Quadratic Stabilizability for Polytopic Uncertain Continuous-Time Switched Linear Systems by Output Feedback

Takuya Soga and Naohisa Otsuka

Abstract—In this paper, we study a quadratic stabilizability problem via output-feedback for uncertain continuous-time switched linear systems whose subsystem’s matrices are represented as a polytopic linear combination of vertex matrices. At first, the conditions for polytopic uncertain continuous-time switched linear systems to be quadratically stabilizable via output-feedback are summarized. After that, sufficient conditions for the same switched linear systems to be quadratically stabilizable via output-feedback are given. Further, a numerical example is also investigated.

I. INTRODUCTION

The so-called switched system is composed of the family of subsystems with switching rule which concerns with various environmental factors and different controllers. The aspect of the switched system is found in various fields such as aircraft industry, mobile robot and animal world [4]. For such switched systems it is important to investigate the stability and stabilizability as fundamental problems. Until now many results on stability and stabilizability problems for various types of switched systems have been studied (e.g., [1], [3]-[7], [11], [14]-[16]). Further, many interesting results for various problems of switched systems have been written in some books (e.g., [8], [9], [13]).

On the other hand, from the practical viewpoint, it is important to investigate switched systems which contain uncertain parameters. Recently, Lin and Antsaklis[10], Zhai, Lin and Antsaklis[17] and Otsuka and Soga[12] investigated the stability and stabilizability problems for continuous-time and / or discrete-time uncertain switched linear systems. Especially, the paper[17] investigated the sufficient conditions for the polypotic uncertain switched linear system which is composed of two subsystems to be quadratically stabilizable via state feedback. After that, necessary and sufficient conditions for the same switched linear system to be quadratically stabilizable via state feedback were proved in the paper[12]. However, the same problem via output-feedback has not been investigated. In order to study the output feedback problem, it is necessary to consider the results of state feedback problem in general.

In this situation, we study quadratic stabilizability problems via state-feedback and output-feedback for uncertain continuous-time switched linear systems whose subsystem’s matrices are represented as a polytopic linear combination of vertex matrices. In Section 2 sufficient conditions for polytopic uncertain switched linear systems to be quadratically stabilizable via state-feedback are summarized.

In section 3 sufficient conditions for the same switched linear systems to be quadratically stabilizable via output-feedback are proved. The obtained results are extensions of the previous results of Feron[5] to a general number of polytopic uncertain subsystems case. Further, a numerical example is also investigated in Section 4. Finally, concluding remarks are given in Section 5.

II. QUADRATIC STABILIZABILITY VIA STATE FEEDBACK

Consider the following continuous-time switched linear system

$$\Sigma_\sigma : \dot{x}(t) = A_\sigma(x,t)x(t), \quad x(0) = x_0$$

where \(x(t) \in \mathbb{R}^n\) is the state vector, \(\sigma(x,t) : \mathbb{R}^n \times \mathbb{R}^+ \rightarrow \{1, 2, \cdots, N\}\) is a switched rule which depends on the state \(x\) and time \(t\), and \(\mathbb{R}^+\) is the set of non-negative real numbers. Then, the above switched system \(\Sigma_\sigma\) is composed of the family of continuous-time subsystems

$$\Sigma_i : \dot{x}(t) = A_ix(t), \quad i = 1, 2, \cdots, N,$$

where \(N \geq 2\) is the number of subsystems. The architecture of the switched system \(\Sigma_\sigma\) is shown as Fig 1.

![Fig. 1 Architecture of switched system \(\Sigma_\sigma\) (Image)](image)

In this section, we consider the case that all subsystems have polytopic uncertainties described as

$$A_i = \sum_{k_i=1}^{N_i} \mu_{i,k_i} A_{i,k_i}, \quad \sum_{k_i=1}^{N_i} \mu_{i,k_i} = 1, \quad (i = 1, 2, \cdots, N),$$

where \(A_{i,k_i} (k_i = 1, 2, \cdots, N_i)\) are the vertex matrices of the polytopic matrix \(A_i\), \(N_i\) is the number of the vertex matrices \(A_{i,k_i}\) and \(\mu_{i,k_i} (k_i = 1, 2, \cdots, N_i)\) are polytopic uncertain parameters.

T. Soga is with Graduate School of Science and Engineering, Tokyo Denki University, Hatayama-Machi, Hiki-Gun, Saitama 350-0394, Japan
N. Otsuka is with Division of Science, School of Science and Engineering, Tokyo Denki University, Hatayama-Machi, Hiki-Gun, Saitama 350-0394, Japan otsuka@mail.dendai.ac.jp
Now, we give the definition of quadratic stabilizability via state feedback for the switched system $\Sigma_\sigma$.

**Definition 2.1:** The switched linear system $\Sigma_\sigma$ is said to be quadratically stabilizable via state feedback if there exist a Lyapunov function of the form $V(x) = x^TPx$ where $P$ is a positive-definite matrix, a positive number $\epsilon (>0)$ and a switched rule $\sigma(x,t)$ such that
\[
\frac{d}{dt} V(x) < -\epsilon x^T x
\]
for all trajectory $x$ of the system $\Sigma_\sigma$.

When we investigate the quadratic stabilizability problem of switched systems, the following assumption is given. Because, if there is a stable subsystem in the family of subsystems, we can always activate the stable subsystem and therefore the problem becomes trivial one.

**Assumption 2.2:** Assume that all subsystems $\Sigma_i$ ($i = 1, 2, \cdots, N$) are asymptotically unstable, which imply there does not exist positive definite matrices $P_i > 0$ ($i = 1, 2, \cdots, N$) such that
\[
A_i^T P_i + P_i A_i < 0 \quad (k_i = 1, 2, \cdots, N_i).
\]

When the number of subsystems with certain matrices is two (i.e., $N = 2$), the sufficiency of the theorem shown by Feron[5] was extended to the polytopic uncertain switched linear systems by Zhai et al. [17]. Further it was stated that this result can be extended to more than two subsystems in [17], but it is not given as a theorem. Now, we give the following theorem without proof.

**Theorem 2.3:** If there exist $\lambda_{k_i} \in [0,1]$ ($k_i = 1, \cdots, N_i$; $i = 1, \cdots, N$), $P > 0$ and $\epsilon > 0$ such that
\[
\sum_{i=1}^{N} \lambda_{k_i} A_i^T P_i + P_i A_i < -\epsilon I,
\]

\[
\sum_{i=1}^{N} \lambda_{k_i} = 1, \quad k_i = 1, \cdots, N_i,
\]

then the switched system $\Sigma_\sigma$ is quadratically stabilizable via state feedback.

In Theorem 2.3, if we consider no uncertainties in the switched system $\Sigma_\sigma$, this theorem is reduced to the result which is given by [5],[14]. Therefore, Theorem 2.3 is an extension to the polytopic uncertain case.

Recently, it was shown that the sufficient conditions of Theorem 2.3 for $N = 2$ are also necessary ones by the present authors [12]. Therefore, the following Theorem 2.4 is an extension of the result of Feron[5] to polytopic uncertain case.

**Theorem 2.4:** [12] Suppose that $N = 2$ and Assumption 2.2 holds. Then, the polytopic uncertain switched linear system $\Sigma_\sigma$ with (1) is quadratically stabilizable via state feedback if and only if there exist $\lambda_{k,j} \in [0,1]$ ($k = 1, \cdots, N_1, j = 1, \cdots, N_2$) such that $\lambda_{k,j} A_{i,k} + (1 - \lambda_{k,j}) A_{i,j}$ are simultaneously asymptotic stable, that is, there exists a positive definite matrix $P > 0$ such that
\[
[\lambda_{k,j} A_{i,k} + (1 - \lambda_{k,j}) A_{i,j}]^T P + P [\lambda_{k,j} A_{i,k} + (1 - \lambda_{k,j}) A_{i,j}] < 0,
\]
equivalently
\[
\lambda_{k,j} (A_{i,k}^T P + PA_{i,k}) + (1 - \lambda_{k,j}) (A_{i,j}^T P + PA_{i,j}) < 0.
\]

### III. QUADRATIC STABILIZABILITY VIA OUTPUT FEEDBACK

Consider the following switched system
\[
\Sigma^o_\sigma : \begin{cases}
\dot{x}(t) = A_{\sigma(\tilde{x},t)} \tilde{x}(t), & x(0) = x_0, \\
y(t) = C \tilde{x}(t),
\end{cases}
\]
where $y(t) \in \mathbb{R}^m$ is the output, $\tilde{x}(t) \in \mathbb{R}^n$ is the state of the observer
\[
\frac{d}{dt} \tilde{x}(t) = A_{\sigma(\tilde{x},t)} \tilde{x}(t) + L \{ y(t) - C \tilde{x}(t) \},
\]
where $\sigma(\tilde{x}, t) : \mathbb{R}^n \times \mathbb{R}^+ \rightarrow \{1, 2, \cdots, N\}$ is a switched rule which depends on the state $\tilde{x}$ and time $t$, and $\mathbb{R}^+$ is the set of non-negative real numbers. Then, the above switched system $\Sigma^o_\sigma$ is composed of the family of continuous-time subsystems
\[
\Sigma_i : \dot{x}(t) = A_i x(t), \quad i = 1, 2, \cdots, N,
\]
where each $A_i$ has a polytopic uncertainties as (1) and $N \geq 2$ is the number of subsystems.

Now, consider the following closed-loop system $\hat{\Sigma}_\sigma$ composed of $\Sigma^o_\sigma$ and the observer (2) as
\[
\hat{\Sigma}_\sigma : \frac{d}{dt} \hat{x}(t) = \begin{bmatrix} A_{\sigma(\tilde{x},t)} & LC \\ 0 & A_{\sigma(\tilde{x},t)} - LC \end{bmatrix} \hat{x}(t),
\]
where $\hat{x}(t) := [\tilde{x}^T(t), (x(t) - \tilde{x}(t))^T]^T \in \mathbb{R}^{2n}$ is the extended state vector. The following figure shows the architecture of the above switched system $\hat{\Sigma}_\sigma$.  

![Architecture of switched system $\hat{\Sigma}_\sigma$](image)
The following definition gives quadratic stabilizability via output feedback for the switched system $\Sigma^g_y$.

**Definition 3.1:** The switched linear system $\Sigma^g_y$ is said to be quadratically stabilizable via output feedback if there exist a Lyapunov function of the form $V(x) = \tilde{x}^T \tilde{P} \tilde{x}$, where $\tilde{P}$ is a positive-definite matrix, a positive number $\epsilon(>0)$ and a switched rule $\sigma(\tilde{x}, t)$ such that
\[
\frac{d}{dt} V(x) < -\epsilon \tilde{x}^T \tilde{x}
\]
for all trajectory $\tilde{x}$ of the system $\tilde{\Sigma}_\sigma$. In this case, The states $x(t)$ and $\tilde{x}(t)$ of the extended switched linear system $\tilde{\Sigma}_\sigma$ converge to 0 as $t$ tends to $\infty$.

The following result is our main result.

**Theorem 3.2:** If the following two conditions (i) and (ii) are satisfied, then the switched linear system $\Sigma^g_y$ is quadratically stabilizable via output feedback by the switching rule
\[
\sigma(\tilde{x}, t) = \min_{1 \leq i \leq N} \{ i \mid 1 \leq i \leq N \}
\]
\[
\sigma(\tilde{x}, t) = \min_{1 \leq i \leq N} \{ i \mid \tilde{x}^T(t)(A_{i,k_i}^T P_1 + P_1 A_{i,k_i}) \tilde{x}(t) < -\epsilon \tilde{x}^T(t) \tilde{x}(t), k_i = 1, 2, \ldots, N_i \},
\]
in this case the states $x(t)$ and $\tilde{x}(t)$ converge to 0 as $t$ tends to $\infty$.

(i) There exist $\lambda_{k_i} \in [0, 1]$ ($k_i = 1, \ldots, N_i; \ i = 1, \ldots, N$), $P_1 > 0$ and $\epsilon > 0$ such that
\[
\sum_{i=1}^N \lambda_{k_i} T A_{i,k_i}^T P_1 + P_1 \sum_{i=1}^N \lambda_{k_i} A_{i,k_i} < -\epsilon I,
\]
(4)

(ii) There exist $P_2 > 0$ and $Y \in \mathbb{R}^{n \times m}$ such that
\[
\begin{cases}
A_{i,k_i}^T P_2 + P_2 A_{i,k_i} - C^T Y T - Y C < -\eta I, \\
Y k_i = 1, \ldots, N_i, \\
A_{i,k_i}^T P_2 + P_2 A_{i,k_i} - C^T Y T - Y C < -\eta I, \\
Y k_N = 1, \ldots, N_N
\end{cases}
\]

for some $\eta > 0$ with $L := P_2^{-1} Y$.

**Proof:** Suppose that statements (i) and (ii) are satisfied. Then, there exist $\lambda_{k_i} \in [0, 1]$ ($k_i = 1, \ldots, N_i; \ i = 1, \ldots, N$), $P_1 > 0$ and $\epsilon > 0$ such that (4) hold and there exist $P_2 > 0$ and $Y \in \mathbb{R}^{n \times m}$ such that (5) hold.

Consider the function $V(\tilde{x}(t)) = \tilde{x}^T(t) \tilde{P} \tilde{x}(t)$ for positive definite matrix
\[
\tilde{P} := \begin{bmatrix}
P_1 & 0 \\
0 & \alpha P_2
\end{bmatrix},
\]
where $\alpha$ is a positive number to be determined later.

Then, we have
\[
\frac{d}{dt} V(\tilde{x}(t)) = \frac{d}{dt} (\tilde{x}^T(t) \tilde{P} \tilde{x}(t) + \tilde{x}^T(t) \tilde{P} \frac{d}{dt} (\tilde{x}(t))
\]
\[
= [A_{\sigma(\tilde{x},t)} \tilde{x}(t) + L C(x(t) - \tilde{x}(t))]^T P_1 \tilde{x}(t) + \alpha \{ [A_{\sigma(\tilde{x},t)} - L C] \}^T P_2 \{ x(t) - \tilde{x}(t) \} + \tilde{x}^T(t) P_1 [A_{\sigma(\tilde{x},t)} \tilde{x}(t) + L C(x(t) - \tilde{x}(t))] + \alpha \{ x(t) - \tilde{x}(t) \}^T P_2 [A_{\sigma(\tilde{x},t)} - L C] \{ x(t) - \tilde{x}(t) \} = \tilde{x}^T(t) \{ A_{\sigma(\tilde{x},t)}^T P_1 + P_1 A_{\sigma(\tilde{x},t)} \} \tilde{x}(t) + 2 \tilde{x}^T(t) P_1 L C \{ x(t) - \tilde{x}(t) \} + \alpha \{ x(t) - \tilde{x}(t) \}^T \{ A_{\sigma(\tilde{x},t)} - L C \}^T P_2 + P_2 \{ A_{\sigma(\tilde{x},t)} - L C \} \{ x(t) - \tilde{x}(t) \} = \tilde{x}^T(t) \{ A_{\sigma(\tilde{x},t)}^T P_1 + P_1 A_{\sigma(\tilde{x},t)} \} \tilde{x}(t) + 2 \tilde{x}^T(t) P_1 L C \{ x(t) - \tilde{x}(t) \} + \alpha \{ x(t) - \tilde{x}(t) \}^T \{ A_{\sigma(\tilde{x},t)}^T P_2 + P_2 A_{\sigma(\tilde{x},t)} - C^T Y T - Y C \} \{ x(t) - \tilde{x}(t) \}. \]

Now, it follows from (4) that for all $\tilde{x}(t)$ the following inequality hold.
\[
\sum_{i=1}^N \lambda_{k_i} \tilde{x}^T(t) (A_{i,k_i}^T P_1 + P_1 A_{i,k_i}) \tilde{x}(t) < -\epsilon \tilde{x}^T(t) \tilde{x}(t), \ k_i = 1, \ldots, N_i,
\]
which imply that there exists an $i$ such that
\[
\tilde{x}^T(t) (A_{i,k_i}^T P_1 + P_1 A_{i,k_i}) \tilde{x}(t) < -\epsilon \tilde{x}^T(t) \tilde{x}(t), \ k_i = 1, \ldots, N_i.
\]

Here, if we consider the switching rule
\[
\sigma(\tilde{x}, t) = \min_{1 \leq i \leq N} \{ i \mid \tilde{x}^T(t) (A_{i,k_i}^T P_1 + P_1 A_{i,k_i}) \tilde{x}(t) < -\epsilon \tilde{x}^T(t) \tilde{x}(t), \ k_i = 1, 2, \ldots, N_i \},
\]
polytopic uncertainties (1), (5), (6) and (7), then we have
\[
\frac{d}{dt} V \{ \tilde{x}(t) \} = 
\sum_{k=1}^{N} \mu_{\sigma,k} \left[ \tilde{x}^T(t) (A_{\sigma,k}^T P_1 + P_1 A_{\sigma,k}) \tilde{x}(t) \right] 
+ 2 \tilde{x}^T(t) P_1 L \sum_{k=1}^{N} \mu_{\sigma,k} \tilde{x}(t) \right] 
+ \alpha \left( \tilde{x}(t) - \tilde{x}(t) \right)^T \left( \tilde{x}(t) - \tilde{x}(t) \right) \right) 
- \alpha \| \tilde{x}(t) \| \left( \tilde{x}(t) - \tilde{x}(t) \right) 
= -e \tilde{x}^T(t) \tilde{x}(t) + 2 \tilde{x}^T(t) P_1 L \left( \tilde{x}(t) - \tilde{x}(t) \right) 
- \alpha \| \tilde{x}(t) \| \left( \tilde{x}(t) - \tilde{x}(t) \right) 
= -e \tilde{x}^T(t) \tilde{x}(t) + 2 \tilde{x}^T(t) P_1 L \left( \tilde{x}(t) - \tilde{x}(t) \right) 
- \alpha \| \tilde{x}(t) \| \left( \tilde{x}(t) - \tilde{x}(t) \right).
\]

Now, choose a parameter \( \alpha \) satisfying \( \alpha > \frac{\| P_1 L \|^2}{\epsilon \eta} \) (\( \| H \| \) is the norm of matrix \( H \)) in (8). Then, we have
\[
\frac{d}{dt} V \{ \tilde{x}(t) \} < -e \tilde{x}^T(t) \tilde{x}(t) + 2 \tilde{x}^T(t) P_1 L \left( \tilde{x}(t) - \tilde{x}(t) \right) 
- \alpha \| \tilde{x}(t) \| \left( \tilde{x}(t) - \tilde{x}(t) \right) 
< -e \tilde{x}^T(t) \tilde{x}(t) + 2 \tilde{x}^T(t) P_1 L \left( \tilde{x}(t) - \tilde{x}(t) \right) 
- \alpha \| \tilde{x}(t) \| \left( \tilde{x}(t) - \tilde{x}(t) \right)
\]
\[
= -e \tilde{x}^T(t) \tilde{x}(t) + 2 \tilde{x}^T(t) P_1 L \left( \tilde{x}(t) - \tilde{x}(t) \right) 
- \alpha \| \tilde{x}(t) \| \left( \tilde{x}(t) - \tilde{x}(t) \right)
\]
\[
= - e \tilde{x}^T(t) \tilde{x}(t) + 2 \tilde{x}^T(t) P_1 L \left( \tilde{x}(t) - \tilde{x}(t) \right) 
- \alpha \| \tilde{x}(t) \| \left( \tilde{x}(t) - \tilde{x}(t) \right)
\]
\[
= - \left( \sqrt{e} \tilde{x}(t) - \frac{\| P_1 L \|}{\epsilon} \left( \tilde{x}(t) - \tilde{x}(t) \right) \right)^T \cdot \left( \sqrt{e} \tilde{x}(t) - \frac{\| P_1 L \|}{\epsilon} \left( \tilde{x}(t) - \tilde{x}(t) \right) \right)
< 0,
\]
which implies \( V \{ \tilde{x}(t) \} \) is a Lyapunov function for the extended switched system \( \Sigma_{\sigma} \). Hence, \( \tilde{x}(t) := \left[ \tilde{x}^T(t) \ (x(t) - \tilde{x}(t))^T \right]^T \to 0 \) as \( t \to \infty \), that is, \( x(t) \) and \( \tilde{x}(t) \to 0 \) as \( t \to \infty \). Thus, the switched system \( \Sigma_{\sigma} \) is quadratically stabilizable via output feedback.

\( \textbf{Corollary 3.3:} \) Suppose that the switched linear system \( \Sigma_{\sigma} \) has no uncertainties. If the following two conditions (i) and (ii) are satisfied, then the system \( \Sigma_{\sigma} \) is quadratically stabilizable via output feedback by the switching rule
\[
\sigma(\tilde{x}, t) = \min_{1 \leq i \leq N} \left\{ i \mid \tilde{x}^T(t) (A_i^T P_1 + P_1 A_i) \tilde{x}(t) < -\epsilon \tilde{x}^T(t) \tilde{x}(t) \right\},
\]
in this case the states \( x(t) \) and \( \tilde{x}(t) \) converge to 0 as \( t \) tends to \( \infty \).

(i) There exist \( \lambda_i \in [0, 1] \), \( P_i > 0 \) and \( \epsilon > 0 \) such that
\[
\sum_{i=1}^{N} \lambda_i A_i^T P_1 + P_1 \left( \sum_{i=1}^{N} \lambda_i A_i \right) < -\epsilon I, \quad \sum_{i=1}^{N} \lambda_i = 1,
\]
which implies switched system \( \Sigma_{\sigma} \) is quadratically stabilizable via state feedback.

(ii) There exist \( P_i > 0 \) and \( Y \in \mathbb{R}^{n \times m} \) such that
\[
A_i^T P_1 + P_2 A_i - C^T Y^T - Y C < -\eta I, \quad \lambda_i A_i^T P_2 + P_2 A_i - C^T Y^T - Y C < -\eta I,
\]
for some \( \eta > 0 \) with \( L := P_2^{-1} Y \).

\( \textbf{Remark 3.4:} \) It is noted that Corollary 3.3 reduced to the results of Feron[5] in the case of \( N = 2 \).

\( \textbf{IV. A Numerical Example} \)

Consider the following two-dimensional switched linear system :
\[
\Sigma_{\sigma} : \left\{ \begin{array}{l}
\dot{x}(t) = A_\sigma(\tilde{x}, t) x(t), \quad x(0) = x_0 \\
y(t) = C x(t),
\end{array} \right.
\]
where \( y(t) \in \mathbb{R} \) is the output, \( \tilde{x}(t) \in \mathbb{R}^2 \) is the state of the observer
\[
\frac{d}{dt} \tilde{x}(t) = A_\sigma(\tilde{x}, t) \tilde{x}(t) + L \{ y(t) - C \tilde{x}(t) \}.
\]
Here, system \( \Sigma_{\sigma} \) is composed of two subsystems and each subsystem’s matrices are represented as three vertex matrices.
\[
A_1 = \mu_{1,1} A_{1,1} + \mu_{1,2} A_{1,2} + \mu_{1,3} A_{1,3}, \quad \mu_{1,1} + \mu_{1,2} + \mu_{1,3} = 1,
A_2 = \mu_{2,1} A_{2,1} + \mu_{2,2} A_{2,2} + \mu_{2,3} A_{2,3}, \quad \mu_{2,1} + \mu_{2,2} + \mu_{2,3} = 1.
\]
where
\[
A_{1,1} = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}, \quad A_{1,2} = \begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}, \quad A_{1,3} = \begin{bmatrix} -1 & 2 \\ 3 & -1 \end{bmatrix},
\]
\[
A_{2,1} = \begin{bmatrix} -1 & -2 \\ -1 & -1 \end{bmatrix}, \quad A_{2,2} = \begin{bmatrix} -1 & -2 \\ -2 & -1 \end{bmatrix}, \quad A_{2,3} = \begin{bmatrix} -1 & -2 \\ -3 & -1 \end{bmatrix}.
\]

Then, it can be easily checked that all vertex matrices \(A_{i,k}\) are not asymptotically stable for \(i = 1, k_1 = 1, 2, 3(= N_1); i = 2, k_2 = 1, 2, 3(= N_2)\), that is, the above two subsystem’s matrices \(A_1\) and \(A_2\) are not asymptotically stable. Thus, Assumption 2.2 is satisfied.

Firstly, if we choose a positive definite matrix \(P_1 = I_2\) and the following parameters \(\lambda_{k_i}(k_i = 1, \cdots, N_i; i = 1, 2)\) as the following table

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and \(\epsilon = 1\), then the condition (i) of Theorem 3.2 (i.e., Theorem 2.3) is satisfied. Therefore, it follows that the switched linear system \(\Sigma^p\) is quadratically stabilizable via state feedback for polytopic uncertain parameters.

Secondly, consider the observer (10) consists of
\[
C = \begin{bmatrix} 4 & 0 \end{bmatrix} \text{ and } L := P_2^{-1}Y,
\]
where \(P_2 := I_2, Y := \begin{bmatrix} 3 \\ 0 \end{bmatrix} \).

Then, if we choose a parameter \(\eta = 1\), then the condition (ii) in Theorem 3.2 is satisfied.

Thus, this switched system \(\Sigma^p\) is quadratically stabilizable via output feedback for polytopic uncertain parameters.

In fact, for example, if we choose uncertain parameters \(\mu_{k_i} (i = 1, 2, 3; k_i = 1, \cdots, N_i)\) as
\[
A_1 = 0.1A_{1,1} + 0.3A_{1,2} + 0.6A_{1,3} = \begin{bmatrix} -1.0 & 2.0 \\ 2.5 & -1.0 \end{bmatrix},
\]
\[
A_2 = 0.2A_{2,1} + 0.4A_{2,2} + 0.4A_{2,3} = \begin{bmatrix} -1.0 & -2.0 \\ -2.2 & -1.0 \end{bmatrix},
\]
and initial states \(x_0 = [3 \\ 7]^T, \hat{x}_0 = [3 \\ 5]^T\), then we have
\[
\begin{cases}
\dot{x}_0^T(A_{1,1}^T P + PA_{1,1}) \hat{x}_0 = 22, \\
\dot{\hat{x}}_0^T(A_{1,2}^T P + PA_{1,2}) \hat{x}_0 = 52, \\
\dot{\hat{x}}_0^T(A_{1,3}^T P + PA_{1,3}) \hat{x}_0 = 82,
\end{cases}
\]
\[
\begin{cases}
\dot{x}_0^T(A_{2,1}^T P_1 + P_1A_{2,1}) \hat{x}_0 = -158, \\
< -\epsilon \hat{x}_0^T \hat{x}_0 = -45, \\
\dot{\hat{x}}_0^T(A_{2,2}^T P_1 + P_1A_{2,2}) \hat{x}_0 = -188, \\
< -\epsilon \hat{x}_0^T \hat{x}_0 = -45, \\
\dot{\hat{x}}_0^T(A_{2,3}^T P_1 + P_1A_{2,3}) \hat{x}_0 = -218, \\
< -\epsilon \hat{x}_0^T \hat{x}_0 = -45,
\end{cases}
\]

According to the switched rule \(\sigma(\hat{x}, t)\) in Theorem 3.2, let us choose \(\sigma(\hat{x}_0, 0) = 2\). Further, if we choose the subsystems according to the switched rule in the same way, then \(x(t)\) and \(\hat{x}(t)\) go to the origin as \(t\) tends to \(\infty\) (see Fig. 3) and euclidean norms of \(x(t), \hat{x}(t)\) converge to 0 as \(t\) tends to \(\infty\) (see Fig. 4).
V. CONCLUSION

In this paper, we investigated quadratic stabilizability problem via output-feedback for polytopic uncertain continuous-time switched linear system in the sense that subsystem’s matrices are represented as a polytope of vertex matrices. At first, the conditions for polytopic uncertain switched linear systems to be quadratically stabilizable via state-feedback were summarized. Next, sufficient conditions for the same switched linear systems to be quadratically stabilizable via output-feedback were proved. The obtained result (Theorem 3.2) is an extension of the result of Feron[5] to a general number of polytopic uncertain subsystems case. Further, a numerical example was also investigated.

REFERENCES


