High Gain Observer Based Nonlinear Position Control for Electro-Hydraulic Servo Systems

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Abstract—We present a high gain observer-based nonlinear position control for electro-hydraulic servo systems. We design passivity based control for improve tracking performance. The passivity based control needs the full states information. Since the electro-hydraulic servo system has nonlinearities, we designed a modified high gain observer to estimate its states without the transformation for normal form. To reduce the effect of nonlinear terms, we use a high gain technique. We analyze the stability of the closed-loop system using the singular perturbation method. Simulations show that the proposed controller has better position tracking performance than a standard PI controller.

I. INTRODUCTION

Electro-hydraulic servo systems are important to modern industrial automation; they are used in many kinds of mechanizations, including robot and aircraft actuators and rolling mills. Electro-hydraulic servo systems are able to rapidly generate very high forces. Compared to their electrical counterparts, they also have a high power to weight ratio [1]. However, electro-hydraulic servo systems exhibit significantly higher nonlinearities in their dynamics because of the compressibility of hydraulic fluid and the complex flow properties of servo valves. Therefore, the investigation of position control for electro-hydraulic servo systems has been of great interest from both academic and industrial perspectives.

Various techniques have been used to control force or position of electro-hydraulic servo systems. Local linearization of the nonlinear dynamics about a nominal operating point was proposed in [1]. However, the nonlinear behavior of the system requires the use of conservative loop gain that sacrifices controller performance in favor of robustness. In [2], the use of pressure feedback was proposed to improve the performance of proportional derivative (PD) controllers in hydraulic systems, but global stability was not proven. The variable structure control (VSC) strategy has been studied for the control of hydraulic servo-systems in [3], [4]. However, chattering in the control action, which is inherent in VSC, can easily excite high frequency modes and may make the system unstable. Feedback linearization has also been used to develop hydraulic actuator control systems [5], [6]. These studies assumed that valve dynamics are negligible. However, for high performance, valve dynamics must be considered. A backstepping approach which develops a Lyapunov-based tracking controller was applied to electro-hydraulic servo systems [7]. Backstepping controllers are, however, long and complex and thus may not be suitable for real-time implementation. Passivity-based control has been successfully applied to a 4th order nonlinear model and shown to be very effective for hydraulic actuator pressure control [8]. Passivity-based control is a design technique that uses passivation to achieve the control objective [9]. Passivity-based controllers need to measure the full state informations of the system. However, due to cost and space limitations, it is not always possible to measure all states of the system.

The main contribution of this paper is the design and implementation of a high-gain observer-based nonlinear controller for position control of an electro-hydraulic servo system. Servo-valve dynamics should be modelled as a 2nd order system to achieve dynamic response on a wider frequency range [10]. Therefore, our electro-hydraulic servo system is modelled as a 5th order nonlinear system. The proposed controller extends the controller proposed in [8]. Since the electro-hydraulic servo system has nonlinearities, we design a modified high gain observer for state estimation without the transformation for normal form. The high gain technique is used to reduce the effects of the nonlinear terms. We prove that the estimation error converges to the interior of a bounded ball, using the boundedness of the nonlinearities. We analyze the stability of the closed-loop system using the singular perturbation method. The closed-loop system is divided into two separate subsystems: a fast subsystem (the estimation error dynamics) and a slow subsystem (the tracking error dynamics). We prove that the closed-loop system is stable. To investigate the performance of the proposed controller, we perform several simulation tests: bandwidth, step response, and path tracking. According to the simulation results, the proposed controller shows improved tracking performance compared with a standard proportional integral (PI) controller.

II. ELECTRO-HYDRAULIC SERVO SYSTEMS

An electro-hydraulic servo system designed for a quadruped robot being developed by the Korea Institute of Industrial Technology depicted in Fig. 1. This system
where $x_1$ is the position of the rotor [rad], $x_2$ is the velocity of the rotor [rad/s], $x_3$ is the load pressure [N/m$^2$], $x_4$ is the spool position of the servo valve [m], $x_5$ is the velocity of the servo valve [m/s], and $u$ is the current input [mA], respectively. $h_1 := \frac{4\rho A C_m}{V_t \sqrt{\beta}}$, $h_2 := \frac{4\rho B C_m}{\sqrt{\beta}}$, $J_m$ is the moment of inertia of the rotor [kg·m$^2$], $\tau_F(\dot{\theta}_m)$ is the friction torque [Nm], and $\tau_L$ is the load torque [Nm], $C_d$ is the orifice flow coefficient, $w$ is the area gradient of the servo-valve spool [m], $P_s$ is the supply pressure of the pump [N/m$^2$], $P_l$ is the differential pressure [N/m$^2$], and $\rho$ is the density of hydraulic oil [kg/m$^3$], $A$ is the effective area of the rotor [m$^2$], $C_m$ is the total leakage coefficient [m$^3$/Ns], $C_v$ is the valve leakage coefficient, $V_t$ is the total actuator volume [m$^3$], and $\beta$ is the effective bulk modulus of the system [N/m$^2$]. $\zeta$ is the damping ratio, $\omega_n$ is the natural frequency [rad/s], and $k_v$ is the torque motor gain [N/m/A], respectively. The friction model equation (2) given in [13] is used as follow

$$\tau_F(x_2) = F_v x_2 + \text{sgn}(x_2) \left[ F_{v0} + F_{v0} e^{-\frac{x_2^2}{C_{v0}}} \right]$$

where $F_v$ is the viscous coefficient, $F_{v0}$ is the parameter for Coulomb friction, and $C_{v0}$ are the parameter for static friction, respectively. Fig. 2 shows the result of the friction model. The load torque model is written as

$$\tau_L(x_1) = mg l \sin(x_1)$$

where $m$ is the mass of the pendulum [kg], $g$ is the acceleration due to gravity [m/s$^2$], and $l$ is the distance from the pivot to the center of mass of the pendulum [m], respectively. In electro-hydraulic servo system, the state is physically bounded. Therefore, we assume $x \in \Omega$ where $\Omega$ is a compact set of $\mathbb{R}^2$.

### III. CONTROL DESIGN

Passivity-based control is a recursive procedure derived from a nonlinear system to guarantee passivity [9]. The controller proposed in [8] is extended for 5th model. The electro-hydraulic servo system (1) can be put into the strict feedback form as shown in (4) [11]

$$\dot{x}_4 = f_4(x_1, \ldots, x_4) + g_4(x_1, \ldots, x_4) x_5$$
$$\dot{x}_5 = f_5(x_1, \ldots, x_5) + g_5(x_1, \ldots, x_5) u$$

where $x_i \forall i \in [1,5]$, $y$, and $u$ are the state, the output and the input of system, respectively. In (1), we get $f_1 = 0, f_4 = 0, g_1(x) \neq 0 \forall i \in [1, 5]$ and we see that

$$\dot{x}_1 = \frac{1}{g_1} x_2$$
$$\dot{x}_2 = \frac{\tau_F(x_2) + \tau_L(x_1)}{J_m} + \frac{D_m}{J_m} x_3$$
$$\dot{x}_3 = \frac{-h_2 x_2 - h_3 x_3 + h_1 x_1 \text{sgn}(x_4) x_3}{J_m}$$
$$\dot{x}_4 = \frac{1}{g_4} x_5$$
$$\dot{x}_5 = \frac{-\omega_n^2 x_4 - 2\zeta \omega_n x_5 + \omega_n^2 K_v u}{J_m}$$

In the electro-hydraulic servo system, the output is the rotor position $x_1$. We define $x_1^d$ as the desired position of the rotor.
Then tracking errors can be written as
\[ e_i = x_i - x_i^d, \quad \forall i \in [1, 5]. \] (6)

We differentiate each tracking error to create the tracking error dynamics as follows
\[ \dot{e}_i = \dot{x}_i - \dot{x}_i^d = f_i + g_i \dot{x}_{i+1} - \dot{x}_i^d, \quad \forall i \in [1, 4] \]
\[ \dot{e}_5 = \dot{x}_5 - \dot{x}_5^d = f_5 + g_5 \dot{u} - \dot{x}_5^d. \] (7)

Since \( e_i = x_i - x_i^d \), (7) can be written as
\[ \dot{e}_i = f_i + g_i e_{i+1} + g_i \dot{x}_{i+1}^d - \dot{x}_i^d, \quad \forall i \in [1, 4] \]
\[ \dot{e}_5 = f_5 + g_5 u - \dot{x}_5^d. \] (8)

Then we can define the desired state and the system input as
\[ x_{i+1}^d = \frac{1}{k_i} (-f_i + \dot{x}_i^d - k_i e_i), \quad \forall i \in [1, 4] \]
\[ u = \frac{1}{k_5} (-f_5 + \dot{x}_5^d - k_5 e_5), \]
\[ k_i > 0, \quad \forall i \in [1, 5]. \] (9)

By defining the positive-definite Lyapunov function as
\[ V_i = \frac{1}{2} e_i^2, \quad \forall i \in [1, 4], \] (11)
we obtain
\[ \dot{V}_i = -k_i e_i^2 + g_i e_{i+1} e_i, \quad \forall i \in [1, 4]. \] (12)

Using \( g_i e_{i+1} \) as the input and \( e_i \) as the output in (12) [8] gives
\[ g_i e_{i+1} e_i = \dot{V}_i + k_i e_i^2 \geq 0. \] (13)

Then (13) shows that the relationship between \( e_i \) and \( e_{i+1} \) is strictly passive [11] and each subsystem is bounded input bounded output (BIBO) stable for \( \forall i \in [1, 4] \). Serial interconnections of strictly passive elements are also strictly passive. Further, the 5th tracking error dynamics becomes
\[ \dot{e}_5 = -k_5 e_5. \] (14)

Consequently \( e_5 \) converges exponentially to zero with convergence rate \( k_5 \). And \( |e_i| \) converges to zero \( \forall i \in [1, 4] \).

For position tracking control, from (5) to (9), we may write the system dynamics and its control law as follows
\[ \dot{x} = f(x, u) \]
\[ \dot{x}_2 = \frac{1}{J_m} (D_m x_3 - \tau_f (x_2) - \tau_e (x_1)) \]
\[ \dot{x}_3 = h_1 x_4 \sqrt{P_2 - \text{sgn}(x_4) x_3 - h_2 x_2 - h_3 x_3} \]
\[ \dot{x}_4 = x_5 \]
\[ \dot{x}_5 = -\omega_n^2 x_4 - 2 \zeta \omega_n x_5 + \omega_n^2 K_u u \]
\[ y = x_1 \] (15)

\[ u = \gamma(x, x_1^d) \]
\[ u = -\frac{1}{\omega_n^2 K_u} (-\omega_n^2 x_4 - 2 \zeta \omega_n x_5 - \dot{x}_5^d + k_5 e_5) \]
\[ x_2^d = -(x_2 - x_2^d + k_1 e_1) \]
\[ x_3^d = -\frac{1}{D_m} (-\tau_f (x_2) - \tau_e (x_1) - \dot{x}_2^d + k_2 e_2) \]
\[ x_4^d = -\frac{1}{h_1 \sqrt{P_2 - \text{sgn}(x_4) x_3}} (-h_2 x_2 - h_3 x_3 - \dot{x}_3^d + k_3 e_3) \]
\[ x_5^d = -(x_5 - \dot{x}_5^d + k_4 e_4) \] (16)

and the tracking error dynamics becomes
\[ \dot{e} = A_e e + B_e(e, x_3, x_4), \] (17)
where \( e = [e_1, e_2, e_3, e_4, e_5]^T \),
\[ A_e = \begin{bmatrix} -k_1 & 0 & 0 & 0 & 0 \\ 0 & -k_2 & 0 & 0 & 0 \\ 0 & 0 & -k_3 & 0 & 0 \\ 0 & 0 & 0 & -k_4 & 0 \\ 0 & 0 & 0 & 0 & -k_5 \end{bmatrix}, \]
\[ B_e(e, x_3, x_4) = \begin{bmatrix} g_1 e_2 \\ g_2 e_3 \\ g_3 (x_3, x_4) e_4 \\ g_4 e_5 \end{bmatrix}^T. \]

### IV. OBSERVER DESIGN AND STABILITY ANALYSIS

To use the control law (16), we need full information on all states. However, only position \( x_1 \) is measurable, so we design a high gain observer to reduce the nonlinearity of the electro-hydraulic servo system. To design the observer, let us rewrite (15) as
\[ \dot{x} = A x + \phi(x) + B u \]
\[ y = C x \] (18)
where
\[ A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{D_m}{J_m} & 0 & 0 \\ 0 & -h_2 & -h_3 & h_1 \sqrt{P_2} & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\omega_n^2 & -2 \zeta \omega_n \end{bmatrix}, \]
\[ \phi(x) = \begin{bmatrix} 0 \\ -\frac{1}{\omega_n^2} (\tau_f (x_2) + \tau_e (x_1)) \\ h_1 (\sqrt{P_2 - \text{sgn}(x_4) x_3} - \sqrt{P_2}) x_4 \\ 0 \\ 0 \end{bmatrix}, \]
\[ B = \begin{bmatrix} 0 & 0 & 0 & 0 & \omega_n^2 K_u \end{bmatrix}^T, \]
and
\[ C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}. \]

Now we replace \( u \) with \( \dot{u} \). Then the controller (16) is rewritten as
\[ \dot{u} = \gamma(x_1, \dot{x}_2, \dot{x}_3, \dot{x}_4, \dot{x}_5, \dot{x}_1^d) \]
\[ \dot{u} = -\frac{1}{\omega_n^2 K_u} (-\omega_n^2 x_4 - 2 \zeta \omega_n x_5 - \dot{x}_5^d + k_5 \dot{e}_5) \]
\[ \dot{x}_2^d = -(\dot{x}_2 - \dot{x}_2^d + k_1 \dot{e}_1) \]
\[ \dot{x}_3^d = -\frac{1}{D_m} (-\tau_f (\dot{x}_2) - \tau_e (\dot{x}_1) - \dot{x}_2^d + k_2 \dot{e}_2) \]
\[ \dot{x}_4^d = -\frac{1}{h_1 \sqrt{P_2 - \text{sgn}(x_4) x_3}} (-h_2 \dot{x}_2 - h_3 x_3 - \dot{x}_3^d + k_3 \dot{e}_3) \]
\[ \dot{x}_5^d = -\dot{x}_5 - \dot{x}_5^d + k_4 \dot{e}_4 \] (19)
where \( \hat{e}_i = \hat{x}_i - \hat{x}_i^d \), \( \forall i \in [2, 5] \). Note that control law (19) uses measured \( x_1 \) except other states. Equation (18) may also be rewritten as
\[
\dot{x} = Ax + \phi(x) + B\hat{u}.
\] (20)
Therefore, the tracking error dynamics (17) becomes
\[
\dot{e} = A_e e + B_e (e, x_3, x_4) + \xi (x, \hat{x}, x_i^d)
\] (21)
where \( \hat{e}_i = [0, 0, 0, 0, g_s u + g_\omega \dot{\theta}]^T \). To estimate \( x \), we design a nonlinear observer such that
\[
\dot{\hat{x}} = A \hat{x} + \phi(\hat{x}) + B \hat{u} + Ly - \hat{y}.
\] (22)
To avoid the peaking phenomenon [11], we need to ensure that \( \hat{x} \) has the same bound as \( x \). The observer gain \( L = [\frac{\alpha_1}{\varepsilon}, \frac{\alpha_2}{\varepsilon^2}, \frac{\alpha_3}{\varepsilon^3}, \frac{\alpha_4}{\varepsilon^4}, \frac{\alpha_5}{\varepsilon^5}]^T \) is designed such that \( A_H \) is Hurwitz:
\[
A_H = \begin{bmatrix}
-\alpha_1 & 1 & 0 & 0 & 0 \\
-\alpha_2 & 0 & P_{nn} & 0 & 0 \\
-\alpha_3 & -h_2 & -h_3 & h_1 \sqrt{P_x} & 0 \\
-\alpha_4 & 0 & 0 & 0 & 1 \\
-\alpha_5 & 0 & 0 & -\omega_n^2 & -2 \zeta \omega_n \\
\end{bmatrix}
\]
Now we will analyze the convergence of the estimation error and stability of the closed-loop system. Let us define the estimation error as
\[
\hat{\eta} = A_\eta \hat{x} + \delta(x, \hat{x})
\] (24)
where
\[
A_\eta = A - LC = \begin{bmatrix}
-\alpha_1/\varepsilon & 1 & 0 & 0 & 0 \\
-\alpha_2/\varepsilon^2 & 0 & P_{nn} & 0 & 0 \\
-\alpha_3/\varepsilon^3 & -h_2 & -h_3 & h_1 \sqrt{P_x} & 0 \\
-\alpha_4/\varepsilon^4 & 0 & 0 & 0 & 1 \\
-\alpha_5/\varepsilon^5 & 0 & 0 & -\omega_n^2 & -2 \zeta \omega_n \\
\end{bmatrix},
\]
\[
\delta(x, \hat{x}) = \phi(x) - \phi(\hat{x})
\] = \[
\begin{bmatrix}
\eta_1 \\
\eta_2 \\
\eta_3 \\
\eta_4 \\
\eta_5 \\
\end{bmatrix}
\]
(25)
By the newly defined variable \( \eta \), the estimation error dynamics (24) is transformed into a singularly perturbed form as
\[
\varepsilon \dot{\eta} = A_\eta \eta + \Delta(\varepsilon, x, \hat{x})
\] (26)
where
\[
\Delta(\varepsilon, x, \hat{x}) = \begin{bmatrix}
0 \\
\varepsilon^2 \delta_2 \\
\varepsilon^3 \delta_3 \\
0 \\
0 \\
\end{bmatrix}
\]
Given \( \alpha_i, i \in [1, 5] \) such that \( A_\eta \) is Hurwitz, there exists a positive definite matrix \( P_1 \) satisfying the following Lyapunov equation
\[
A_\eta^T P_1 + P_1 A_\eta = -I.
\] (27)

**Theorem 1:** If there exist \( \alpha_i, i \in [1, 5] \) such that \( A_\eta \) is Hurwitz and \( x, \hat{x} \in \Omega \), the estimation error \( \eta \) goes into the inside of a ball
\[
B_r(\varepsilon) = \{ \eta \in \mathbb{R}^5 \mid \| \eta \| \leq 2r_\varepsilon \| P_1 \| \}
\]
where
\[
r_\varepsilon = \sup_{x, \hat{x} \in \Omega} \| \Delta(\varepsilon, x, \hat{x}) \|.
\]
In finite time \( \tilde{t} \) and stays in it for \( t > \tilde{t} \).

**Proof:** Consider the following Lyapunov candidate function
\[
V_1(\eta) = \varepsilon \eta^T P_1 \eta.
\] (28)
Differentiating \( V_1(\eta) \) with respect to time, we get
\[
\dot{V}_1 = \varepsilon \dot{\eta}^T P_1 \eta + \varepsilon \eta^T P_1 \dot{\eta} = \eta^T (A_\eta^T P_1 + P_1 A_\eta) \eta + 2\Delta(\varepsilon, x, \hat{x}) P_1 \eta \leq -\|\eta\|^2 + 2\|\Delta(\varepsilon, x, \hat{x})\|\|P_1\|\|\eta\|.
\] (29)
If \( x, \hat{x} \in \Omega \), then \( \delta_2 \) and \( \delta_3 \) are bounded. Thus, we can conclude that \( \Delta(\varepsilon, x, \hat{x}) \) is bounded. Equation (29) becomes
\[
\dot{V}_1 \leq -\|\eta\|^2 + 2r_\varepsilon \| P_1 \| \|\eta\|
\]
where
\[
r_\varepsilon = \sup_{x, \hat{x} \in \Omega} \| \Delta(\varepsilon, x, \hat{x}) \| \| P_1 \|.
\]
If \( \|\eta(t)\| > 2r_\varepsilon \| P_1 \| \), then \( \dot{V}_1 < 0 \). Therefore, if \( \|\eta(t)\| > 2r_\varepsilon \) \( \|\eta(t)\| \) goes into the inside of the ball \( B_r \) in finite time \( \tilde{t} \).
And \( \|\eta\| \) stays in the inside of the ball \( B_r \) for \( t > \tilde{t} \).

**Remark 1:** In the system (18), \((A, C)\) is detectable, so it is possible to design \( \alpha_i, i \in [1, 5] \) such that \( A_\eta \) is Hurwitz. Thus, according to Theorem 1, \( \dot{\eta} \) converges to the inside of the ball \( B_r \). \( B_r \) can be shrunk by decreasing \( \varepsilon \). Ideally \( \dot{\eta} \) can be 0 by setting \( \varepsilon = 0 \). Therefore, if \( \dot{\eta} \) is 0, \( \dot{x} \) converges to 0.

The tracking error dynamics (21) and the estimation error dynamics (26) can be represented in the singularly perturbed form as follows
\[
\dot{e} = A_e e + B_e (e, x_3, x_4) + \xi (x, \hat{x}, x_i^d) \\
\varepsilon \dot{\eta} = A_\eta \eta + \Delta(\varepsilon, x, \hat{x})
\] (30)
with the equilibrium point \((e, \eta) = (0, 0)\). The tracking error dynamics (21) have slow dynamics and the estimation error
dynamics (26) have fast dynamics. Making $\varepsilon = 0$ implies that
\[
\begin{align*}
\eta_1 &= 0 = x_1 - \hat{x}_1 \\
\eta_2 &= 0 = \varepsilon(x_2 - \hat{x}_2) \\
\eta_3 &= 0 = \varepsilon^3(x_3 - \hat{x}_3) \\
\eta_4 &= 0 = \varepsilon(x_4 - \hat{x}_4)
\end{align*}
\]
has an equilibrium point $[x_1, x_2, x_3, x_4]^T = [\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_4]^T$. Substituting the equilibrium point into the slow dynamics, we obtain the quasi-steady-state model as
\[
\dot{\varepsilon} = A_{\varepsilon}\varepsilon + B_{\varepsilon}(\varepsilon, x_3, x_4)
\] (31)
On the other hand, the boundary layer system of (30) is:
\[
d\frac{d}{d\tau} \eta = A_{\eta} \eta
\] (32)
where $\tau = \frac{t}{\varepsilon}$ and $A_{\eta}$ is defined as in (26). The following theorem will show the stabilities of the equilibrium points $e = 0$ of (31) and $\eta = 0$ of (32).

**Theorem 2**: The equilibrium points $e = 0$ of (31) and $\eta = 0$ of (32) are asymptotically stable.

**Proof**: According to the passivity theorem of section III, $e = 0$ of quasi-steady-state (31) is asymptotically stable. For a boundary layer system, because $A_{\eta}$ is Hurwitz, there exists a positive definite matrix $P_{\eta}$ such that
\[
A_{\eta}^T P_{\eta} + P_{\eta} A_{\eta} = -I.
\]
Let us consider the Lyapunov function
\[
V_2(\eta) = \eta^T P_{\eta} \eta.
\]
Then we see that the derivative of Lyapunov $V_2$ becomes
\[
d\frac{d}{d\tau} V_2(\eta) = \eta^T (A_{\eta}^T P_{\eta} + P_{\eta} A_{\eta}) \eta = -\eta^T \eta < 0.
\]

**Remark 2**: From the perspective of the singular perturbation analysis, the estimation error dynamics (26) has a faster dynamics than the tracking error dynamics (21). Under the assumption that $\varepsilon = 0$, the estimation error $\eta$ and tracking error $e$ are asymptotically stable if the position of the rotor is measurable.

In practice, it is not possible for an observer (22) to have $\varepsilon = 0$. Thus it is necessary to find a positive value of $\varepsilon$, that is $\varepsilon^*$ that makes the system stable.

**Theorem 3**: If there exist a continuous interval $\Gamma = (0, \varepsilon)$ such that for all $\varepsilon \in \Gamma$ satisfies
\[
f(\varepsilon) = (1 - d)dt_1 T_3 \varepsilon^7 - 2(1 - d)dt_1 T_4 \varepsilon^5 - (1 - d)^2 T_2^2 < 0
\] (33)
where $T_1$, $T_2$, $T_3$, and $T_4$ are positive constants, $0 < d < 1$, $\varepsilon$ is the positive root of (33), then the origin $(e, \eta) = (0, 0)$ of (30) is asymptotically stable.

**Proof**: Let us select a Lyapunov candidate function of (30) as
\[
V_{cl} = (1 - d)\varepsilon^T P_{cl} e + d\eta^T P_{\eta} \eta.
\] (34)
Since $A_e$ and $A_\eta$ are Hurwitz, there exist positive definite $P_e$ and $P_\eta$ such that
\[
\begin{align*}
A_e^T P_e + P_e A_e &= -Q_e, \\
A_\eta^T P_\eta + P_\eta A_\eta &= -Q_\eta.
\end{align*}
\]
Differentiating $V_{cl}$ with respect to time $t$ gives us
\[
\dot{V}_{cl} = -(1 - d)\varepsilon e^T Q_e e + 2(1 - d)\varepsilon^2 T_e B_e + 2(1 - d)^2 T_e^2 e^T P_e \varepsilon + \frac{2 - d}{\varepsilon} e^T Q_\eta \varepsilon + 2e^T \eta^T P_\eta \Delta.
\] (35)
Since the nonlinearity functions of the system (1) are local Lipshitz, we draw three conclusions as follows
(a) Since $B_e$ is bounded, there exists $\gamma > 0$ such that
\[
\|B_e\| \leq \gamma \|\varepsilon\|.
\] (36)
Therefore, if there exists $\gamma$ such that $\gamma < \lambda_{min}(Q_e)/2\|P_e\|$ where $\lambda_{min}(A)$ is the minimum eigenvalue of the matrix $A$, then
\[
-(1 - d)\varepsilon e^T Q_e e - 2e^T P_e B_e \leq -(1 - d)\lambda_{min}(Q_e) - 2\gamma \|P_e\| \|\varepsilon\|^2.
\] (37)
(b) We know that $e_i - \hat{e}_i = \tilde{x}_i$, $i \in [1, 5]$ and $e^4 \|\tilde{x}\| \leq \|\eta\| \|u\|$ (16) and $\hat{u}$ (19) consist of $x, \dot{x},$ and $x^2$. But $x^2$ disappears in $u - \hat{u}$. Therefore, there exists $H_1$ such that
\[
\|\xi\| = \|0 0 0 0 -g(u - \hat{u})\|^T \leq \|H_1\| \|\xi\| \leq \frac{1}{\varepsilon} \|H_1\| \|\eta\|.
\] (38)
(c) There exists $H_2$ satisfying that
\[
\|\Delta\| = \|0 \varepsilon^2 \delta_2 \varepsilon^3 \delta_3 0 0\|^T \leq \varepsilon^2 \|H_2\| \|\tilde{x}\| \leq \frac{\varepsilon^2}{\varepsilon} \|H_2\| \|\xi\| \leq \frac{1}{\varepsilon} \|H_2\| \|\eta\|.
\] (39)
Applying (a), (b), and (c) to (35) gives us
\[
\dot{V}_{cl} \leq -(1 - d)T_1 \|\varepsilon\|^2 + 2(1 - d)T_2 \varepsilon \|\eta\| + \frac{(1 - d)T_3}{\varepsilon^2} \|Q_\eta\| \|\eta\| \|\Delta\| \leq -\frac{(1 - d)T_3}{\varepsilon^2} \|Q_\eta\| \|\eta\|^2 + \frac{\varepsilon^2}{\varepsilon^2} (\frac{1 - d}{\varepsilon} T_2 - T_4) \|\eta\|^2
\] (40)
where
\[
T = \begin{bmatrix}
-(1 - d)T_1 & \frac{(1 - d)}{\varepsilon^2} T_2 \\
\frac{(1 - d)}{\varepsilon^2} T_2 & -\frac{d}{\varepsilon} T_3 + 2\frac{d}{\varepsilon} T_4
\end{bmatrix}.
\]
If there exists a continuous interval $\Gamma = (0, \varepsilon)$ such that for all $\varepsilon \in \Gamma$ satisfies
\[
f(\varepsilon) = (1 - d)dt_1 T_3 \varepsilon^7 - 2(1 - d)dt_1 T_4 \varepsilon^5 - (1 - d)^2 T_2^2 < 0,
\]
we can guarantee $T$ is a negative definite. The $\varepsilon$ will be the upper bound of the continuous interval $\Gamma$. \square
Remark 3: Since (33) has 7 possible solutions such that \( f(\epsilon) < 0 \), there exists a positive real root of (33) such that in the interval \((0, \bar{\epsilon})\). The condition (33) is only sufficient.

The estimation error shrinks to zero as the gain of the observer grows to infinity \((\epsilon \to 0)\) if ideally no measurement noise is present. When noise is present, the noise is increased due to high-gain of the observer [14]. Hence, the observer gain should be designed by considering to trade-off between the error in the absence of noise and the amplification of the noise.

V. SIMULATION RESULTS

We used simulations to investigate the performance of the proposed controller. We compared its position tracking to that of the standard PI controller often used to control electro-hydraulic servo systems. For this comparison, we tested bandwidth, step response and path tracking performance. The following system parameters were used:

\[
\begin{align*}
J_m &= 0.179, \quad D_m = 2.5 \times 10^{-5}, \quad \beta_e = 7.138 \times 10^7, \\
V_t &= 8.0 \times 10^{-5}, \quad C_m = 0.6, \quad w = 0.022, \\
P_t &= 1.85 \times 10^6, \quad \rho = 840.0, \quad C_{im} = 2.21 \times 10^{-11}, \\
\zeta &= 0.5, \quad \omega_n = 950.0, \quad K_v = 4 \times 10^{-5}, \quad m = 10, \quad l = 0.3.
\end{align*}
\]

In numerical simulations, we define the maximum magnitude of the input noise and output noise as \(0.1 \text{mA}\) and \(\pm 1.74 \times 10^{-4} \text{rad}\).

A. BANDWIDTH

Figure 3 compares the position tracking performance of the proposed controller to that of a PI controller for sinusoidal references \(f = 2\text{Hz}, f = 5\text{Hz}, \text{ and } f = 10\text{Hz}\). The PI has been tuned to reduce the position tracking error. The tracking performance of the PI degrades at higher frequencies: When the frequency of the reference signal is above 5Hz, phase lag and decreased magnitude are observed. However, even at high frequencies, the proposed controller has only a small phase shift. We found that the proposed controller’s tracking performance begins to degrade from a frequency of 11 Hz.

B. STEP RESPONSE

We also tested step response by using a square wave. The step response test can identify several important characteristics of a controller, including speed of response, overshoot, and settling time. Figure 4 plots the results of square wave tracking. Overshoot, oscillation, and steady state error are observed when the PI controller is applied to the system. However, with the proposed method, reduced overshoot or no oscillation are observed. Furthermore, the proposed method reduces the steady state error.

C. PATH TRACKING

The bandwidth and step response tests are performed on the system with only simple and small amplitude reference signals. In this section, we show the performance of the controller for the path tracking. The position tracking performance of the proposed controller is compared with the PI
VI. CONCLUSIONS

In this paper, the high gain observer-based nonlinear control was proposed for the position control of an electro-hydraulic servo system. To improve the system dynamic performance, the controller was designed. The high gain observer was designed for the estimation of states of the high nonlinear system. High gain technique was used to reduce the effect of the nonlinear terms. The stability of closed-loop system was proven via singular perturbation analysis. The proposed method increased the bandwidth of the system. The step response and the position tracking performance are improved by the proposed controller.

REFERENCES