Architectures for Phase Variation Compensation in AFR Control

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Abstract—To counteract the binary nature of an exhaust gas oxygen sensor and to improve the efficiency of a three way catalyst, the delivered fueling quantity is typically dithered around stoichiometry for gasoline internal combustion engines. Determining whether the switching sensor reads rich or lean because of the dithering or a disturbance is very difficult. This problem is often overcome by estimating the plant delay and tracking a reference signal. Without an accurate delay estimate, however, the benefits of this type of control architecture are limited. This paper compares two switching sensor based control architectures, phase lock loop and duty cycle control, for tracking a periodic reference air-to-fuel ratio signal while overcoming the uncertainty of a dynamic plant delay estimate.

I. INTRODUCTION

Air-to-fuel ratio (AFR) control has evolved significantly since the internal combustion engine was first applied to mobile applications. Originally, before fuel economy and emissions were important, AFR was controlled strictly using feed forward control. In the modern era, fuel economy and emissions requirements have mandated the use of feedback control. Nearly every gasoline engine uses at least one oxygen sensor and one three-way catalyst (TWC) to reduce the tailpipe-out emissions. For a TWC to efficiently reduce all three of the important pollutant species (hydrocarbons, carbon dioxide and nitrogen oxides), the AFR of the combustion mixture must be stoichiometric. Dithering the AFR around stoichiometry extends the efficiency range of a TWC [1], and it is helpful for AFR controllers that use switching exhaust gas oxygen (EGO) sensors. Because of its binary nature, an EGO sensor can only be used to determine the relative AFR composition of the exhaust stream. The sensor output indicates whether the AFR is rich or lean, but not by how much. The AFR magnitude is only known when the sensor switches from rich to lean or from lean to rich. During a transition, the exhaust stream is at least briefly stoichiometric. Dithering the AFR command artificially increases the probability of an EGO switching event and thus enables better estimation of the AFR.

Although wide range universal exhaust gas oxygen (UEGO) sensors provide magnitude information, they are more expensive and slower than standard EGO sensors. For these reasons, many production vehicles still use EGO sensors. However, most of the AFR control techniques described in the open literature are designed around UEGO sensors. This disparity is a strong motivator of this work. Advanced control formulations such as $H_{\infty}$ [2] and Lyapunov [3] based controllers have been designed for UEGO sensor feedback. Other papers including [4] and [5] have also developed estimators and controllers that require state feedback via a UEGO sensor.

Without magnitude information, these advanced control techniques are not as successful when they are applied to switching sensor systems. As demonstrated in [6], an observer developed for a linear UEGO sensor can be applied to a standard EGO sensor, however the resulting controller exhibits an uncontrolled limit cycle type behavior. Although the AFR fluctuates near stoichiometry, this type of control cannot regulate the average AFR to exactly stoichiometry. Depending on the duty cycle of the limit cycle oscillations, the average AFR may be slightly rich or slightly lean. To prevent these small but significant (in terms of emissions) errors, AFR oscillations must be controlled. Sliding mode control has been applied in [7] and [8] to control the EGO switching. Similarly, a prescribed dither applied to the baseline AFR command can be tracked as in [6]. The biggest obstacle for tracking-based AFR controllers is the variable plant delay. If the delay is estimated incorrectly, then the predicted AFR will not be in-phase with the measured AFR. Because the plant delay also prevents a standard PID controller from removing phase errors, delay estimation errors limit the performance of PID based tracking controllers. Recently, a phase lock loop (PLL) technique was applied to AFR control [9]. With this control architecture it is possible to eliminate phasing errors caused by delay estimation errors. This paper introduces a new duty cycle control architecture which is based on a PLL and is also capable of overcoming the variable plant delay. It will be shown that a duty cycle controller outperforms a PLL in both steady-state tracking and disturbance rejection performance.

II. EFFECT OF DELAY ESTIMATION ERRORS

When tracking a sinusoidal reference, it is imperative that the controller be robust to phase errors; a simple PID controller is not. Consider the simplest tracking control architecture where the input to the controller is the difference between the predicted and measured AFR. In this case the reference AFR is simply the desired sinusoidal AFR signal delayed according to a plant delay estimate. For this paper all numerical values of AFR are provided in terms of a fuel-to-air equivalence ratio (EQR) because in this domain the nominal control target, stoichiometry, is unity. The EQR of a combustion mixture can be calculated using

$$EQR = \frac{f_{\text{trap}}}{a_{\text{trap}}} AFR_s$$  (1)
where $f_{\text{trap}}$ is the trapped fuel mass, $\alpha_{\text{trap}}$ is the trapped air mass and $A\text{F}R_{s}$ is the stoichiometric AFR. As illustrated in Fig. 1, the proportional gain of a PID controller can actually generate additional errors. In this example, the plant delay is assumed to be 10 events. The unity gain proportional controller is not applied until event 20. At event 20, the measured EQR is larger (richer) than the reference, so the controller subtracts fuel. However, when this correction appears in the output 10 events later, the measured EQR is smaller (leaner) than the reference. The correction applied by the proportional controller is therefore in the wrong direction. Because of the plant delay, proportional corrections cannot correct for phase errors. Instead, the corrections will tend to exaggerate the phase error and increase the amplitude of the oscillations. Integral control action which is based on the same error signal cannot correct phase errors either. Derivative control action will tend to slightly reduce the amplitude of the oscillations but will not affect the phasing. For switching EGO sensor based feedback systems, phase errors pose even greater problems, because the error magnitudes are unknown. To prevent a PID controller from amplifying phasing errors, the control gains must be selected very conservatively which in turn reduces the disturbance rejection performance.

III. Phase Lock Loops

A PLL is a feedback structure that compares an input sinusoidal signal to a synthesized sinusoidal signal. The error between these two signals drives a controlled oscillator to produce a signal that matches the input. The key components are a phase detector, a loop filter and a controlled oscillator. This structure is illustrated in Fig. 2 where the multiplication junction represents the phase detector. The theory behind phase lock loops is similar to amplitude modulation and is based on the simple trigonometric identity

$$A\sin(\omega_1 t)B\sin(\omega_2 t + \theta) = \frac{AB}{2}\left(\cos((\omega_1 - \omega_2) t - \theta) - \cos((\omega_1 + \omega_2) t + \theta)\right),$$

for frequencies $\omega_1$, $\omega_2$ and phase $\theta$ ($A$ and $B$ are generic coefficients).

Amplitude modulation is a technique that facilitates the transportation of a data signal. At the data source location, a high frequency sinusoid known as the carrier signal multiplies the lower frequency data signal. At the destination location, the same carrier signal is multiplied with the amplitude modulated signal. After low pass filtering, the original data signal is recovered. A similar process is used for phase lock loops, but the goal is to match the frequency of the two signals. When this occurs (2) becomes

$$A\sin(\omega t)B\sin(\omega t + \theta) = \frac{AB}{2}\left(\cos(-\theta) - \cos(2\omega t + \theta)\right).$$

When the resulting signal is conditioned with a low pass filter, the output depends only on the phase error, $\theta$. The phase error continues to drive the controlled oscillator until phase error is reduced to zero.

For the AFR control problem, the engine acts as a digitally controlled oscillator (DCO) and the dithered reference AFR signal represents the input signal. Both the dither signal and the PLL controller operate in the event domain where the number of events per engine cycle is equal to the number of cylinders. The phase and frequency of the engine DCO can be calculated from the EGO sensor as long as the sensor is switching. To ensure that the EGO sensor switches, an additional controller must be integrated into the system. This controller monitors the number of events between switches. If too many events pass without a switch, then this controller applies an integral type action to enforce a switching event.

The motivation for using a PLL to control AFR lies in its ability to remove phase errors caused by delay estimation errors. In steady-state the phase errors are reduced to zero, so any AFR errors that appear must be caused by disturbances (magnitude errors), thereby allowing larger gains for improved disturbance rejection capability. Although a PLL is a nonlinear system, the essential elements of its behavior can be captured with a linear representation and therefore analyzed using linear techniques [10]. Outside the linear range, the PLL is supplemented with a high gain return-to-switching controller. The overall control structure shown in Fig. 3 is an adaptation of the one described in [9].

To estimate the overall delay between fuel injection and EGO measurement, an exhaust geometry based delay model based on [11] and [12] is used. This model captures the physical periodic delay between the fueling and exhaust events as well as the exhaust transport delay and sensor dynamics. The transport delay is calculated using a plug flow model of the exhaust system. This type of model is
able to predict both steady-state and transient delays very well. A third order fuel dynamics model is used in the PLL (as well as in the duty cycle controller of the next section). The goal of the fuel dynamics model is to exactly cancel the physical fuel dynamics of the engine. This model is able to cancel most of the fuel dynamics effects caused by engine transients as well as dithering the AFR quantity.

To observe the phase and frequency of the exhaust AFR signal, the EGO sensor voltage is quantized such that voltages above 450 mV are defined as rich and voltages below 450 mV are defined as lean. With this convention, the period (and thus frequency) of the AFR signal can be calculated by considering the number of events between switching. This calculation is made every time the EGO switches, which is ideally half of the dither period. The phase refers to the number of events since the last switch and is updated every event. When the PLL is locked, the engine will switch between slightly rich and slightly lean AFR values at the same frequency and phase as the reference signal.

IV. DUTY CYCLE CONTROL

A duty cycle controller is similar to a PLL except that only the frequency of the EGO switching is considered. Physically, the phasing of a sinusoidal dither does not affect the emissions performance. When the reference signal is perfectly reproduced, the only factors that affect the engine-out and tailpipe-out emissions are the dither amplitude, which cannot be measured with an EGO sensor, and the dither frequency [13]. Therefore, regulating the frequency of EGO switching is sufficient, given the sensor limitations.

The term duty cycle generally refers to the ratio of time a signal is “on” ($t_{on}$) to the total window of time ($T_{window}$) in the manner

$$D_{on} = \frac{t_{on}}{T_{window}}.$$  

(4)

For a feedback control application, waiting for a complete dither period before calculating the duty cycle would result in a slow control response. If a disturbance occurs just after the EGO switches, it could take as much as one and a half dither periods to detect the error. Additionally, having a 50% duty cycle does not guarantee that the reference sinusoid is being tracked. To combat these issues, only half of the dither period is considered and a variable duty cycle window size is used. When the EGO switches from rich to lean, the duty cycle is calculated using

$$\bar{T}_{on} = \frac{2T_{on}}{T_{dither} + 2t_{on}}.$$  

(5)

where $T_{on}$ is the number of consecutive events the EGO measured rich and $T_{dither}$ is the period of the sinusoidal dither signal. It is assumed that the preceding lean to rich transition took exactly one half of the dither cycle. For a lean to rich transition, the duty cycle can be calculated similarly with

$$\bar{T}_{on} = \frac{T_{dither}}{T_{dither} + 2t_{off}}.$$  

(6)

where $t_{off}$ is the number of consecutive events the EGO measured lean. Here, the preceding rich to lean transition is assumed to have taken exactly one half of the dither cycle. Assuming the previous switching lasted exactly half of a dither period is equivalent to assuming the previous control action perfectly removed any prior disturbance. This assumption also allows the controller to take action every EGO switching event just like the PPL controller.

The leading advantage over a PLL structure is that a delay model is not needed for a duty cycle controller. A block diagram of the duty cycle control architecture is provided in Fig. 4. Fundamentally, the duty cycle controller compares the number of events between EGO switchings to the desired period. This comparison is made on two timescales, every event and every switch. To ensure EGO switching, the late switching compensator is updated every event with the
following error signal:
\[ e_{late} = \begin{cases} 
\zeta & \text{if } n_{switch} > \left(\frac{T_{dither}}{2} + \delta\right) \\
0 & \text{otherwise}
\end{cases} \] (7)

where
\[ \zeta = \begin{cases} 
-1 & \text{if rich} \\
1 & \text{if lean}
\end{cases} \] (8)

\( n_{switch} \) is the number of events since the last EGO switching and \( \delta \) is length of the deadband. If the EGO sensor indicates that too many events have passed since the last switching (more than half the dither period plus the deadband length), then the late switching compensator begins to drive the AFR back to stoichiometry. This control action is similar to the return-to-switching controller of the PLL design.

The early switching compensator is only triggered with the EGO switches. If the switch occurs too soon, then the early switching compensator offset is updated with
\[ e_{early} = \begin{cases} 
-\zeta \Delta_{switch} & \text{if } \Delta_{switch} > 0 \\
0 & \text{otherwise}
\end{cases} \] (9)

where
\[ \Delta_{switch} = \left(\frac{T_{dither}}{2} - \delta\right) - \epsilon_{switch} \] (10)

and \( \epsilon_{switch} \) is the number of events between EGO switches. The early switching compensator holds this offset correction like an integrator. Note that even without a deadzone, the early and late switching controllers act independently. Applying a deadband further isolates the controllers and allows a small range of switching frequencies to go unpunalyzed by the controllers.

It is assumed that an EGO switches early only when the measured AFR oscillations are slightly off center from stoichiometry. For example, if the EGO switches from rich to lean early, then the measured oscillations are concluded to have a lean bias and fueling is increased. The early switching correction is assumed to return the mean of the AFR oscillations perfectly to stoichiometry.

Unlike the early switch compensator, the late switch compensator must be able to correct for slightly non-stoichiometric dither offsets as well as large AFR disturbances. When sufficiently large, AFR disturbances can cause the EGO sensor to stop switching. Therefore, the late switching compensator must be capable of applying corrections that return the AFR back to stoichiometry under a wide range of disturbance magnitudes. By construction, the late compensator will continue to push the AFR back to stoichiometry until the EGO switches. Once the EGO switches, it is assumed that the disturbance has been completely rejected and will not reappear. This is achieved with a PI controller which is dominated by integral action. The proportional gain improves the response slightly and helps account for the plant delay. Because of the plant delay, the integrator will continue to “wind up” until the effect of the control action is measured by the EGO sensor. Having a small proportional gain helps mitigate this effect. With this control structure, each EGO switching event is independent because the necessary disturbance rejection control action is carried over. As such, the assumptions required for the duty cycle calculations in (5) and (6) are met.

In steady-state the AFR errors caused by air and fuel estimation errors are constant. When either the PLL or duty cycle controller achieves regular EGO switching, these constant errors are necessarily canceled. Since the fuel dynamics effects are mostly canceled with the fuel dynamics model, the amplitude of the actual AFR oscillations must be nearly equal to the reference signal, even though the amplitude cannot be measured. Therefore, when the PLL controller is locked or when the duty cycle controller achieves tracking, the engine reproduces the reference AFR trajectory. This claim will be demonstrated in the next section.

V. PERFORMANCE COMPARISONS

This section compares the disturbance rejection and tracking performance of the PLL and duty cycle controllers on a model of a 2.4 liter 4-cylinder port injected gasoline engine. The engine plant model consists of an volumetric efficiency based air estimation model, a third order wall wetting based fuel dynamics model and a combined delay model and EGO sensor model based on [12]. For either controller to be effective, the desired control behavior (PLL locking and duty cycle tracking) must correspond to tracking the dithered AFR request. Aside from this prerequisite, the three most important metrics were deemed to be: (i) the overall AFR distribution over a drive cycle, (ii) the number of events needed to recover from a step magnitude disturbance, and (iii) the number of events needed to recover from a step delay disturbance.

A. Calibration

Because the major error evaluation of these controllers happens only twice every dither period, the length of the dither period is very important to AFR regulation. As the period is decreased, the number of events needed to observe an AFR error decreases, but the signal also becomes more difficult to track. The dither signal with the shortest period that could still be tracked for this application is 25 events for both controllers, and is therefore adopted for the simulations to follow. For this dither period length, a 1.5 event deadband for the duty cycle controller was found to have the best performance, allowing the EGO to switch with a period of between 11 and 14 events without invoking a penalty. As the amplitude of the dither signal is decreased, the AFR distribution narrows and the driveability improves, but the dither signal is more difficult to track. Therefore, an EQR amplitude of 0.015 was chosen because it provided the best trade off. Each controller was optimized in simulation using the same set of drive cycle data.

B. Controller Comparison

To evaluate the effectiveness of the two controller architectures, they were compared in simulation. The possible sources of EQR errors considered are delay estimation errors, fuel dynamics modeling errors, fuel injector errors (slope and
offset), air estimation errors, and unknown fuel contributions from the evaporative canister system. To evaluate the transient performance of each controller, the federal emission certification test cycle (FTP-75) was simulated. Two types of EQR errors were applied, baseline estimation errors and artificial errors used to test the robustness of each controller. The baseline EQR errors were found by driving the physical engine on an FTP-75 cycle under open loop control and measuring the resulting EQR with a UEGO sensor. The artificial errors which represent the errors described above included step, ramp and impulse changes in EQR.

Without feedback control, these combined disturbances would result in an average EQR error (distance from stoichiometry) of 6.21%. For each controller, both the locking/tracking behavior and the overall EQR distribution were analyzed. Fig. 5 shows the EQR distributions for the PLL controller. The larger distribution includes all of the operating points contained in the FTP-75 cycle and the smaller distribution contains only those where the PLL was locked. When locked (about 28% of the operating conditions) the EQR distribution narrows substantially. The average EQR error goes from 2.71% overall to 1.89% when locked. These results show that the desired EQR behavior is achieved when the PLL is locked.

As shown in Fig. 6, the EQR distributions for the duty cycle controller are narrower than the PLL controller. The overall average EQR error is 2.24%, which is a reduction in error of more than 60% compared to the open loop case. When the controller is tracking (about 22% of the operating conditions), the average error (1.37%) approaches the average error due to the oscillating dither reference (0.95%). The EQR error statistics for all of the cases are reported in Table I, where “perfect” control corresponds to the dither signal being replicated exactly.

One of the most important benefits of these controllers is their ability to overcome the variable plant delay. To demonstrate this advantage, the plant delay was artificially increased by 10 events via a step change. For the PLL controller, this causes a phase error between the reference and measured EQR. During this transient shown in Fig. 7, the amplitude and frequency of the measured signal remains nearly constant, and in about six dither cycles the phase error is removed. The duty cycle controller does not incorporate a delay model (whereas the PLL controller does), so a change in delay only affects the frequency of the measured controller for one cycle. After that cycle, the desired dither waveform is reproduced but with a slight offset which takes about six dither cycles to be removed (Fig. 8). These trajectories not only demonstrate the robustness of each controller to phase errors, but also the faithfulness of the dither reproduction.

### TABLE I

<table>
<thead>
<tr>
<th>Controller</th>
<th>Data</th>
<th>Std (%)</th>
<th>Mean(%)</th>
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</thead>
<tbody>
<tr>
<td>Open loop</td>
<td>All</td>
<td>6.53</td>
<td>6.21</td>
</tr>
<tr>
<td>PLL</td>
<td>All</td>
<td>3.97</td>
<td>2.71</td>
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<tr>
<td>PLL</td>
<td>Locked</td>
<td>2.68</td>
<td>1.89</td>
</tr>
<tr>
<td>Duty cycle</td>
<td>All</td>
<td>3.36</td>
<td>2.24</td>
</tr>
<tr>
<td>Duty cycle</td>
<td>Tracking</td>
<td>1.97</td>
<td>1.37</td>
</tr>
<tr>
<td>Perfect</td>
<td>Steady-state</td>
<td>1.06</td>
<td>0.95</td>
</tr>
</tbody>
</table>

![Fig. 5. EQR distribution comparison for PLL controller: All cases (dark) versus locked cases (light)](image1)

![Fig. 6. EQR distribution comparison for duty cycle controller: All cases (dark) versus duty cycle tracking cases (light)](image2)

![Fig. 7. PLL controller disturbance rejection performance: Delay change at event 50](image3)
VI. CONCLUSIONS

Both PLL and duty cycle based AFR control can be used to track a periodic reference AFR trajectory using only a switching EGO sensor. Unlike most tracking architectures, these controllers are able to overcome predicted and un-predicted changes in the plant delay. In steady-state each controller is able to reproduce reference AFR signals which have a period as short as 25 events and an amplitude as low as 0.015 EQR. In terms of disturbance rejection and transient performance, the duty cycle controller outperforms the PLL controller. However, the PLL controller is able to lock onto the reference slightly more frequently than the duty cycle controller during an FTP-75 drive cycle. The amount of correction required to lock the phasing could be used to dynamically correct the delay estimate. Other control or diagnostics problems such as cylinder imbalance can rely heavily on the plant delay. In these cases, it might make sense to use a PLL structure, otherwise the duty cycle structure should be chosen.

REFERENCES