Iterative Learning Air-Fuel Ratio Control with Adaptation in Spark Ignition Engines

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Abstract—The paper addresses the problem of air-fuel ratio (AFR) stabilization applying learning techniques. The proposed strategy consists in iterative redesign of control coefficients based on the control obtained in a previous step in conjunction with an adaptive scheme. The interesting feature of the proposed solution is that the adaptive control attempts to match not the AFR dynamics but rather the error dynamics originated by the substitution of the feedforward control in the real system. Results of application are reported and discussed.

I. INTRODUCTION

A special class of engine control problems deals with reliable adaptation of on-board control systems to changes in the engine parameters, caused in normal usage and by engine deterioration. This class of problems also includes “vehicle-to-vehicle” control variations. Designed for a nominal vehicle in factory conditions, a controller may not provide the same performance for the same vehicle in the real environment due to some mismatches in the operation conditions. To compensate these deviations from the reference case, an adaptive or learning technique can be implemented in the on-board control systems. In this case, the main attention would be on safety and reliability of the adaptation algorithms as well as the quality of improvement in the learning process.

The field of air-fuel ratio regulation constitutes one of the main engine control problems and is originated by the growing ecological requirements on engine characteristics. The ecological cleanliness of engines is maintained by the three-way catalytic converter (TWC) which oxidizes HC and CO and reduces NOx species. However, efficiency of TWC is guaranteed if AFR is close to the stoichiometric value as the conversion efficiencies of TWC are significantly reduced away from the stoichiometry. This is why the primary objective of the AFR control system is to track the fuel injection in stoichiometric proportion to the ingested air flow. The problem of AFR regulation has attracted large attention during the last three few decades [5]. Adaptive control theory [3], [6], [14], [15], robust control approaches [4], fuzzy control systems theory [7] and neural network techniques [10], [16] are successfully tested for this application. However, the complexity of the problem and growing demands on AFR regulation quality require new and more sophisticated solutions.

In this work we solve the problem of AFR regulation applying learning approaches. There exist many papers devoted to the application of learning methods for AFR control systems design (see [1], [2], [8]–[13], [16] and references therein). Almost all of them are based on neural networks, which are used for feedforward part of control realization, with PID controls in feedback. The feedback controls improve the quality of feedforward part of control regulation with posterior retraining of the feedforward algorithm for the next iteration. In the present paper it is assumed that an initial feedforward control is provided and it is necessary to improve and adapt this control with respect to the current operating conditions. The control is plugged-in with the engine and, to ensure its adaptation to AFR dynamics fluctuation, an additional adaptive loop is designed. The adaptation should ensure against transient quality loss in new operating conditions. Model-based adaptive control approach is applied for this purpose. The new combined control is used as the reference for off-line learning and the calculation of feedforward control for the next iteration.

In section 2 detailed problem statement and some preliminaries are presented. Section 3 contains description of the control algorithms. Results of the system implementation are reported in section 4.

II. PROBLEM STATEMENT

Spark ignition engines are highly nonlinear multi-variable systems and the derivation of their precise models is a complex process. Moreover, even when detailed models are derived, the models may be not suitable for control design purposes. The reasons for this are the high dimensionality of an accurate model, its hybrid nature (it has to include continuous time operating blocks as well as discrete or event-based subsystems) and the intractability of its identification (especially onboard due to the time, memory and computational power restrictions). This is the reason why simplified models of SI engine are very popular in practice. These models can take into account the main
features of engine processes which are important for control
design or fault detection applications such as time delays
and nonlinearities. However, usually the simplified models
may guarantee acceptable accuracy of representation of
engine dynamics only locally (e.g. for some regimes or
modes of operation) and it is rather hard to design global
engine models (by global we mean a model covering all
operating conditions for a fixed set of coefficients of the
model).

In this work the following generic model is chosen for
AFR dynamics description (in this context AFR refers to the
non-dimensional engine-out air-fuel ratio sometimes known
as λ):

\[ y(m) = f(y(m-1), ..., y(m-k); u(m), ..., u(m-p); d(m)), \]

where \( y \in R \) is the regulated variable (in our case we take
fuel-to-air ratio as \( y \)), \( u \in [u_{\text{min}}, u_{\text{max}}] \) is the control input
(electrical fuel pulsewidth – FPW – in this work,
0 < \( u_{\text{min}} < u_{\text{max}} < +\infty \) are actuator constraints), \( d \in R^n \)
is the vectors of inputs (they contain physical engine variables
products at the current and past time instants), \( k \geq 1 \) and
\( p \geq k-1 \) are the model polynomials degrees, \( m \) is the
number of current event (discrete time);
\( f : R^k \times R^{p+1} \times R^n \rightarrow R \) is an unknown nonlinear function.

It is assumed that a dataset is given that contains the
measured information on \( y \), \( u \) and other engine variables
involved in the vector \( d \) for various regimes of engine
operation. The problem is to propose a procedure for
improvement of available in the dataset control
operation. The problem is to propose a procedure for
improvement of available in the dataset control
algorithm (4) and the engine (structure scheme for
experimental setup is shown in Fig. 1) the new dataset has to
be the control law approximated basing on the dataset
information. The low index \( ff \) signifies that this control is
feedforward and it does not contain any feedback errors, the
upper index defines the number of the current iteration, zero
states for the initial step. Note that by construction \( U_{ff}^0(i) \in [u_{\text{min}}, u_{\text{max}}], \ i \geq 0 \).

The scheme of the learning control system is presented in
Fig. 1, where feedforward control is given by (2) and
feedback control is typically realized as a nonlinear PID
control:

\[ U_{PID}(m) = k_1 e(m) + k_2 \sum_{i=0}^{m} e(i) + \]

\[ + k_3 [e(m) - e(m-1)] + k_4 \text{sign}(e(m)) + k_5 e(m)^3, \]

where \( e = y_d - y \) is the regulation error, \( k_j, \ j = 1,5 \) are
control parameters, which have to be determined based on
real or computer experiments. The feedback control (3)
improves the quality of regulation of the feedforward
control algorithm (2). Due to the presence of actuator
constraints, the final form of the control is

\[ u(m) = \text{sat}(U_{ff}^0(m) + U_{PID}(m)), \]

where the saturation function is used to ensure that
\( u(m) \in [u_{\text{min}}, u_{\text{max}}] \). Performing experiments for the
control algorithm (4) and the engine (structure scheme for
experimental setup is shown in Fig. 1) the new dataset has to
be saved. Then,

\[ U_{ff}^1(m) = U_{ff}^0(y_d(m); y(m), ..., y(m-k)); \]

\[ u(m-1), ..., u(m-p); d(m)) = \]

\[ = \text{sat}(U_{ff}^0(m) + U_{PID}(m)) + \varepsilon(m) \]

is the approximated feedforward control on the next step,
\( \varepsilon(m) \) is the approximation error, whose norm can be made
arbitrarily small by adjusting the chosen approximation
approach [1], [2], [10]–[13]. Next, the experiments should be
carried out again.

Repeating this strategy on step \( s > 0 \) we obtain the
formula for feedforward control

### III. MAIN RESULTS

The model (1) has a generic form and, as mentioned, the
optimal choice of its structure and values of parameters for
its identification is a rather complex process. An advantage
of learning approaches consists in the possibility of model-free
design, especially in the case when an initial control is
given (as it is assumed the dataset with a control is
provided).

#### A. Iterative learning

Let

\[ U_{ff}^0(m) = U_{ff}^0(y_d(m); y(m), ..., y(m-k)); \]

\[ u(m-1), ..., u(m-p); d(m)) \]

be the control law approximated basing on the dataset
information. The low index \( ff \) signifies that this control is
feedforward and it does not contain any feedback errors, the
upper index defines the number of the current iteration, zero
states for the initial step. Note that by construction
\( U_{ff}^0(i) \in [u_{\text{min}}, u_{\text{max}}], \ i \geq 0 \).
where all variables are from the dataset obtained in experiments for the control $U_{ff}$. On each step a performance functional $J_s$ is computed for the evaluation of the current control quality. For instance,

$$J_s = \alpha J_s^{\text{int}} + \beta J_s^{\text{max}},$$

$$J_s^{\text{int}} = M^{-1} \sum_{i=1}^{M} e(i)^2, \quad J_s^{\text{max}} = \max_{1 \leq i \leq M} \{ e(i) \},$$

where $\alpha > 0$, $\beta > 0$ are some weights and $M > 0$ is the number of stored measurements in the dataset. While $J_s \leq J_{s-1}$ the iterative learning procedure is repeated, the condition of the procedure termination is as follows:

$$(J_s > J_{s-1}) \lor (J_s \leq J_{\text{min}}),$$

where $0 < J_{\text{min}}$ is the goal value of the performance functional $J_s$, when $J_s$ is approached the control is assumed possessing required quality of AFR regulation.

As it is previously reported [1], [2], [10]–[13] such a scheme of learning control has rather good performance, acceptable quality and the ability to improve the initial control. However, there exist cases when the proposed scheme stops before the desired performance constraint $J_s \leq J_{\text{min}}$ is reached (i.e. in (6) we have $J_s > J_{s-1}$ for some $s > 0$). The reason for this is hidden in (3): PID control compensates the first order error dynamics with respect to the control (5). Due to the high order dynamics, PID feedback may not provide further improvement of AFR regulation for the approximated feedforward control (5). Therefore, a more complex feedback control algorithm has to be designed.

The idea of this work is to introduce adaptive control with adjustable nonlinear model to compensate dynamic mismatches of the learning control algorithm. The structure of this system is shown in Fig. 2. Adaptive control $u_a(m)$ is added to the feedforward and feedback controls, at the input it receives the regulated output $y(m)$, the reference $y_r(m)$ and the vector of engine variables $d(m)$.

![Fig. 2. The structure of the learning control system with adaptation](image)

### B. Adaptive control

To design the control $u_a$ it is assumed that input-output dynamics from the input $u_a$ to the output $y$ has form:

$$y(m) = \sum_{i=1}^{k} a_i y(m-i) + \sum_{j=0}^{p} b_j u_a(m-j) + \sum_{j=0}^{p} r_j^T d(m-j),$$

where $k \geq 1$ and $p \geq k - 1$ are the model polynomial degrees, $a = [a_1 \ldots a_k]^T \in \mathbb{R}^k$, $b = [b_0 \ldots b_p] \in \mathbb{R}^{p+1}$ and $R = [r_0 \ldots r_p] \in \mathbb{R}^{m(p+1)}$ are the model parameters, which are assumed unknown. This model describes the residual dynamics of the engine (1) closed by the learning control (4).

Since the values of parameters $a$, $b$ and $R$ are unknown the following adaptive observer is applied:

![Fig. 3. The scheme for the iterative learning with the adaptation algorithm](image)
\[
\hat{y}(m) = \sum_{i=1}^{k} \hat{a}_i(m) y(m-i) + \sum_{j=0}^{p} \hat{b}_j(m) u_a(m-j) + \sum_{j=0}^{p} \hat{r}_j T(m) d(m-j),
\]

where \( \hat{y}(m) \) serves as an estimate of \( y(m) \) and
\[
\mathbf{a} = [\hat{a}_1 \ldots \hat{a}_k] \in \mathbb{R}^k, \quad \mathbf{b} = [\hat{b}_0 \ldots \hat{b}_p] \in \mathbb{R}^{p+1} \quad \text{and} \quad \mathbf{R} = [\hat{r}_0 \ldots \hat{r}_p] \in \mathbb{R}^{p+1}
\]
are adjustable parameters of the observer (8). The model (7) can be rewritten as follows
\[
y(m) = \mathbf{a}(m)^T \theta,
\]
where \( \theta = [\mathbf{a}^T \mathbf{b}_0 \ldots \mathbf{b}_p \mathbf{r}_0^T \ldots \mathbf{r}_p^T]^T \) and
\[
\mathbf{a}(m) = [y(m-1) \ldots y(m-k) \ldots u_a(m-p) \ldots d(m-p) \ldots d(m-p)^T]
\]
is the vector of regressors. Then we obtain the following parameterization for the identification error
\[
\epsilon(m) = y(m) - \hat{y}(m) = \mathbf{a}(m) \theta - \hat{\theta}(m),
\]
where \( \theta = [\mathbf{a}^T \mathbf{b}_0 \ldots \mathbf{b}_p \mathbf{r}_0^T \ldots \mathbf{r}_p^T]^T \) is the adjustable vector of estimates for \( \theta \). To design adaptation algorithm for \( \hat{\theta} \) let us choose two error functionals, the first one is local
\[
Q_1(m) = 0.5 \epsilon(m)^2,
\]
and the second functional for some number of points \( N > 0 \):
\[
Q_2(m) = 0.5(N+1)^{-1} \sum_{j=0}^{N} \epsilon(m-j)^2.
\]
Minimization of the functional \( Q_1 \) is ensured by the gradient adaptation algorithm
\[
\hat{\theta}(m) = \hat{\theta}(m-1) + \gamma(m) \mathbf{a}(m-1)^T \epsilon(m-1),
\]
where \( \gamma(m) > 0 \) is a design parameter. To minimize the functional \( Q_2 \), the least-square algorithm is used:
\[
\hat{\theta}(m) = R(m)^{-1} S(m),
\]
\[
S(m) = \sum_{j=0}^{N} \mathbf{a}(m-j)^T y(m-j),
\]
\[
R(m) = \sum_{j=0}^{N} \mathbf{a}(m-j)^T \mathbf{a}(m-j).
\]
The algorithm (10) can be applied each \( N \) events improving the accuracy and the rate of convergence for the algorithm (9). Algorithm (10) plays the role of global adaptation algorithm, it adjusts vector \( \hat{\theta} \) based on not only local measured data at event \( m \) as in (9), but also on all measured information during the last \( N \) events. Choosing different lengths \( N \) of identified intervals it is possible to evaluate parameter changes originated by different time scale processes. The coefficients calculated by algorithm (10) serve as the new initial conditions for algorithm (9) for the next step. Of course, computations in algorithm (10) are performed if the matrix \( R(m) \) is non-singular (for any persistently exciting signal \( \omega \)). A drawback of the algorithm (10) consists in its computational complexity, thus, it should not be called too frequently.

A supervision algorithm has to be designed providing switching between the adaptation algorithms (9) and (10). Assume that \( Q_{\text{max}} > 0 \) is the maximum admissible value of functional \( Q_g \), exceeding which it is necessary to call the algorithm (10). The global identification algorithm has not to be called too frequently since, it may require significant amount of computations. Additionally, it helps waiting some time (say \( T_g > 0 \)) after the last call allowing the accumulation of new data matrices \( R(m) \) and \( S(m) \) after the previous retraining of the vector \( \hat{\theta}(m) \). In this case \( T_g \) is a minimal sample time for algorithm (10). It is minimal since the algorithm (10) is activated when \( Q_g(m) > Q_{\text{max}} \) and the previous call of this algorithm was performed more than \( T_g \) steps back in time. If these conditions are failed to satisfy, then the local algorithm (9) is used to adjust \( \hat{\theta}(m) \) at the time instant \( m \).

Thus, the formal definition of the hybrid adaptation algorithm is as follows:
\[
\hat{\theta}(m) = \begin{cases} 
\hat{\theta}(m-1) + \gamma(m) \epsilon(m-1) & \text{if } Q_g(m) \geq Q_{\text{max}} \wedge T_s - t \geq T_g, \\
\hat{\theta}(m) + \gamma(m) \epsilon(m-1) \mathbf{a}(m-1) & \text{otherwise,}
\end{cases}
\]
\[
S(m) = \sum_{j=0}^{N} \mathbf{a}(m-j)^T y(m-j),
\]
\[
R(m) = \sum_{j=0}^{N} \mathbf{a}(m-j)^T \mathbf{a}(m-j),
\]
where \( T_s \) is the time of the previous call of the adaptation algorithm (10).

For adjusted parameters \( \hat{\theta}(m) \) the control for the system (7) has the form:
\[
u_a(m) = \hat{b}_0^{-1}(m) \times \left[ y_d(m) - \sum_{i=1}^{k} \hat{a}_i(m) y(m-i) - \sum_{j=0}^{p} \hat{r}_j T(m) d(m-j) - \sum_{j=0}^{p} \hat{b}_j(m) u_a(m-j) \right].
\]
This control provides the following closed loop dynamics in the system (7) and (12) for the case \( \theta(m) = \hat{\theta} \):
\[
y(m) = y_d(m).
\]

Additional feedback may be implemented in (12) to ensure the robustness with respect to noise and adaptation errors. Projection algorithm has to be implemented in (12) to ensure that \( \hat{b}_0(i) \neq 0 \) for all \( i \geq 0 \).

Finally, the block of adaptive control in Fig. 2 contains the observer (8), the hybrid adaptation algorithm (11) and
the control (12). This block improves the quality of AFR 
regulation and aims at the condition $J_s \leq J_{\min}$ realization 
after some step in the iterative learning control. Let us 
consider the results of this approach for engine control 
application.

IV. THE APPLICATION

The proposed switching control has been tested in a 
vehicle with a 5.7 ℓ V8 engine. The experiments performed 
for AFR regulation without the adaptation loop (the system 
with the structure presented in Fig. 1) did not demonstrate 
improvements in the regulation. The iterative learning with 
the adaptation was applied with the schedule of testing as 
follows. In this scheme, FPW designates the fuel pulsewidth 
as the control variable and AFR represents the normalized 
air-fuel ratio as the regulated variable.

![AFR and FPW graphs](image)

Fig. 4. Trajectories on the last iteration (x-axis: number of 
engine events)

On the first step, a control $U_j^{0}$ was approximated based 
on the available data for a different FTP (Federal Test 
Procedure) cycle previously performed for the vehicle. 
Values of the coefficients $k_j$, $j = 1, 5$ were tuned to ensure 
the stability of the system presented in Fig. 1. Next, the 
adaptive control block was activated and parameters $\gamma$, $N$ 
and $T_\gamma$ were tuned to ensure the stability of the system and 
the best approximation of the residual dynamics by the 
observer (8).

<table>
<thead>
<tr>
<th>Step</th>
<th>$J_{\text{int}}, 10^{-2}$</th>
<th>$J_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.1613</td>
<td>0.2201</td>
</tr>
<tr>
<td>2</td>
<td>3.8502</td>
<td>0.1873</td>
</tr>
<tr>
<td>3</td>
<td>4.006</td>
<td>0.1896</td>
</tr>
</tbody>
</table>

Table 1. The values of the performance functionals

After tuning of all blocks in the scheme presented in Fig. 
2, the iterative learning procedure began as it is described in 
Fig. 3. On each iteration several FTP tests were performed 
for the feedforward control (5), the feedback control (3) and 
adaptive control (12), the results of the experiments were 
saved in the database and the performance functional $J_s$ 
was calculated (values of $J_s$ are presented in Table 1). The 
procedure stops after three steps. The corresponding 
trajectories on the last step are shown in Fig. 4.

V. CONCLUSION

In the present paper a new iterative learning control 
design is proposed for the problem of air-fuel ratio 
regulation. A special additional adaptive loop is designed 
which improves the quality of control used for the 
approximations on successive iterations. The results of 
implementation confirm applicability of the approach.

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