Abstract—The problem of air-fuel ratio (AFR) stabilization in spark ignition engines is addressed in this paper. The proposed strategy consists of proper switching among two control laws to improve the quality of the closed-loop system. The first control law is based on the a priori off-line identified engine model and ensures robust and reliable stabilization of the system at large, while the second control law is adaptive, provides on-line adaptive adjustment to the current fluctuations and improves the accuracy of the closed-loop system. The supervisor realizes the switching rule between these control laws providing better performance for AFR regulation. Results of application are reported and discussed.

I. INTRODUCTION

VEHICLE tailpipe emissions requirement is one of the main restrictions for engine development and certification. Three-way catalytic converters (TWC) installation in exhaust manifold aims at oxidizing HC and CO and reducing NOx species. Usually TWC peak efficiency is guaranteed if air-fuel ratio (AFR) is close to the stoichiometric value and the conversion efficiencies of TWC are significantly reduced away from the stoichiometric value. Therefore, the primary objective of the AFR control system is to maintain the fuel injection in stoichiometric proportion to the ingested air flow (exception to this occurs in heavy load situations where a rich mixture is required to avoid premature detonation or for more power). Variations in the air flow affected by the driver serve as the disturbance to the system.

Due to its importance, the problem of AFR regulation has attracted significant attention during the last few decades [3]. Adaptive control theory [1], [6], [16], [17], robust control approaches [2], fuzzy control systems theory [7], neural network techniques [11], [18] are successfully tested in this particular application. However, the complexity of the problem and growing demands on AFR regulation quality require new solutions, which can combine reliability and performance of robust control approaches and the accuracy and insensitivity to dynamics changes of adaptation methods. Switching control theory gives a solution to this problem.

There exist many good reasons and practical motivations to use a set of controllers to regulate a single plant as opposed to one controller [10], [11], [15]. In such a case the natural question arises as how to trade off the advantages and disadvantages of each subsystem for modeling and control. The theory of switched systems addresses this issue regarding the proper switching laws between controllers. Application of a supervisory (switched) control algorithm may seriously improve performance of the system regulation [4].

In this work we solve the problem of AFR regulation considering switching between two control laws. The first one is based on robust model-based control algorithm, which ensures stability for all ranges of the system parameters and inputs, but may have shortcomings in satisfying the required accuracy. The second control law is adaptive and is directed at improving the quality of transient response for the cases of dynamic fluctuation around the reference model (used in the first control). Supervisor performs activation of the adaptive control when unsatisfactory quality of the reference model is detected and, hence, improvement of the robust control is needed.

In section 2 detailed problem statement and some preliminaries are presented. Section 3 contains a description of the control algorithms. Supervisor equations are introduced in section 4. Results of the system implementation are reported in section 5.

II. PROBLEM STATEMENT

It is a well-known fact that an automotive engine is a highly nonlinear multi-variable system and derivation of its precise model is a complex process. This is one reason for the simplified models of engines being very popular in practice. These models can take into account the main features of engine processes in the presence of time delays and nonlinearities, which are important for controller design or fault detection applications. In this work nonlinear autoregressive (NARX) model is chosen for AFR dynamics description (in this context AFR refers to the non-dimensional engine-out air-fuel ratio sometimes known as $\lambda$):
where \( y \in R \) is the regulated output (in our case we take fuel-to-air ratio as \( y \)), \( u \in [u_{\text{min}}, u_{\text{max}}] \) is the control input (fuel pulsewidth in this work, \( 0 < u_{\text{min}} < u_{\text{max}} < +\infty \) are actuator constraints), \( d \in R^n \) and \( f \in R^p \) are the vectors of inputs (may contain physical engine variables products), \( k \geq 1 \) and \( p \geq k-1 \) are the model polynomial degrees, \( m \) is the number of current event (discrete time); \( v \in R \) is a disturbance acting on the system; \( a = [a_1 ... a_k] \in R^k \), \( B = [b_0 ... b_p] \in R^{p \times (p+1)} \) and \( R = [r_0 ... r_p] \in R^{n \times (p+1)} \) are the model (1) constant parameters. The advantage of NARX models consists in availability of various methods for their approximation and simplicity of controls design.

It is assumed that a dataset is given, that a priori has collected measured information on \( y, u \) and other state variables involved in the vectors \( d, f \) for various regimes of engine operation. Then, applying standard approaches [14] it is possible to obtain off-line the vectors of coefficients \( a, B \) and \( R \) such that the model (1) represents dynamics of AFR loop in the engine with sufficient accuracy. The residual error can be assumed bounded and modeled as a part of the exogenous disturbance \( v \). The coefficients \( a, B \) can be derived ensuring stability of the model (1) as well as stability of its inverse with respect to the control input.

**Assumption 1. Polynomials \( a \) and \( b_j \), \( 0 \leq j \leq p \) have all zeros with norms smaller than one.**

Under this assumption, stabilizing controls for the system (1) can be designed applying simple inversion of its equations (inverse system is stable and, thus, the control algorithm will be realizable). Based on the given dataset, the compact sets \( D \subset R^n \) and \( F \subset R^p \) can be computed which define admissible values for the vectors \( d, f \) respectively.

The problem is to design control \( u(i) \in [u_{\text{min}}, u_{\text{max}}] \), \( i \geq 0 \) ensuring practical output regulation to a given reference \( y_d(i), i \geq 0 \), i.e., the property \( |y(i) - y_d(i)| \leq \Delta \) should be satisfied for all \( i \geq 0 \) and \( d \in D, f \in F \) for some prescribed \( \Delta > 0 \) providing that \( |y(0) - y_d(0)| \leq \Delta \).

To this end, recall that a continuous function \(\sigma: R_{+} \rightarrow R_{+} \) belongs to class \( K \) if it is strictly increasing and \( \sigma(0) = 0 \); additionally it belongs to class \( K_{\infty} \) if it is also radially unbounded; and continuous function \(\beta: R_{+} \times R_{+} \rightarrow R_{+} \) is from class \( \mathcal{KL} \), if it is from class \( K \) for the first argument for any fixed second one, and it is strictly decreasing to zero by the second argument for any fixed first one.

### III. CONTROL ALGORITHM

In this section descriptions of robust model-based and adaptive controls are presented.

#### A. Model-based control algorithm

Under assumption 1 this algorithm is chosen as a simple inversion of the model (1) with respect to the control:

\[
\begin{align*}
  u(m) &= U(m) = \frac{1}{b_0} f(m) \\
  &\times \left[ y_d(m) - U_{\text{PID}}(m-1) - \sum_{i=1}^k a_i y(m-i) - \sum_{j=0}^p r_j^T d(m-j) - \sum_{j=1}^p [b_j^T f(m-j)] u(m-j) \right] \\
&\quad + k_1 e(m) + k_2 \sum_{i=0}^m e(i) + k_3 [e(m) - e(m-1)] + k_4 \text{sign}(e(m)) + k_5 e(m)^3 \tag{3}
\end{align*}
\]

where due to the presence of the disturbance \( v \) (which reflects possible unmodeled dynamics, measurement noise and approximated model errors) it is required to use an internal feedback in the form of the nonlinear PID:

\[
U_{\text{PID}}(m) = k_1 e(m) + k_2 \sum_{i=0}^m e(i) + k_3 [e(m) - e(m-1)] + k_4 \text{sign}(e(m)) + k_5 e(m)^3,
\]

where \( e = y_d - y \) is the regulation error, \( k_j, j = 1,5 \) are control parameters, which have to be determined based on real or computer experiments. The control (2) ensures the model inversion and the following closed loop dynamics:

\[
y(m) = y_d(m) - U_{\text{PID}}(m-1) + v(m).
\]

Without \( U_{\text{PID}} \) the control (2) forms the so-called feedforward part of the control, which does not contain any deviation errors (it depends on the current and past values of the inputs and outputs of the engine dynamics and the approximated coefficients of the model).

The following is the condition of the control (2) applicability.

**Assumption 2. For all \( f \in F \) it holds \( b_0^T f \neq 0 \).**

Since the vector \( f \) is composed of physical engine variables or their nonlinear functions and products, which all have some sets of admissible values, then assumption 2 can be easily checked for \( f \in F \) and the coefficients \( b_0 \). For instance, \( f(i), i \geq 0 \) and elements of \( b_0 \) can be all positive (that may be guaranteed by proper approximation of (1)).

The control (2) can not be realized in practice since there exist constraints on admissible control amplitudes, i.e. it should be within the following bounds: \( u_{\text{min}} \leq u \leq u_{\text{max}} \).

The implementation of a simple saturation

\[
u(m) = \text{sat} \left[ U(m) \right], \quad \text{sat}(x) = \begin{cases} u_{\text{min}} & \text{if } x < u_{\text{min}} \\ u_{\text{max}} & \text{if } x > u_{\text{max}} \\ x & \text{otherwise} \end{cases}\tag{4}
\]

for stable plants provides a solution to the problem. Define the control actuator error as follows
\[
\hat{\delta}(m) = b_0^T f(m) [u(m) - U(m)],
\]
then the closed loop dynamics of (1), (4) takes the form:
\[
y(m) = y_d(m) - U_{pp}(m-1) + \hat{v}(m),
\]
\[
\hat{v}(m) = v(m) + \hat{\delta}(m).
\]

**Proposition 1.** Under assumptions 1 and 2 there exist constants \(k_j, j = 1, 5\) such that for any solutions of the system (1), (4) with \(d \in D\), \(f \in F\) for all \(i \geq 0\):
\[
|e(i)| \leq \beta_i(|e(0)| + i) + \gamma_i(||\hat{v}||_{0,1}),
\]
\[
||\hat{v}||_{0,1} = \sup_{0 \leq j \leq i} ||\hat{v}(j)||, \beta_i \in \mathbb{K}^L, \gamma_i \in \mathbb{K}.
\]

All proofs are omitted due to space limitations.

Additionally adjusting values of the coefficients \(k_j, j = 2, 5\) one can improve the performance of the control (4). For example, coefficient \(k_2\) provides insensitivity to static errors (integral part of PID), coefficient \(k_4\) cancels disturbances with amplitudes less than \(k_4\), and coefficient \(k_5\) may help on large deviations of the error.

The estimate obtained in proposition 1 is close to input-to-state stability property introduced in the paper [8].

### B. Adaptive control algorithm

Despite the fact that the control (4) has feedbacks aimed at disturbances, approximation error and measurement noise attenuation, in some cases an additional adaptation of the control is further needed. An important issue is that the model (1) has been approximated on a large *a priori* collected dataset, and the coefficients \(a, B\) and \(R\) suit well for all \(d \in D\), \(f \in F\), but for some operating conditions, which were not well presented in the dataset, there exists another set of coefficients \(\tilde{a}, \tilde{B}\) and \(\tilde{R}\) which represents dynamics of AFR more accurately. In fact, for almost all modes of engine operation there exist such coefficients locally working better than global ones \(a, B\) and \(R\).

Thus, the problem of the coefficients \(\tilde{a}, \tilde{B}\) and \(\tilde{R}\) identification with posterior update of the control can be posed. To solve the problem it is proposed to use the conventional identification algorithm [5] denoting
\[
y(m) = \alpha(m)^T \theta,
\]
where
\[
\theta = [\tilde{a}^T \, \tilde{b}_0^T \, \tilde{b}_p^T \, \tilde{r}_0^T \, \tilde{r}_p^T]^T
\]
and
\[
\alpha(m) = [y(m-1), \ldots, y(m-k) \ldots, f(m)^T \ldots, f(m-p)^T \ldots, u(m)^T \ldots, u(m-p)^T]^T
\]
is the vector of regressors. Then we obtain the following parameterization for the identification error
\[
e(\theta)(m) = y(m) - \hat{y}(m) = \alpha(m) [\theta - \hat{\theta}(m)],
\]
where \(\hat{\theta} = [\hat{a}^T \, \hat{b}_0^T \, \hat{b}_p^T \, \hat{r}_0^T \, \hat{r}_p^T]^T\) is the adjustable vector of estimates for \(\theta\) and \(\hat{y}(m)\) is the output of the adaptive observer:
\[
\hat{y}(m) = \sum_{i=1}^{p} \hat{u}_i(m) y(m-i) + \sum_{j=0}^{p} \tilde{r}_j^T(m) d(m-j) + \sum_{j=0}^{p} [\tilde{b}_j^T(m) f(m-j)] u(m-j).
\]

Note that the model (6) has a form similar to (1), however, for the coefficients \(\tilde{a}, \tilde{B}\) and \(\tilde{R}\) it is assumed that \(\tilde{v}(i) = 0, \, i \geq 0\) (the coefficients locally approximate the system dynamics exactly). The observer (7) also has the form (1) under \(\hat{\theta}\) substitution instead of \(\theta\).

To design the adaptation algorithm for \(\hat{\theta}\) let us choose the conventional quadratic error functional
\[
\mathcal{Q}_\theta(m) = 0.5 \epsilon(m)^2,
\]
whose minimization is ensured by the following gradient adaptation algorithm
\[
\hat{\theta}(m) = \hat{\theta}(m-1) + \gamma(m) \alpha(m-1)^T \epsilon(m-1),
\]
where \(\gamma(m) > 0\) is a design parameter. To specify conditions of the algorithm (8) applicability we impose the following restrictions on the engine dynamics.

**Assumption 3.** For all \(d \in D\), \(f \in F\) it holds:

- there exist series \(\tilde{a}_k, \tilde{B}_k\) and \(\tilde{R}_k\) and \(\Delta_k \geq 0\) such that the model (6) is valid with \(\tilde{a}_k, \tilde{B}_k\) and \(\tilde{R}_k\) for all \(k \leq i \leq k+1\) for all \(k \geq 0\) with \(j_{k+1} = j_k + \Delta_k\), \(j_0 = 0\);
- \(|\alpha(i)| \neq 0, \, i \geq 0\);
- for any \(i \geq 0\) there exist \(K \geq 0\) and \(0 < \rho < 1\) such that
\[
\prod_{j=i}^{i+K-1} P_j \leq \rho I, \quad P_j = I - |\alpha(j)|^2 \alpha(j)^T \alpha(j),
\]
where \(I\) is the identity matrix of corresponding dimension.

This assumption has three parts. First, it is assumed that the time range of the system operation can be decomposed on subintervals \(j_k \leq i \leq j_{k+1}\), \(k \geq 0\), where the model (6) is valid for some \(\tilde{a}_k, \tilde{B}_k\) and \(\tilde{R}_k\). Secondly, it is assumed that the regressor \(\alpha\) norm differs from zero (i.e. \(\gamma(i) \neq 0\) for all \(i \geq 0\)). The last part is a variant of persistency of excitation condition required for the convergence of the adjusted parameters to
\[
\theta_k = [\tilde{a}_k^T \tilde{b}_{0,k}^T \tilde{b}_{p,k}^T \tilde{r}_{0,k}^T \tilde{r}_{p,k}^T]^T.
\]

**Proposition 2.** Let assumption 3 hold, then observer (7) with adaptation algorithm (8) for
\[
\gamma(m) = |\alpha(m-1)|^2
\]
has the following estimate on the parameters identification error
\[
|\hat{\theta}_k(i)| \leq |\hat{\theta}_k(j_k)| \exp[\ln(\rho)(i-j_k) \mod K],
\]
under conditions of proposition 2 the parametric error \(\hat{\theta}_k(m)\) asymptotically converges to zero, then taking control.
\[ u(m) = \text{sat}(U(m)), \quad U(m) = \frac{1}{b_i^j f(m)} \times \]
\[ \sum_{i=1}^{5} y_i^j(m) - U_{PID}(m-1) - \sum_{i=1}^{k} a_i^j y(m-i) - \]
\[ - \sum_{j=0}^{p} r_j^f (d(m-j)) + \sum_{j=0}^{p} r_j^f (d(m-j)) u(m-j) \]  

it is possible to ensure the model (6) stabilization, where \( U_{PID}(m-1) \) is defined by (3).

**Proposition 3.** Under assumption 3 there exist constants \( k_j, j = 1, 5 \) such that for any solutions of the system (1), (7), (8), (10) with \( d \in D, f \in F \) for all \( j_k \leq i \leq j_{k+1} \), \( k \geq 0 \) (\( \beta_2 \in \mathcal{KL}, \gamma_2 \in \mathcal{K} \)):
\[ |e(i)| \leq \beta_2(|e(j_k)|, |i-j_k|) + \gamma_2(|\tilde{\theta}_k(j_k)|). \]

The advantage of the control (10) is that \( \tilde{\theta}_k(i) \to 0 \) with \( i \to \infty \) according to proposition 2, therefore, if the adaptive algorithm is active for sufficiently long time (constants \( \Delta_k \geq 0 \) from assumption 3 are large enough), then the adaptive control (7), (8) and (10) ensure exact regulation of AFR dynamics at the stoichiometric value.

Thus, in this section two control algorithms are proposed which ensure input-to-state stabilization of AFR dynamics. Both controls are based on the measurement information (the first one designed for the approximated off-line model, the second one for the on-line model). The issue of the improvement of the closed-loop system quality with the use of special switching between these laws is discussed in the following section.

**IV. THE SUPERVISOR**

Either of the proposed algorithms from section 3 possess its own advantages. The control algorithm designed off-line is rather reliable (it ensures stability for all operating modes of the engine) and robust (it is not sensitive to disturbances and unmodeled dynamics), but it may fail to ensure good accuracy over the entire range of operating conditions. The adaptive control has some transients after which it is tuned to compensate for all the errors at a particular engine operating condition. The switching algorithm executed in the supervisor has to combine the advantages of these controls neglecting their shortcomings and provides the closed loop control with an improved performance. For this purpose, note that the main difference between these controls consists in the models on which they are based. The following performance functionals evaluate the models accuracy on the last \( L \geq 0 \) steps:
\[ J_1(m) = L^1 \sum_{m=L}^{m=L} e_i^2(q), J_2(m) = L^1 \sum_{m=L}^{m=L} e_i^2(q), \]
\[ e_i(q) = y(q) - \sum_{i=1}^{5} a_i^j y(q-i) - \]
\[ - \sum_{j=0}^{p} b_j^f (d(q-j)) u(q-j) - \sum_{j=0}^{p} r_j^f (d(q-j)), \]
\[ e_i(q) = y(q) - \sum_{i=1}^{5} a_i^j y(q-i) - \]
\[ - \sum_{j=0}^{p} b_j^f (d(q-j)) u(q-j) - \sum_{j=0}^{p} r_j^f (d(q-j)). \]

In this case activation of the control with the most accurate model looks reasonable, that is the idea of supervision algorithm in this work, but the switching among nonlinear systems is not so trivial. Even if the systems are asymptotically stable or input-to-state stable as in our case, an inappropriate switching strategy may lead to instability [12].

The problem of switching among input-to-state systems has been addressed in the previous works [4], [9], [10]. The main idea there consists in the dwell-time mechanism application. Dwell-time constant restricts the rate of switching between the controls and for a sufficiently slow rate the stability of the closed loop system is guaranteed. For the rest of the section let \( u_1(m) \) be defined by (2) and \( u_2(m) \) be given as in (10).

**Theorem 1.** Let assumptions 1-3 hold and there exist dwell-time constant \( \tau_D > 0 \) such that \( \beta_1^2(r, \tau_D) \leq \lambda r \), \( 0 \leq r \) for some \( 0 < \lambda < 1 \). If \( s_{w+1} - s_w \geq \tau_D \), \( w \geq 0 \), where \( s_w \) is the instant of \( w \)th switch, then in the system (1) with control
\[ u(m) = u_{z(s_w)}(m), \quad z(s_w) \in \{1, 2\} \]

for any solutions the following estimate is satisfied:
\[ |e(i)| \leq \beta(|e(0)|, i) + \gamma(l)l_{(0,i)}, \quad \beta \in \mathcal{KL}, \quad \gamma \in \mathcal{K}, \]
\[ l(i) = \begin{cases} 1 & \text{if } i \geq s_w = 1; \\ \tilde{\theta}_k(s_w) & \text{if } i \geq s_w = 2. \end{cases} \]

In the case of theorem 1, switching of the controls results in changing of the disturbance. The dwell-time switching algorithm ensures boundedness of the system trajectories and the theorem presents worst-case estimate on the closed loop error behavior. Dwell-time switching algorithm still leaves room to further supervisor rule design focusing on the improvement of transient AFR behavior. Keeping in mind the performance functionals (11) and dwell-time rule from theorem 1, the following supervision algorithm is proposed:
\[ u(m) = u_{z(s_w)}(m), \quad z(s_w) \in \{1, 2\}, \quad w \geq 0, \]
\[ s_{w+1} = \begin{cases} \arg \inf_{m \geq s_w} \{ J_2(m) < J_1(m) \} & \text{if } s_w = 1; \\ \arg \inf_{m \geq s_w} \{ J_1(m) < J_2(m) \} & \text{if } s_w = 2, \end{cases} \]
\[ z(s_{w+1}) = 3 - z(s_w), \quad z(s_0) = 1, \quad s_0 = 0, \]
where \( s_w, w \geq 0 \) determines the time instant of the last switch; \( \tau_D > 0 \) is the dwell-time that prevents chattering (high frequency switching between the control algorithms), and from theorem 1 ensures the closed-loop stability. Since \( s_0 = 1 \) the system starts with model-based control (2), then after dwell-time if accuracy of the adaptation model (7) improves (\( J_2(m) < J_1(m) \)) the adaptive control (10) has to be
activated. If after dwell-time the accuracy of the model (1) again becomes better $(J_1(m) < J_2(m))$ the control (2) would be switched on.

V. APPLICATION RESULTS

The proposed switching control has been tested in two vehicles with V8 engines: one with 5.7ℓ engine and another with a 5.3ℓ engine. The schedule of testing was as follows.

At the first step based on the database of previous experiments the model (1) was derived for both vehicles (the coefficients $a$, $B$ and $R$ satisfy assumption 1). The results of model (1) tests are shown in Fig. 1.a, 1.c and Fig. 1.b, 1.d for 5.7ℓ and 5.3ℓ engines, respectively (the figures 1.c and 1.d demonstrate zoomed plots of the figures 1.a and 1.b). As we can deduce from these plots the quality of both models are of sufficient accuracy for the control (2) design.

At the second step the controls (2) and (10) are calculated. Assumption 2 is verified for the control (2) on the a priori collected dataset. Coefficients $k_j$, $j = 1, 5$ are assigned as zero initially, and after some experimentations they are tuned to some values providing acceptable performance. For the control (10) the same values of coefficients $k_j$, $j = 1, 5$ are chosen. The assumption 3 is taken to be valid (the first part and the last two parts can be verified on the dataset). After that, the system is ready for experimental testing.

Fig. 1. AFR model accuracy verification

The results of the tests are shown in figures 2 and 3 for the vehicles with the 5.7ℓ and 5.3ℓ engines, respectively. As we can conclude from these results the adaptive model provides better quality of approximation of AFR dynamics very frequently, but changes in modes of engines operation results in backward activation of the model (1) based controls. For both vehicles, experiments confirm applicability of the
proposed approach.

VI. CONCLUSION
Switching control algorithm for air-fuel ratio regulation is developed and practically tested for two vehicles. The controller contains three parts: robust model-based control, adaptive control and the supervisor. The first control is designed for approximated off-line using a priori available experimental dataset, the adaptive control is based on the adjusted (i.e. tuned) real-time model. Both models and controls have similar structure and the only difference is the type of information used for their design (off-line or on-line measurements). The supervisor provides switching between these controls taking into account the current accuracy of the models. If off-line approximated model has better quality, then the robust control is active. In situations when the adaptively adjusted model has better accuracy of AFR dynamics representation during some number of previous events, the adaptive control is switched on. Such an scheme allows the controller designer to combine the reliability of robust control (which was intensively tested and it ensures admissible quality of AFR regulation for all operating regimes) and the flexibility of the adaptive control (which can improve the performance due to the higher local accuracy of the AFR dynamics approximation). Practical implementation and intensive tests has demonstrated the applicability of the approach.

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