Output Feedback Stabilization for Linear Impulsive Systems

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Abstract—In this paper, we address an output feedback stabilization problem for a class of linear impulsive systems that accommodate arbitrarily-spaced impulse times and possibly singular state transition matrices. By combining recent results for state feedback stabilization and state estimation, we show that a separation property holds and formulate an output feedback compensation scheme in which the feedback loop is closed between a discrete-time measurement and a continuous-time control input. Rather than directly adopting an observer-based structure involving the time-varying gains associated with the separate stabilization and estimation problems, we construct a purely discrete-time compensator followed by a memoryless generalized hold device that achieves closed-loop exponential stability.

I. INTRODUCTION

The separation property of observer-based output feedback compensation is ubiquitous in linear systems and control theory with recent extensions to various classes of hybrid systems such as sampled-data systems represented as impulsive systems [8], [9], [10], therein referred to as systems with jumps, and switched linear systems [2], [11], [12]. In this paper, we investigate an output feedback stabilization problem for a general class of linear impulsive systems. Our approach combines recent results for state feedback stabilization [6] and state estimation [5] in order to formulate a stabilizing output feedback compensator, thereby establishing that a separation property holds for this system class as well. The compensator naturally incorporates a static state feedback law with time-varying gain in which the state is replaced by an estimate produced by an impulsive observer. We also present an alternative compensator structure that consists of a purely discrete-time compensator followed by a memoryless generalized hold device.

The linear impulsive systems we consider are described by

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) & t \in \mathbb{R} \setminus T \\
x(\tau_k) &= x_k & \tau_k \in T \\
y[k] &= Cx(\tau_k)
\end{align*}
\]

(1)

where \(x(t)\) is the continuous-time state that undergoes instantaneous changes at the impulse times, \(u(t)\) is a continuous-time input, and \(y[k]\) is a discrete-time measurement. The state space for (1) is denoted by \(\mathcal{X}\).

In addition, \(T\) is a countably infinite set of strictly increasing impulse times assumed to contain a finite number of elements on any finite time interval. We further assume that

**Assumption 1.1:** For a countably infinite impulse time set \(T\), the spacing between consecutive impulse times \(\delta_k = \tau_{k+1} - \tau_k\) satisfies

\[
\delta := \inf \delta_k > 0 \quad \text{and} \quad \bar{\delta} := \sup \delta_k < \infty.
\]

Given an initial time \(t_0\) and final time \(t_f \geq t_0\), if \(T \cap (t_0, t_f) = \emptyset\) then the state transition matrix for (1) is simply

\[
\Phi(t_f, t_0) = e^{A(t_f - t_0)}
\]

Otherwise, we denote the non-empty subset of impulse times \(T \cap (t_0, t_f)\) by \(\{\tau_1, \tau_2, \ldots, \tau_k\}\). Then, to simplify certain expressions in the sequel, we let \(\tau_0 = t_0\) and \(\tau_{k+1} = t_f\) in terms of which we define \(\delta_j = \tau_{j+1} - \tau_j\) for \(j = 0, \ldots, k\). We adopt this notational convention throughout the remainder of the paper unless explicitly stated otherwise.

The state transition matrix for (1) is then given by

\[
\Phi(t_f, t_0) = e^{A_{\tau} \delta_0} + e^{A_{\tau} \delta_{k-1}} + \cdots + e^{A_{\tau} \delta_1} A_{\tau} e^{A_{\tau} \delta_0}
\]

(2)

We call the impulsive state equation (1) reversible if the state transition matrix (2) is invertible for all \(t_f \geq t_0\). Clearly, nonsingularity of \(A_{\tau}\) is necessary and sufficient for reversibility of (1).

II. STRONG REACHABILITY AND STATE FEEDBACK STABILIZATION

In this section, we review the discussion in [6] that focuses on uniform exponential stabilization via state feedback control laws

\[
u(t) = K_{\mathcal{C}}(t)x(t)
\]

(3)

for a bounded piecewise-continuous feedback gain matrix \(K_{\mathcal{C}}(t)\). The feedback gain construction presented in [6] involves reachability properties of (1) which we briefly summarize. First, the set of states that are reachable from the origin when the initial time, final time, and impulse times are fixed is given by

\[
\mathcal{R}_{\text{fixed}}(t_0, t_f, T) = \{x_f \in \mathcal{X} \mid \exists \text{ p.c. } u(\cdot) \text{ s.t. } x(t_0) = 0 \text{ and } x(t_f) = x_f\}
\]

In terms of this, we define

**Definition 2.1:** (Strongly Reachable System) The linear impulsive system (1) is strongly reachable if there exists a positive integer \(\ell\) such that for all impulse time sets \(T\) satisfying Assumption 1.1, \(\mathcal{R}_{\text{fixed}}(t_0, t_f, T) = \mathcal{X}\) for any finite interval \((t_0, t_f)\) containing at least \(\ell\) impulse times in \(T\).
The reachability analysis conducted in [3] establishes that the reachable set $R_{\text{fixed}}(t_0, t_f, T)$ is contained in a certain invariant subspace for (1). Consequently, having this invariant subspace coincide with the entire state space is necessary for strong reachability. Reversibility of (1) makes this condition almost sufficient in the sense that the reachability property described in Definition 2.1 holds for almost all impulse time sets satisfying Assumption 1.1. The characterization of pathological impulsive time sets analogous to pathological sampling periods in sampled-data control is currently under investigation. On the other hand, reversibility is not necessary for strong reachability and it is shown in [4] that the reversibility requirement can be weakened to a feedback-reversibility condition that can be characterized in geometric terms.

**Definition 2.2:** For the impulsive system (1), the weighted reachability gramian $W_\alpha(t_0, t_f)$ is defined as

$$W_\alpha(t_0, t_f) = \int_{t_0}^{t_f} e^{2\alpha(t_3 - \tau)} \Phi(t_3, \tau)B_CB_C^T(\Phi(t_3, \tau)) d\tau$$

where $\alpha$ is a finite nonnegative constant.

The weighted reachability gramian of a strongly reachable linear impulsive system (1) with associated positive integer $\ell$ is such that $W_\alpha(\tau, \tau_{\ell+1})$ is uniformly positive definite and bounded with respect to all $T$ satisfying Assumption 1.1 and all $t_0 \in T$. In terms of this, the bounded piecewise-continuous feedback gain matrix defined by

$$K_c(t) = -e^{2\alpha(\tau + \ell + 2 - t)}B_C^T \times \Phi^T(\tau_{k+\ell+2}, t)W_\alpha^{-1}(t, \tau_{k+\ell+2})\Phi(\tau_{k+\ell+2}, t)$$

for $t \in [\tau_k, \tau_{k+1})$ is shown in [6] to achieve uniform exponential stability for the closed-loop impulsive state equation

$$\dot{x}(t) = \hat{A}_c(t)x(t), \quad t \in \mathbb{R} \setminus T$$

$$x(\tau_k) = A_Ix(\tau_k^-) \quad \tau_k \in T$$

where

$$\hat{A}_c(t) = A_c + B_cK_c(t)$$

### III. STRONG OBSERVABILITY AND STATE ESTIMATION

Here we briefly review several key points of the analysis in [5]. For the linear impulsive system (1), impulsive observers of the form

$$\dot{x}(t) = A_c\hat{x}(t) + B_cu(t), \quad t \in \mathbb{R} \setminus T$$

$$\hat{x}(\tau_k) = A_I\hat{x}(\tau_k^-) + L_I[k](C_I\hat{x}(\tau_k^-) - y[k]) \quad \tau_k \in T$$

are considered in which an observer gain $L_I[k]$ is sought that yields uniformly exponentially stable error dynamics

$$\dot{x}(t) = A_c\hat{x}(t), \quad t \in \mathbb{R} \setminus T$$

$$\hat{x}(\tau_k) = (A_I + L_I[k]C_I)\hat{x}(\tau_k^-) \quad \tau_k \in T$$

The observer gain construction derived in [5] relies on the following observability properties of (1). In terms of the set of unobservable states on a finite interval with fixed impulse times given as

$$Q_{\text{fixed}}(t_0, t_f, T) = \{ x_0 \in \mathcal{X} \mid C_I\Phi(\tau_k^-, t_0)x_0 = 0 \text{ for all } \tau_k \in T \cap (t_0, t_f) \}$$

we define

**Definition 3.1:** (Strongly Observable System) The linear impulsive system (1) is strongly observable if there exists a positive integer $\ell$ such that for all impulse time sets $T$ satisfying Assumption 1.1, $Q_{\text{fixed}}(t_0, t_f, T) = 0$ for any finite interval $(t_0, t_f)$ containing at least $\ell$ impulse times in $T$.

It follows from the observability analysis in [3] that the unobservable set $Q_{\text{fixed}}(t_0, t_f, T)$ contains a certain invariant subspace for (1). Thus, having this invariant subspace equal the zero subspace is necessary for strong observability. By duality, reversibility of (1) makes this condition almost sufficient for strong observability. However, reversibility is not necessary for strong observability and the results in [4] can be dualized to derive geometric conditions that are necessary and almost sufficient for strong observability.

**Definition 3.2:** For the impulsive system (1), the weighted observability gramian $M_\beta(t_0, t_f)$ is defined by

$$M_\beta(t_0, t_f) = \sum_{j=1}^{k} \beta^{2(j-k)}\Phi(\tau_j^-, t_0)C_I^TC_I\Phi(\tau_j^-, t_0)$$

for a finite constant $\beta > 1$.

It is shown in [5] that a strongly observable linear impulsive system (1) is such that, for any impulse time set $T$ satisfying Assumption 1.1, the weighted observability gramian $M_\beta(t_0, t_f)$ is uniformly positive definite and bounded on any interval $(t_0, t_f)$ containing at least $\ell$ impulse times in $T$ for any finite $\beta > 1$. This, in turn, leads to the construction of the bounded observer gain

$$L_I[k] = -\beta^{2\alpha}\Phi(\tau_k, \tau_{k-\ell})M_\beta^{-1}(\tau_k, \tau_{k-\ell})C_I^TC_I\Phi(\tau_k^-, t_0)$$

for which the impulsive observer (7) yields uniformly exponentially stable impulse error dynamics (8). We remark that the formula (10) reflects a correction to the corresponding expression in [5] in which $\ell$ has been replaced by $\ell + 1$ in the above.

### IV. OUTPUT FEEDBACK STABILIZATION

For a linear impulsive system that is both strongly reachable and strongly observable, it is straightforward to show by adapting standard results for time-varying linear systems (7) that the observed-based compensator

$$\dot{x}(t) = (A_c + B_cK_c(t))\hat{x}(t), \quad t \in \mathbb{R} \setminus T$$

$$\hat{x}(\tau_k) = (A_I + L_I[k]C_I)\hat{x}(\tau_k^-) \quad \tau_k \in T$$

achieves uniform exponential stability for the associated closed-loop impulsive system. Here we pursue an alternate compensator construction that does not directly involve the
time-varying feedback gain \( K_C(t) \) and instead features a purely discrete-time compensator followed by a memoryless generalized hold device.

As an intermediate step, we show that the state feedback law presented in Section II admits the sampled-state implementation

\[
\dot{\lambda}(t) = -(2\alpha I + A_C^T(t))\lambda(t) \quad t \in \mathbb{R} \setminus \mathcal{T}
\]

\[
\lambda(\tau_k) = A_I x(\tau_k^-) \quad \tau_k \in \mathcal{T}
\]

\[
u(t) = -B_C^T \lambda(t)
\]

in which

\[
\Lambda(t) := e^{2\alpha(\tau_k + \tau_{k+2} - t)}\Phi^T(\tau_k + \tau_{k+2}, t)W_\alpha^{-1}(t, \tau_k + \tau_{k+2}) \times \Phi(\tau_k + \tau_{k+2}, t)
\]

for \( t \in [\tau_k, \tau_{k+1}] \) so that \( K_C(t) = -B_C^T \Lambda(t) \). We observe that \( \Lambda(t) \) is symmetric and uniformly bounded as a consequence of Assumption 1.1 and strong reachability of (1). The continuous-time component of (12) can be implemented as a memoryless time-varying gain that generates the continuous-time control signal \( u(t) \) via

\[
u(t) = H(t) \lambda(\tau_k)
\]

where

\[
H(t) = -B_C^T e^{-(2\alpha t + A_C^T(t - \tau_k))} \quad t \in [\tau_k, \tau_{k+1}]
\]

and thereby acts as a generalized hold device [1]. The gain (15) can be pre-computed off-line for \( t \in [0, T] \). The impulsive part of (12) can be interpreted as the time-varying discrete-time feedback law whose output \( \lambda(\tau_k) \) drives the generalized hold.

We next show that the closed-loop impulsive state equation resulting from the interconnection of (1) and (12) given by

\[
\begin{bmatrix}
\dot{x}(t) \\
\dot{\lambda}(t)
\end{bmatrix} =
\begin{bmatrix}
A_C & -B_C B_C^T \\
0 & -(2\alpha I + A_C^T)
\end{bmatrix}
\begin{bmatrix}
x(t) \\
\lambda(t)
\end{bmatrix} \quad t \in \mathbb{R} \setminus \mathcal{T}
\]

\[
\begin{bmatrix}
x(\tau_k) \\
\lambda(\tau_k)
\end{bmatrix} =
\begin{bmatrix}
A_I \\
\Lambda(\tau_k) A_I
\end{bmatrix}
\begin{bmatrix}
x(\tau_k^-) \\
\lambda(\tau_k^-)
\end{bmatrix} \quad \tau_k \in \mathcal{T}
\]

is uniformly exponentially stable.

**Theorem 4.1:** For a strongly reachable linear impulsive state equation (1) with impulse time set that satisfies Assumption 1.1 and finite constant \( \alpha > 0 \), the sampled-state feedback law (12) yields a uniformly exponentially stable closed-loop impulsive state equation (16).

**Proof.** We begin with a change of variables given by

\[
\eta(t) := \lambda(t) - \Lambda(t) x(t)
\]

so that

\[
\begin{bmatrix}
x(t) \\
\eta(t)
\end{bmatrix} =
\begin{bmatrix}
-I & 0 \\
-\Lambda(t) & I
\end{bmatrix}
\begin{bmatrix}
x(t) \\
\lambda(t)
\end{bmatrix}
\]

in which the transformation matrix is uniformly bounded with uniformly bounded inverse. Therefore, (17) defines a stability preserving coordinate transformation.

A straightforward, though tedious, calculation shows that

\[
\dot{\eta}(t) = -(2\alpha I + \dot{A}_C^T(t))\eta(t) \quad t \in \mathbb{R} \setminus \mathcal{T}
\]

\[
\eta(\tau_k) = 0 \quad \tau_k \in \mathcal{T}
\]

which leads to the transformed closed-loop impulsive state equation

\[
\begin{bmatrix}
\dot{x}(t) \\
\dot{\eta}(t)
\end{bmatrix} =
\begin{bmatrix}
\dot{A}_C(t) & -B_C B_C^T \\
0 & -(2\alpha I + A_C^T(t))
\end{bmatrix}
\begin{bmatrix}
x(t) \\
\eta(t)
\end{bmatrix} \quad t \in \mathbb{R} \setminus \mathcal{T}
\]

\[
\begin{bmatrix}
x(\tau_k) \\
\eta(\tau_k)
\end{bmatrix} =
\begin{bmatrix}
A_I \\
\Lambda(\tau_k) A_I
\end{bmatrix}
\begin{bmatrix}
x(\tau_k^-) \\
\lambda(\tau_k^-)
\end{bmatrix} \quad \tau_k \in \mathcal{T}
\]

To show that (19) is uniformly exponentially stable, uniform exponential stability of (6) yields the bound

\[
||\hat{\Phi}(t, t_0)|| \leq \hat{\gamma} e^{-\hat{\alpha}(t-t_0)} \quad t \geq t_0
\]

for finite positive constants \( \hat{\gamma}, \hat{\alpha} \). Next, for any initial time \( t_0 \), let \( \tau_1 \) denote the first impulse time following \( t_0 \). For any \( \eta(t_0) \), \( \eta(t) \) evolves on \([t_0, \tau_1]\) according to \( \eta(t) = e^{2\alpha(t-t_0)}\hat{\Phi}^T(t_0, t)\eta(t_0) \) in which \( \hat{\Phi}(. , .) \) is the state transition matrix for the impulsive state equation (6) which reduces to the continuous-time state transition matrix for \( \dot{A}_C(t) \) on \([t_0, \tau_1]\) and is therefore invertible at each \( t \in [t_0, \tau_1] \). Since \( t - t_0 \leq \delta \), there is a finite positive constant \( \delta \) such that \( ||\eta(t)|| \leq k ||\eta(t_0)|| \) for \( t \in [t_0, \tau_1] \). Then from \( \eta(\tau_1) = 0 \), we have \( \eta(t) = 0 \) for \( t \geq \tau_1 \) and so (18) is uniformly exponentially stable with arbitrary rate of decay. In particular, \( ||\eta(t)|| \leq ke^{\alpha \delta} e^{-\hat{\alpha}(t-t_0)} ||\eta(t_0)|| \) for all \( t \geq t_0 \).

From

\[
x(t) = \hat{\Phi}(t, t_0)x(t_0) - \int_{t_0}^{t} \hat{\Phi}(t, \tau) B_C B_C^T \eta(\tau) \ d\tau
\]

standard manipulations, with \( b := ||B_C B_C^T|| \), yield

\[
||x(t)|| \leq \hat{\gamma} e^{-\hat{\alpha}(t-t_0)} ||x(t_0)|| + \int_{t_0}^{t} \hat{\gamma} e^{-\hat{\alpha}(t-\tau)} b ||\eta(\tau)|| \ d\tau
\]

\[
\leq \hat{\gamma} e^{-\hat{\alpha}(t-t_0)} \left( ||x(t_0)|| + \frac{b}{\alpha} (e^{\alpha \delta} - 1) ||\eta(t_0)|| \right)
\]

from which, setting \( \hat{\gamma} = \max \{\hat{\gamma}, ke^{\alpha \delta} (1 + \frac{b}{\alpha} (1 - e^{-\alpha \delta}))\} \),

\[
||x(t)|| \quad \eta(t) \quad t \geq |t-x(t)||+||\eta(t)||
\]

\[
\leq \sqrt{2} \hat{\gamma} e^{-\hat{\alpha}(t-t_0)} ||x(t_0)|| \quad \eta(t_0)
\]

thereby establishing that (19) is uniformly exponentially stable, implying the same for (16).

We now formulate a dynamic output feedback compensator by replacing the sampled-state measurement in (12) with an estimate produced by the impulsive observer (7).
This yields
\[
\begin{bmatrix}
\dot{\lambda}(t) \\
\dot{\hat{x}}(t)
\end{bmatrix} =
\begin{bmatrix}
-(2\alpha I + A_T^T \Lambda) & 0 \\
0 & -(2\alpha I + A_T^T \Lambda)
\end{bmatrix}
\begin{bmatrix}
\lambda(t) \\
\hat{x}(t)
\end{bmatrix},
t \in \mathbb{R} \setminus T
\]
\[
\lambda(\tau_k) = 0.1 \pi > 0 \text{ s}.
\]
Towards realizing the sampled-state feedback law (12), direct computations for \(\alpha = 0.25\) yield
\[
\dot{x}(\tau_k+1) = \Phi[k] \hat{x}(\tau_k) + \Gamma[k] y[k] \\
\lambda(\tau_k) = \Delta[k] \hat{x}(\tau_k)
\]
in which
\[
\Phi[k] = e^{A_T(k+1) - \tau_k} (A_T + L_T[k] C_T) \\
- \int_{\tau_k}^{\tau_{k+1}} e^{A_T(\tau_k - \tau)} B_C C_T e^{(2\alpha I + A_T^T \Lambda) \tau} d\tau \Lambda(\tau_k) A_T \\
\Gamma[k] = -e^{A_T(k+1) - \tau_k} L_T[k] \\
\Delta[k] = \Lambda(\tau_k) A_T
\]
This, viewed as a time-varying discrete-time compensator, followed by the generalized hold device (14), (15) specifies an output feedback compensator that achieves closed-loop uniform exponential stability.

\begin{theorem}
For a strongly reachable and strongly observable linear impulsive state equation (1) with impulse time set that satisfies Assumption 1.1 and finite constants \(\alpha > 0\) and \(\beta > 1\), the impulsive compensator (20) yields a uniformly exponentially stable closed-loop impulsive state equation (21).
\end{theorem}

\begin{proof}
We now adopt the stability preserving change of variables
\[
\eta(t) := \lambda(t) - \Lambda(t) \hat{x}(t) \quad \hat{x}(t) := x(t) - \hat{x}(t)
\]
which yields the transformed closed-loop impulsive state equation
\[
\begin{bmatrix}
\dot{x}(t) \\
\dot{\hat{x}}(t) \\
\dot{\eta}(t)
\end{bmatrix} =
\begin{bmatrix}
A_C & -B_C B_C^T & -B_C K_C(t) \\
0 & -(2\alpha I + A_T^T \Lambda(t)) & 0 \\
0 & 0 & A_C
\end{bmatrix}
\begin{bmatrix}
x(t) \\
\hat{x}(t) \\
\eta(t)
\end{bmatrix},
t \in \mathbb{R} \setminus T
\]
\[
\begin{bmatrix}
x(\tau_k) \\
\eta(\tau_k) \\
\hat{x}(\tau_k)
\end{bmatrix} =
\begin{bmatrix}
A_T & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & A_T + L_T[k] C_T
\end{bmatrix}
\begin{bmatrix}
x(\tau_k^-) \\
\eta(\tau_k^-) \\
\hat{x}(\tau_k^-)
\end{bmatrix},
\tau_k \in T
\]
The \((x, \eta)\)-dynamics are uniformly exponentially stable as a consequence of Theorem 4.1 as are the impulsive observer error dynamics. The cascade interconnection of these two uniformly exponentially stable subsystems with bounded coupling is uniformly exponentially stable and, therefore, so is (21).
\end{proof}

The impulsive compensator (20) can be reformulated as a discrete-time compensator followed by the previously-specified generalized hold (14), (15) as follows. On the interval \([\tau_k, \tau_{k+1})\), the control signal \(u(t)\) is generated from \(\lambda(\tau_k)\) which, in turn, depends on the observer state value \(\hat{x}(\tau_k^-)\). Thus, the feedback law only requires the observer state at the impulse times and the impulse observer can therefore be discretized according to
\[
\hat{x}(\tau_{k+1}) = \Phi[k] \hat{x}(\tau_k) + \Gamma[k] y[k]
\]
\[
\lambda(\tau_k) = \Delta[k] \hat{x}(\tau_k)
\]

V. Example

We consider the linear impulsive state equation (1) specified by
\[
A_C =
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix} \quad B_C =
\begin{bmatrix}
1 \\
0 \\
0 \\
0
\end{bmatrix}
\]
\[
A_T =
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 2 & 0 \\
2 & 0 & 0 & 0 \\
0 & 2 & 0 & 0
\end{bmatrix} \quad C_T =
\begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
which is shown in [6] to be strongly reachable with \(\ell = 3\) provided that impulse time sets satisfy Assumption 1.1 for \(\delta < \frac{\pi}{\beta}\) s.

A non-uniformly spaced impulse time set was created by setting \(t_0 = \tau_0 = 0\) and \(\tau_{k+1} = \tau_k + \delta_k\) with
\[
\delta_k = h \left[ 1 + a \sin \left( \frac{\pi}{N} k \right) \right] \quad k \geq 0
\]
for parameter values \(h = \frac{\pi}{4}\) s, \(a = 0.9\), and \(N = 4\). We observe that for these parameter values we may take \(\delta = h(1 + a) = 1.9 \frac{\pi}{4} < \frac{\pi}{\beta}\) s and \(\tau = h(1 - a) = 0.1 \frac{\pi}{4} > 0\) s.

Towards realizing the sampled-state feedback law (12), direct computations for \(\alpha = 0.25\) yield
The state variable responses shown in Figures 1 through 4 indicate exponentially stable behavior for both feedback laws although we observe in Figure 5 that in both cases $u(t) = 0$ for $t \in [0, \tau_1)$ ($\tau_1 = \frac{T}{2}$ s) so that the response is initially governed by the unforced continuous-time dynamics in (1). The slight difference between the responses for sampled-state feedback and output feedback can be attributed to the transient introduced by the exponentially stable observer error response which has nearly died out by $t = \tau_0 = 5.42$ s.

**VI. Concluding Remarks**

This paper has considered an output feedback stabilization problem for a class of linear impulsive systems. Previous results for state feedback stabilization and asymptotic state estimation are brought to bear on the formulation of a dynamic output feedback compensator. Rather than directly adopting an observer-based structure involving the time-varying gains associated with the separate stabilization problems, a compensator structure has been presented that consists of a discrete-time compensator followed by a memoryless generalized hold device. The advantage of this approach is that the time-varying continuous-time gain associated with the state feedback stabilization problem need not be implemented.
The compensator construction presented herein applies to linear impulsive systems that are both strongly reachable and strongly observable. Of interest are appropriate notions of stabilizability and detectability for linear impulsive systems that are less restrictive than strong reachability and strong observability, respectively, and together are still sufficient for output stabilizability. This is the subject of ongoing investigation.

REFERENCES