① 2023-9-26

$$\min_{x} f(x)$$

$$g(x) \leq 0$$

$$h(x) = 0$$

✓ linear → finite # steps

✓ quadratic → finite # steps

convex

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nonlinear

$$\nabla f(x_0) \rightarrow \frac{\partial f}{\partial x_i}(x_0) = \lim_{\delta \to 0} \frac{f\left(x_0 + \delta \begin{array}{c} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \end{array} \begin{array}{c} \\ \\ \leftarrow i \\ \\ \\ \end{array}\right) - f(x_0)}{\delta}$$

$\downarrow$

$i = 1, \dots, n$

$f \in \mathbb{R}^n$

→ analytically (only if possible)

→ numerically

$i: 1 \rightarrow n$ $$\frac{\partial f}{\partial x_i}(x_0) \approx \frac{f\left(x_0 + \delta \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}\right) - f(x_0)}{\delta}$$

δ small

$$x \rightarrow \boxed{f?} \rightarrow$$

→ n function evaluations to compute gradient

(2)

→ issue for computing ∇f / H numerically

if n is very large

or/and evaluating f takes a lot of time

$\pm \frac{n^2}{2}$ # evaluations of f

→ H →

$$\begin{array}{cc} & n \\ n & \begin{bmatrix} \square & \square & \square & & \square \\ & \square & \square & & \square \\ & & & & \square \\ & & & \ddots & \square \end{bmatrix} \end{array}$$

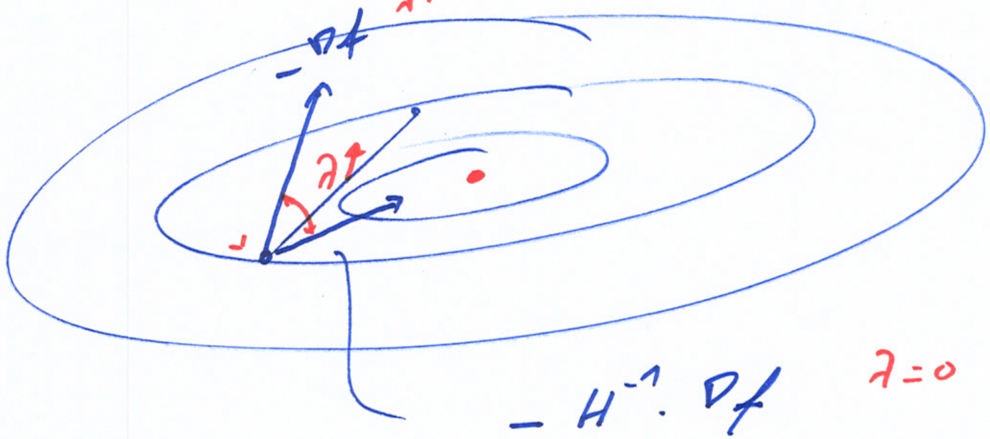
symmetric

$\frac{n(n+1)}{2}$

③ Levenberg - Marquardt

$$\hat{H} = H + \lambda I$$

$\lambda \gg 1$, steepest descent



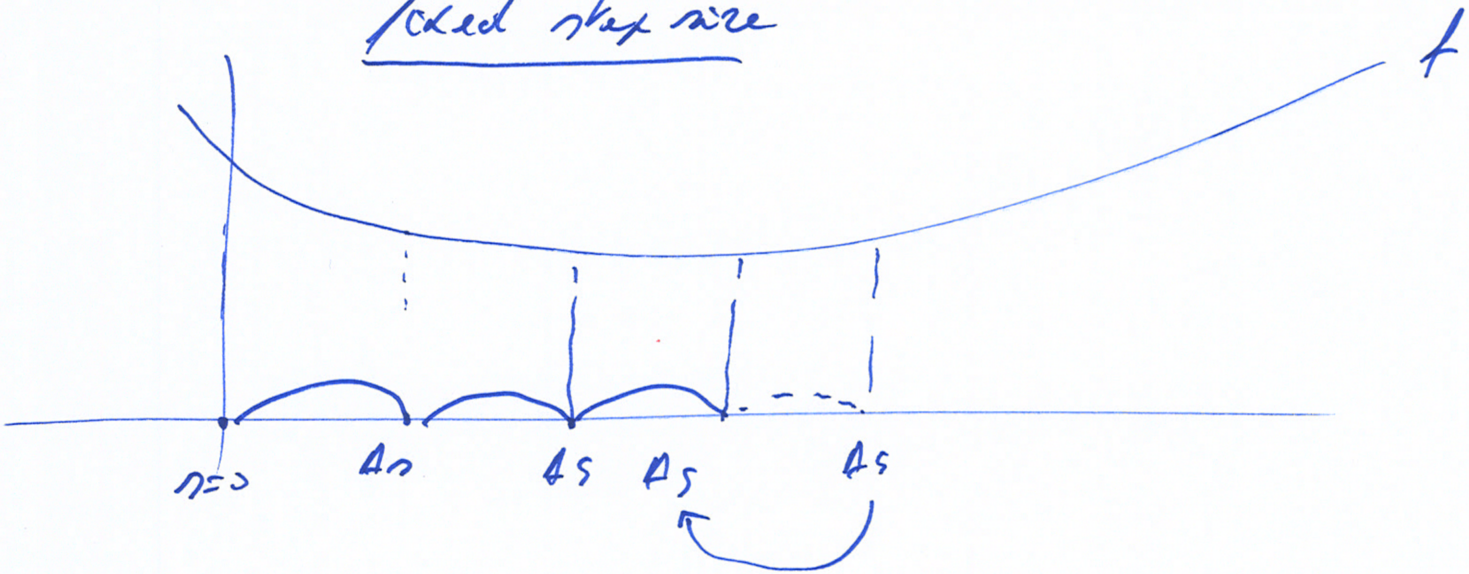
$-H^{-1} \cdot \nabla f$ $\quad$ $\lambda = 0 \rightarrow$ Newton step

$$x_{k+1} = x_k - \hat{H}^{-1} \nabla f = x_k - \frac{1}{\lambda} \nabla f$$

$\lambda \gg 1 \qquad \hat{H} \approx \lambda I$

(4)

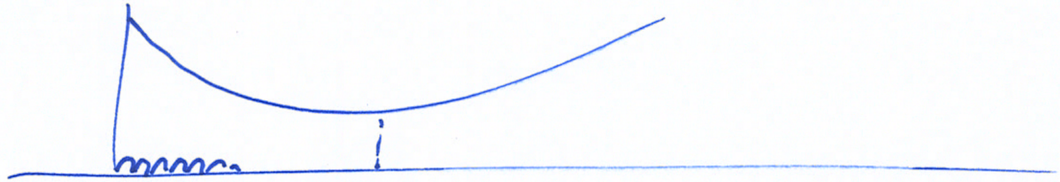
fixed step size



hidden assumption

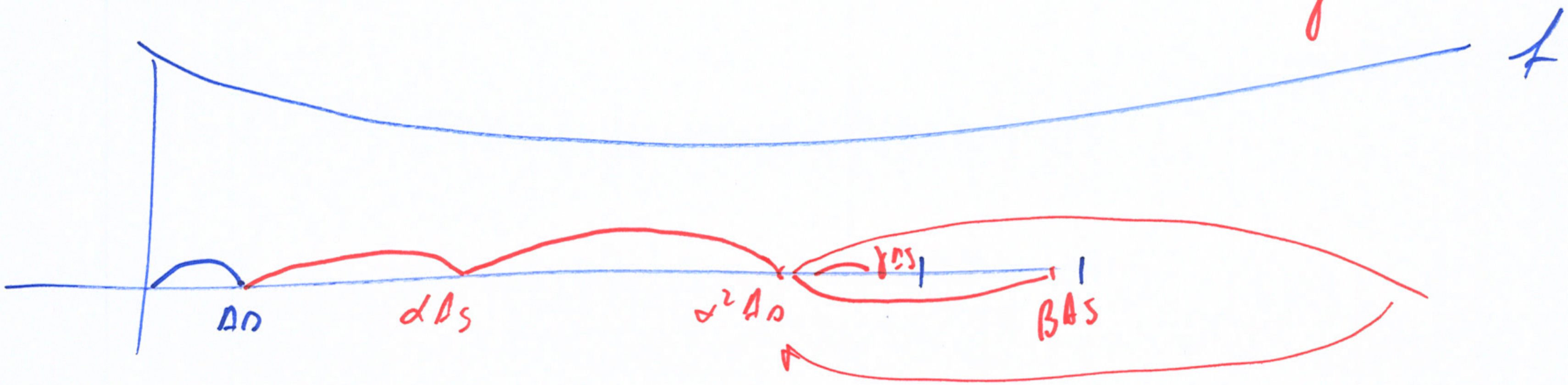
locally function unimodal

size of Δs?

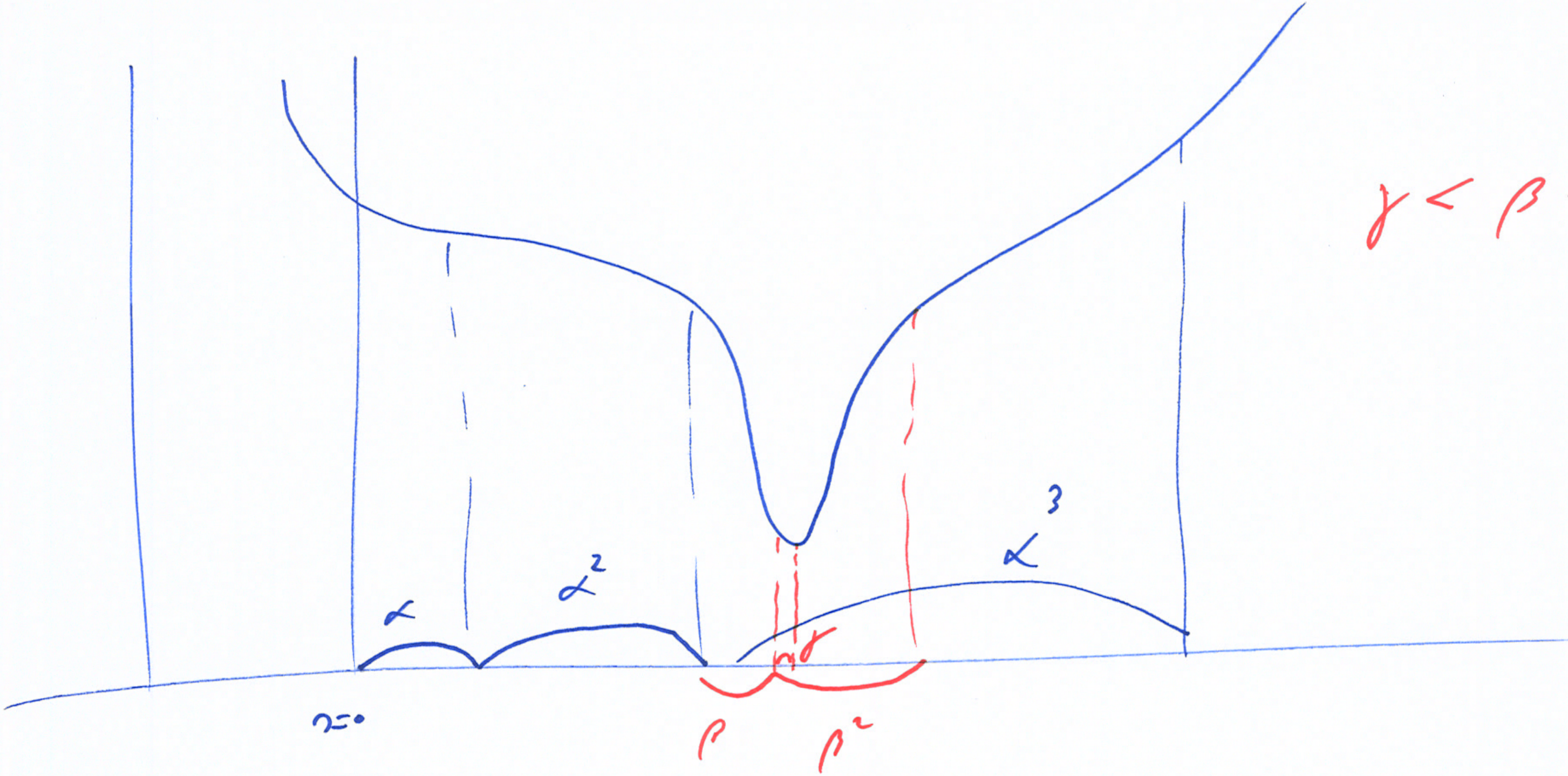


variable step size

$\alpha \downarrow \beta \downarrow \gamma$

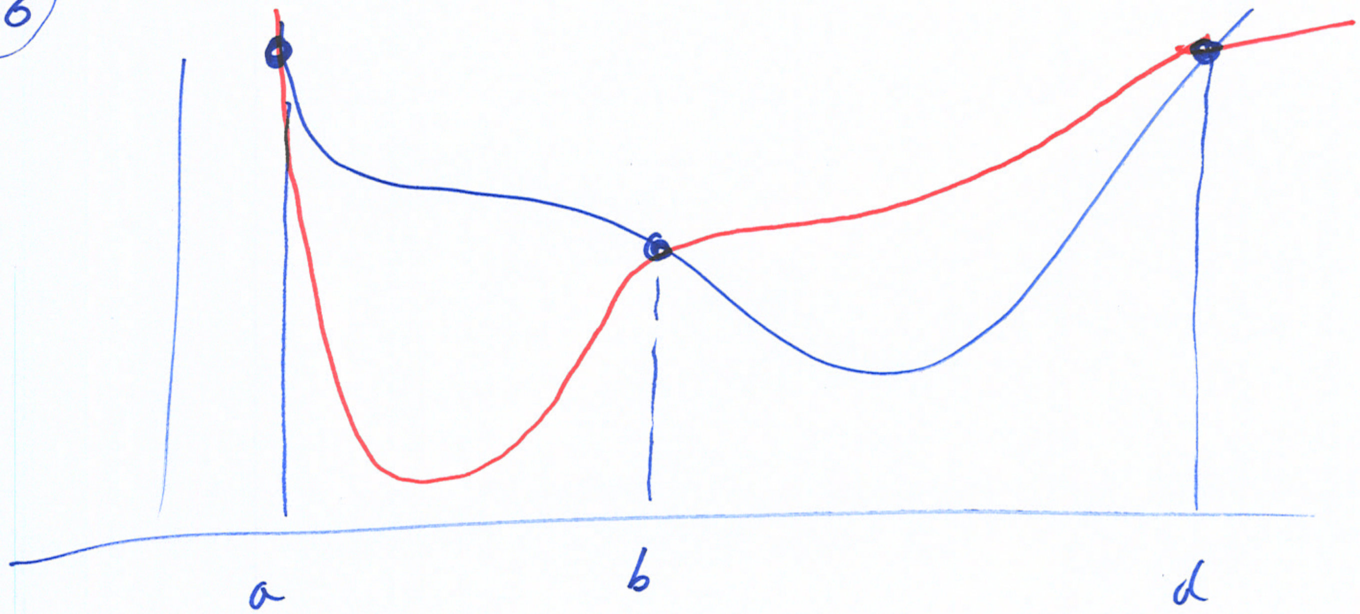


x



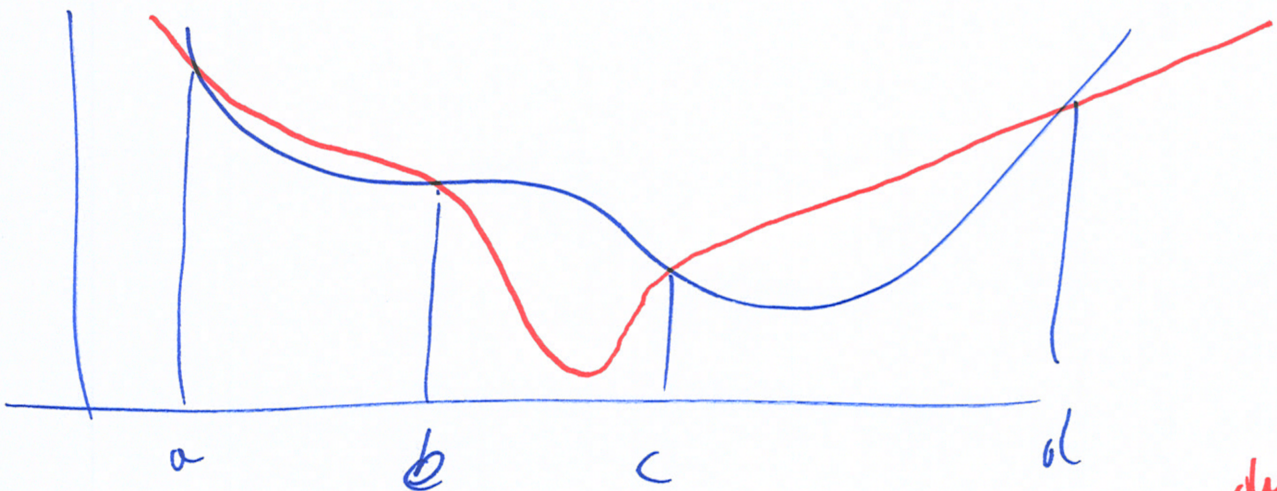
$\gamma < \beta < \alpha$

6



→ 1 point extra not sufficient to reduce interval

unimodal



f(a) ↓ f(b) ↓ f(c) ↗ f(d)

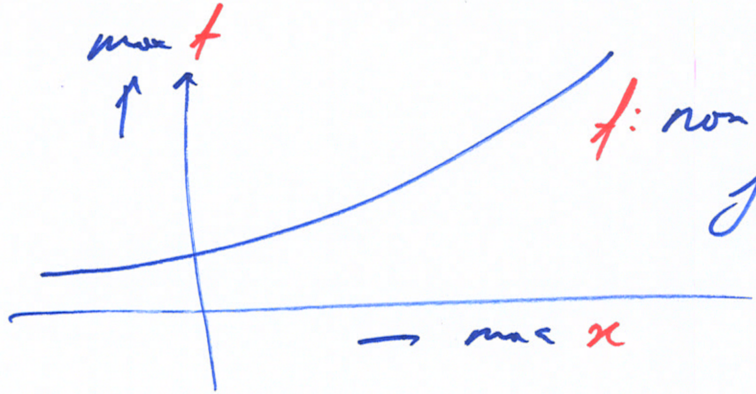
minimum is [cd] or [bc]

~~ab~~

↑ due to unimodality

(7) $\max_{x_1,x_2} \exp(x_1 - x_2)$

$|2x_1 + 3x_2 - 5x_3| \leq 1$



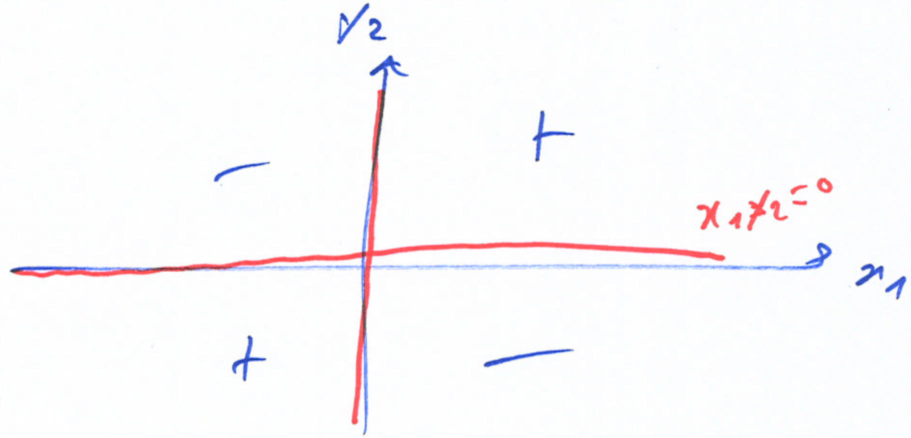
f: nondecreasing function

$\rightarrow \max_{x_1,x_2} \; x_1 - x_2$

$-1 \leq 2x_1 + 3x_2 - 5x_3 \leq 1$ $\rightarrow$ LP

(8)

$$\max_{x_1 x_2 x_3} \; x_1 x_2 + x_2 x_3$$



$$\min_{x_1 x_2 x_3} \; x_1 x_2 + x_2 x_3$$

→ nonlinear programming

→ indefinite function

$$H = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

0

-1

$0+0+0-0-0-0$

~~OK~~ → H should positive (semi) definite

H is not positive definite

$$-H = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

0

-1

0

$-H$ is not positive definite

H indefinite