Nonlinear design of excitation controller and power system stabilizer for voltage regulation and transient stability

Giuseppe Fusco and Mario Russo, Members IEEE
Università degli Studi di Cassino, via G. Di Biasio 43, 03043 Cassino (FR), Italy
{fusco, russo}@unicas.it

Abstract—This paper presents the nonlinear design of excitation controller and power system stabilizer (PSS) aimed at ensuring transient stability and satisfactory post-fault voltage regulation following a large disturbance. The proposed design offers the possibility to select the PSS gains so as to achieve both swings damping in an assigned time interval and desired attenuation of disturbances arising from the excitation control loop. As concerns the excitation controller design it is based on the nonlinear compensation technique and ensures a quick sustainment of the regulated voltage after that the fault is cleared. Simulation results carried out on a four-nodes power system model show the goodness of the proposed design methodology with a reference to a severe large disturbance represented by a three-phase short-circuit near the generator terminals.

I. INTRODUCTION

In power system control, transient stability deals with the problem of avoiding that synchronous generators loose synchronism following a large disturbance, while voltage regulation aims to restore the pre-fault voltage set-point [1]. Moreover, it is well known that the control through the excitation loop of the synchronous generator represents an effective way for improving dynamic performance and voltage control in the power system [5]. To ensure transient stability and voltage regulation, two main approaches have been proposed. The first one introduces a supplementary control loop, in addition to the voltage control loop, in which the power system stabilizer (PSS) damps the electromechanical oscillations by generating a stabilizing signal which is added to the reference signal. In the voltage control loop the Automatic Voltage Regulator (AVR) guarantees voltage regulation, see [10], [11], [13], [14]. The second approach is based on a single controller design, usually developed in the state-space, which achieves both transient stability and voltage regulation [2], [3], [6], [7], [8], [9].

To avoid performance degeneracy due to the nonlinear nature of the power system and to guarantee system stability, the design approaches for power system controllers are based on the adoption of nonlinear control techniques, whereas the power system is usually represented by the single-machine infinite-bus model (SMIBm).

According to the first approach, this paper presents the nonlinear design of the excitation controller and power system stabilizer (PSS) without using the usually employed simplified SMIBm. It requires only four gains whose values can be easily chosen since they are strictly related to system performance.

The design of the excitation control loop, developed starting from the synchronous generator model and the Thevenin equivalent circuit representing the electrical network, cancels the inherent system nonlinearity to alleviate the uncertainties caused by operating point variations. The obtained compensated model is subsequently used to design a simple feedback controller which guarantees that the voltage regulation error goes to zero, differently from the designs illustrated in the reported references in which it is consider the first order variation of the voltage terminal. Furthermore, differently from input-output AVR heuristic design, the parameters of the proposed feedback controller can be easily set on the basis of requirements imposed on the voltage regulation error transient, typically the settling time.

Also the PSS is designed according to a new methodology developed in the input-output frame. The PSS output is a linear function of speed deviation and of its time-derivative by means of two gains which are designed to meet a double objective. The first one is to attenuate the influence of the voltage regulation loop on the PSS loop. To neglecting this influence could cause a worsening of the swings damping. The second one is to ensure that the mechanical oscillations are damped within a prescribed time interval.

The results of detailed numerical simulations confirm the effectiveness of the proposed design with reference to a four-nodes power system connected to a large network with low short-circuit power assigned to stress the effects of a three-phase short circuit near the synchronous generator busbar.

II. MODELING

The synchronous generator can be represented by the following third-order simplified dynamic model [1]

\[
\frac{d\delta}{dt}(t) = \Delta \omega(t) \quad (1)
\]

\[
\frac{d\Delta \omega}{dt}(t) = -\frac{D}{H} \Delta \omega(t) - \frac{\omega_0}{H} (P_e(t) - P_m) \quad (2)
\]

\[
\frac{de'_q}{dt}(t) = -\frac{1}{T_{d0}} \left( e'_q(t) + (x_d - x'_d) i_d(t) - k_v v_f(t) \right) \quad (3)
\]

where \( \delta(t) \) is the rotor angle; \( \Delta \omega(t) = \omega(t) - \omega_0 \) the speed deviation; \( \omega(t) \) the rotor speed; \( \omega_0 \) the synchronous speed; \( P_m \) the mechanical input power; \( P_e(t) \) the active electrical power; \( e'_q(t) \) the transient emf in the quadrature axis; \( i_d(t) \) the direct axis current; \( v_f(t) \) the control input of the excitation system; \( D \) the damping torque constant; \( H \)
the inertia constant; $T_{d0}'$ the direct axis transient open circuit time constant; $x_d$ the direct axis reactance; $x_d'$ the direct axis transient reactance and $k_c$ the gain of the excitation amplifier. In addition, the control input $v_f(t)$ is subject to physical limits represented by

$$V_m \leq v_f(t) \leq V_M.$$

(4)

The classical electrical equations representing the synchronous generator are

$$v_d(t) = x_q i_q(t)$$  \hspace{1cm} (5)

$$v_q(t) = e'_q(t) - x'_q i_d(t)$$  \hspace{1cm} (6)

$$P_e(t) = v_d(t) i_d(t) + v_q(t) i_q(t)$$  \hspace{1cm} (7)

where $x_q$ is the quadrature axis reactance, $i_q(t)$ the quadrature axis current and $v_d(t) = v(t) \sin \phi(t)$ and $v_q(t) = v(t) \cos \phi(t)$ are the voltage component of the terminal voltage of the synchronous generator, namely $v(t)$, along the $d$-axis and $q$-axis, respectively, see Figure 1, where a bar on top of a symbol denotes a phasor.

Concerning the model of the electrical network, it can be represented by the Thevenin equivalent circuit [12] seen from the synchronous generator terminals. Since in high-voltage transmission networks the equivalent resistance $r_{eq}$ can be neglected with respect to the equivalent reactance $x_{eq}$, see [2], [7], the Thevenin model is given by

$$v_d(t) = v_{0d}(t) - x_{eq} i_q(t)$$  \hspace{1cm} (10)

$$v_q(t) = v_{0q}(t) + x_{eq} i_d(t)$$  \hspace{1cm} (11)

with

$$v_{0d}(t) = v_0 \sin \delta(t)$$  \hspace{1cm} (12)

$$v_{0q}(t) = v_0 \cos \delta(t)$$  \hspace{1cm} (13)

being $v_{0d}(t)$ and $v_{0q}(t)$ the components along the $d$-axis and $q$-axis of the no-load voltage $v_0$. The Thevenin equivalent circuit is shown in Fig. 2. The values of $v_0$ and $x_{eq}$ are unknown and change due to variations of power system operating conditions. They can be either determined by off-line procedure such as power flow studies or on-line evaluated by estimation algorithm [4]. Choosing $v_0$ along the real axis in the real-imaginary frame, $\delta(t)$ represents the phase displacement between the quadrature axis and $v_0(t)$, see Fig. 1.

### III. Excitation controller design

To design the excitation controller it is necessary to appropriately merging and arranging the generator model together with the Thevenin equivalent circuit describing the electrical system. To this aim let’s start by deriving $i_d(t)$ and $i_q(t)$ from (10) and (11) as

$$i_d(t) = \frac{1}{x_{eq}} (v_q(t) - v_{0q}(t))$$  \hspace{1cm} (14)

$$i_q(t) = -\frac{1}{x_{eq}} (v_d(t) - v_{0d}(t))$$  \hspace{1cm} (15)

which, substituted into (5) and (6), give

$$v_d(t) = \frac{x_q}{x_q + x_{eq}} v_{0d}(t)$$  \hspace{1cm} (16)

$$v_q(t) = \frac{x_{eq}}{x'_q + x_{eq}} e'_q(t) + \frac{x'_d}{x'_d + x_{eq}} v_0 \cos \delta(t).$$  \hspace{1cm} (17)

Using (13), from (17) one has

$$e'_q(t) = \frac{(x'_d + x_{eq})}{x_{eq}} v_0 \cos \delta(t) - \frac{x'_d}{x'_d + x_{eq}} v_0 \cos \delta(t).$$  \hspace{1cm} (18)

Differentiating (17), using (1), (3) and (13) one obtains

$$\frac{dv_q}{dt}(t) = -\frac{x_{eq}}{(x'_d + x_{eq}) T_{d0}'} \left( e'_q(t) + (x_d - x'_d) i_d(t) - k_c v_f(t) \right) - \frac{x'_d}{x'_d + x_{eq}} v_0 \sin \delta(t) \Delta \omega(t).$$  \hspace{1cm} (19)

Now inserting (14) and (18) in (19) yields

$$\frac{dv_q}{dt}(t) = \frac{1}{(x'_d + x_{eq}) T_{d0}'} \left( - (x_d + x_{eq}) v_q(t) + x_d v_{0q}(t) + x_{eq} k_c v_f(t) - x'_d T_{d0}' v_0 \sin \delta(t) \Delta \omega(t) \right).$$  \hspace{1cm} (20)

The next step is to obtain the expression of $dv_q(t)/dt$ to insert at the left-hand side of (20). To this aim, differentiating (8) and (9) one has

$$\frac{dv_d}{dt}(t) = \frac{dv}{dt}(t) \sin \phi(t) + v(t) \cos \phi(t) \frac{d\phi}{dt}(t)$$  \hspace{1cm} (21)

$$\frac{dv_q}{dt}(t) = \frac{dv}{dt}(t) \cos \phi(t) - v(t) \sin \phi(t) \frac{d\phi}{dt}(t)$$  \hspace{1cm} (22)
while differentiating (16) and using (12) one obtains
\[
\frac{dv_d}{dt}(t) = \frac{x_q}{x_q + x_{eq}} v_0 \cos \delta(t) \Delta \omega(t).
\] (23)

Substituting (23) in
\[
v(t) \frac{d\phi}{dt}(t) = \frac{1}{\cos \phi(t)} \frac{dv_d}{dt}(t) - \tan \phi(t) \frac{dv}{dt}(t)
\]

obtained from (21), yields
\[
v(t) \frac{d\phi}{dt}(t) = \frac{x_q}{x_q + x_{eq}} v_0 \cos \delta(t) \Delta \omega(t) - \tan \phi(t) \frac{dv}{dt}(t)
\]
which inserted in (22) gives the sought expression
\[
\frac{dv_q}{dt}(t) = \frac{1}{\cos \phi(t)} \frac{dv}{dt}(t) - \frac{x_q v_0}{x_q + x_{eq}} \tan \phi(t) \cos \delta(t) \Delta \omega(t).
\] (24)

Substituting (24) in (20), using (9), after some trivial mathematical manipulations one has
\[
\frac{dv}{dt}(t) + \frac{1}{T_d^*} v(t) = k_1 \cos \delta(t) \cos \phi(t) + k_2 \cos \phi(t) v_f(t) + \alpha(t) \Delta \omega(t)
\] (25)

where
\[
T_d^* = \frac{(x_d' + x_{eq}) T_d}{(x_d' + x_{eq}) T_{d0}} \frac{1}{\cos^2 \phi(t)}
\] (26)
\[
k_1 = \frac{x_d}{(x_d' + x_{eq}) T_{d0}} v_0
\]
\[
k_2 = \frac{x_{eq}}{(x_d' + x_{eq}) T_{d0}} k_c
\]
\[
\alpha(t) = v_0 \left( \frac{x_q}{x_q + x_{eq}} \sin \phi(t) \cos \delta(t) - \frac{x'_{eq}}{x'_{eq} + x_{eq}} \cos \phi(t) \sin \delta(t) \right).
\] (27)

Letting
\[
m(t) = k_1 \cos \delta(t) \cos \phi(t) + k_2 \cos \phi(t) v_f(t) + \alpha(t) \Delta \omega(t)
\] (28)
model (25) is compensated into the following
\[
\frac{dv}{dt}(t) + \frac{1}{T_d} v(t) = m(t)
\] (29)
where \(m(t)\) is selected as the new input.

From (26) it is easy to recognize that \(T_d^*\) is constant if \(x_{eq}\) and \(\phi(t)\) are constant, that is only during steady-state operating conditions. From the mathematical point of view, it is possible to verify that
\[
\lim_{x_{eq} \to 0} \frac{1}{T_d^*} = \frac{\cos^2 \phi(t)}{T_d^*}
\] (30)
\[
\lim_{x_{eq} \to \infty} \frac{1}{T_d^*} = \frac{\cos^2 \phi(t)}{T_d}
\] (31)
in which
\[
T_d' = \frac{x_d' T_{d0}}{x_d} < T_d^{'*}
\] (32)
is the direct axis short circuit transient time constant. Considering (30)-(32), it is possible to state that
\[
0 < \frac{1}{T_d^{'*}} < \frac{1}{T_d^*}.
\] (33)

Since typical values of the machine parameter \(T_d^{'}\) range around unity, \(1/T_d^{'}\) varies in a relatively small interval lower limited by zero.

From (28), the compensating law is obtained as
\[
v_f(t) = \frac{1}{k_2 \cos \phi(t)} \left( m(t) - \alpha(t) \Delta \omega(t) \right) - \frac{k_1}{k_2} \cos \delta(t)
\] (34)
in which \(\delta(t)\) is computed as [5]
\[
\delta(t) = \tan^{-1} \left( \frac{v_i(t) + x_q i_r(t)}{v_r(t) - x_q i_r(t)} \right)
\]
while
\[
\phi(t) = \delta(t) - \tan^{-1} \left( \frac{v_i(t)}{v_r(t)} \right)
\]
where \(v_r(t)\) and \(v_i(t)\) represent the components of \(v(t)\) in the real and imaginary frame estimated by a Kalman filter [4].

Compensating law (34) is singular if \(\phi(t) = 90^\circ\). During normal operating conditions or in presence of small disturbances \(\phi(t)\) is smaller than \(90^\circ\); consequently, law (34) is not singular. Viceversa, in the presence of a fault, shortly after that the fault is cleared, due to the mechanical swings \(\phi(t)\) may assume the value \(90^\circ\). However, in such conditions, the control input \(v_f(t)\) switches between its limits to quickly recover the pre-fault value of \(v(t)\) from the transient voltage depression.

A simple controller which fulfills the voltage regulation objective is
\[
C(s) = \frac{k(1 + s \tau)}{s}
\] (35)
with \(k > 0\) and \(\tau < T_d\). To avoid the wind-up of the integrator due to the presence of the saturation imposed on the control input of the excitation system which is forced to satisfy (4), the signal \(m(t)\) is restricted to \(M_m \leq m(t) \leq M_M\) where \(M_m\) (respectively \(M_M\)) is determined replacing \(v_f(t)\) by \(V_m\) (respectively \(V_M\)) in (28).

A simplified block diagram of the excitation control loop is reported in Figure 3. It is trivial to demonstrate that under controller (35), parameter uncertainty of \(1/T_d^{'}\) in the range (33) does not affect the excitation control loop stability.
IV. PSS DESIGN

In presence of a large disturbance, such as for example a short-circuit, the mission of the PSS is to provide a stabilizing signal, in the remainder denoted by \(v_{pss}(t)\), which damps the electromechanical oscillations. Considering a first-order approximation, signal \(v_{pss}(t)\) can be expressed by

\[
v_{pss}(t) \approx \left(\frac{\partial P_e}{\partial v}\right)^{-1} P_{pss}(t)
\]

in which \(P_{pss}(t)\) represents the contribution, during the fault, of the PSS to the rotor angle dynamics given by (2). The proposed expression for \(P_{pss}(t)\) is

\[
P_{pss}(t) = f(\Delta \omega(t)) + g(\Delta \dot{\omega}(t))
\]

where \(f\) and \(g\) are two functions to design and \(\Delta \dot{\omega}(t) \triangleq d\Delta \omega(t)/dt\). The excitation control loop with PSS is reported in Figure 4.

As concerns the active electrical power \(P_e(t)\), from (7) and using (5), (6), (8), (9) and (18) it can be expressed by

\[
P_e(t) = \frac{(x_q + x_{eq})}{2 x_q x_{eq}} v^2(t) \sin(2 \phi(t)) - \frac{v_0}{x_{eq}} v(t) \sin(\phi(t)) \cos(\delta(t))
\]

which allows to compute

\[
\frac{\partial P_e}{\partial v}(t) = \frac{(x_q + x_{eq})}{2 x_q x_{eq}} v(t) \sin(2 \phi(t)) - \frac{v_0}{x_{eq}} \sin(\phi(t)) \cos(\delta(t)).
\]

Adding \(P_{pss}(t)\) to \(P_e(t)\) in (2), since

\[
\frac{dP_e}{dt}(t) = \frac{\partial P_e}{\partial v}(t) \frac{dv}{dt}(t) + \frac{\partial P_e}{\partial \omega}(t) \frac{d\omega}{dt}(t) + \frac{\partial P_e}{\partial \delta}(t) \Delta \omega(t)
\]

and

\[
\frac{dP_{pss}}{dt}(t) = \frac{\partial f}{\partial \Delta \omega} \Delta \dot{\omega}(t) + \frac{\partial g}{\partial \Delta \omega} \Delta \dot{\omega}(t)
\]

with \(\Delta \dot{\omega}(t) \triangleq d\Delta \omega(t)/dt\), the differentiation of (2) yields

\[
(H + \omega_0 \frac{\partial g}{\partial \Delta \omega}) \Delta \dot{\omega}(t) + (D + \omega_0 \frac{\partial f}{\partial \Delta \omega}) \Delta \omega(t) + \omega_0 \frac{\partial P_e}{\partial \delta}(t) \Delta \omega(t) + \omega_0 \frac{\partial P_e}{\partial \delta}(t) \frac{d\delta}{dt}(t) \Delta \omega(t)
\]

Finally, (36) becomes

\[
v_{pss}(t) = \left(\frac{\partial P_e}{\partial v}(t)\right)^{-1} (k_f \Delta \omega(t) + k_g \Delta \dot{\omega}(t))
\]

with \(k_f\) and \(k_g\) given by, respectively, (39), (45) and (42).

Signal \(v_{pss}(t)\) requires the availability of \(\Delta \dot{\omega}(t)\). It can be obtained, for example, by using a tracking differentiator which can track the differential of the input signal with high precision and fast converging speed [15].

V. NUMERICAL SIMULATION

The power system shown in Figure 5 is employed to evaluate the effectiveness of the proposed design. It is a three-phase 220 kV - 50 Hz system which is connected to a larger network at node 1; the network equivalent presents a short-circuit power equal to 1,000 MVA and the open-circuit voltage equal to 1.01 p.u. (per unit) on a 220 kV base. It is important to remark that the assumed short-circuit power value is much lower than the typical values in HV nodes, so as to considering critical conditions for stability study. Three-phase simulation is performed accounting for \(\Delta/Y\) transformer winding connections. The lumped parameter of the elementary two-port circuits, modeling the 220 kV
overhead transmission lines, are evaluated assuming the following values per length unit: series resistance equal to 84 mΩ/km, series inductance equal to 1.3 mH/km and shunt capacitance equal to 8.6 nF/km. At node 3 and 4 two loads (named, respectively, $Q_2$ and $Q_1$) are connected both with a power factor equal to 0.9 and rated power equal to 50 MW. In addition, a capacitor bank, with rated power equal to 50 MVAR, is connected at node 4.

At node 2 a 180 MVA synchronous generator, coupled with a 150 MW turbine, is connected to the network by a 200 MVA–20/220 kV step-up transformer. A full eighth-order synchronous generator Park’s model is simulated; the machine parameters are reported in Table I. In the following ‘per-unit’ (p.u.) values refer to the generator rated power as basis.

The limits in (4) are respectively set to $-V_m = V_M = 6$ p.u.; in addition $k_c = 1$, $x_{eq} = 0.24$ and $v_0 = 1.01$. The value of $x_{eq}$ has been assumed equal to the sum of the short-circuit reactance of the network equivalent and of the L3 line reactance. Such value is used for the design and it is different from the actual value of $x_{eq}$, which is unknown and varies due to the changes in the operating conditions (see the following simulated sequences).

The desired voltage set-point $v_{des}$ is set equal to 1.03 p.u. while the controller parameters are $\tau = 0.1$ s and $k = 120$ p.u.. Concerning the PSS design it is set $\alpha_{ad} = 0.6$, and $\tau_d = 0.75$ s, which corresponds to impose, respectively, a 40% attenuation of the disturbances in (43) and a damping of mechanical oscillations within about 3 s. Accordingly it is obtained $k_f = 0.0934$ and $k_g = 0.0159$. Furthermore the stabilizing signal is limited to $|v_{pss}(t)| < 0.3$ p.u..

The symmetrical three-phase short circuit fault occurs at the beginning of the transmission line L3 next to the generator busbar 2. The following operating sequences are simulated:

1: the power system at $t < 0.1$ s is in a pre-fault steady-state operating condition;
2: the fault occurs at $t = 0.1$ s;
3: the fault is cleared by opening the breakers of line L3 at $t = 0.3$ s;
4: the power system remains in a post-fault configuration with line L3 disconnected for $t > 0.3$ s.

The closed-loop system responses are shown in Figures (6)-(9), reporting respectively the generator busbar voltage, the control input of the excitation system, the stabilizing signal and the rotor speed deviation. From the simulation results it is possible to state that voltage regulation objective as well as swings damping are satisfactory achieved in less than 3 s after fault clearance.

To verify that, in presence of this severe fault, compensating law (34) is never singular, Figure 10 shows the time evolution of $\phi(t)$ evidencing that it remains quite lower than 90° during all the time.

VI. CONCLUSIONS

One of the main problem in power system control is to assure that in presence of a large disturbance the PSS must realize effective swings damping whereas the excitation controller must sustain the regulated voltage so that it can reassert the desired set-point. With reference to this problem the presented paper has developed a nonlinear design
which is articulated in two steps. In the first, the excitation controller is designed using a nonlinear compensating law to obtain a simple linear power system model which is subsequently employed to design a feedback controller. The second step deals with the PSS design, which is developed starting from a proposed expression of the power output by the PSS; such a power is function of the speed deviation and of its derivative. Subsequently, the stabilizing signal is given in terms of the designed PSS power. The overall design requires only four gains whose values can be easily chosen since they are strictly related to system performance. The proposed design gives satisfactory swings damping and effective recovering of the desired voltage set-point, as demonstrated by the results of numerical simulations output in the case of a three-phase fault near the generator terminal. The extension of the proposed design to the multi-machine case is currently under study.

REFERENCES