^2)$$

$$H = \begin{bmatrix} 3 & 8 \\ 8 & 6 \end{bmatrix} \rightarrow \frac{3}{2} x_1^2 + \frac{16}{2} x_1 x_2 + \frac{6}{2} x_2^2$$

$$= \frac{3}{2} x_1^2 + 8 x_1 x_2 + 3 x_2^2$$

⑥

$$f = \frac{1}{2} h x^2 + c x$$

$$h, c \in \mathbb{R}$$

$$f' = 0 \quad h x + c = 0 \quad \rightarrow \quad x = -h^{-1} c$$

$$\frac{1}{2} x^T H x + c^T x \quad \rightarrow \quad x = -H^{-1} \cdot c$$

$A > 0$ leading principal minor > 0

$A < 0$ $\rightarrow (-A) > 0$ neg. definite

$A \underset{>}{<} 0?$ \rightarrow $A > 0?$ ~~no~~ \rightarrow $-A > 0!$ ~~no~~ \rightarrow in definite

2

$$y(n+1) + a y(n) = b u(n) + e(n)$$

a, b?

$$y(n+1) - [-y(n) \quad u(n)] \cdot \begin{bmatrix} a \\ b \end{bmatrix} = e(n)$$

$$\begin{bmatrix} y(2) \\ y(3) \\ \vdots \\ y(n) \end{bmatrix} - \begin{bmatrix} -y(1) & u(1) \\ -y(2) & u(2) \\ \vdots & \vdots \\ -y(n-1) & u(n-1) \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} e(1) \\ e(2) \\ \vdots \\ e(n-1) \end{bmatrix}$$

y

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Φ

α

=

E