Altitude Control of Flapping-wing MAV Using Vision-Based Navigation

S.H. Lin, F.Y. Hsiao*, C.L. Chen, and J.F. Shen

Abstract—The altitude control of flapping-wing micro-aerial vehicles (MAVs) is discussed in this paper. The Tamkang University (TKU) has been devoted to the development of flapping-wing robots for a long period, including design, fabrication and control. On the basis of the earlier knowledge on the Golden Snitch, a flapping-wing MAV in TKU, we develop a non-intrusive navigation methodology by using stereo vision, and stabilize the flight trajectory with a modified P-control. Different from other mechanical or aerial systems, the selections in control signal are limited in this problem due to the restrictions in carry-on weight. Numerical simulations and flight tests are also provided to demonstrate the robustness of our control law and the system structure. We are confident that this should be the first flapping-wing MAV under 10 grams capable of autonomous flight. The ultimate goal of this work is to realize fully autonomous flight of flapping-wing MAVs.

I. INTRODUCTION

This paper studies the altitude control of a flapping-wing MAV, especially the MAV developed in the Tamkang University. Flight in flapping is a very efficient way to transport a unit of mass over a unit of distance, even though it requires extremely high power output [9]. For this reason, it is an interesting field and a new generation technology for the researchers to investigate.

The TKU MEMS Laboratory has been developing bird-like flapping MAVs for several years, and the most recent prototype, “Golden Snitch”, is a 8-gram-weight and 20-cm-wingspan aircraft including the fuselage, flapping wings, tail wing, battery, motor and a set of gear system. The flapping wing is driven by a motor with a four-bar linkage system. By adjusting the lengths of the four bars, various stroke angles can be designed. the stroke angle of Golden Snitch is designed around 53° [5].

Researchers have been devoted themselves into the development of flapping-wing MAVs. Many theories about dynamics and control laws are come up with. Lighthill [8] performed some of the earliest theoretical studies on the aerodynamics of insect flight and Weis-Fogh and Jensen [12] determined the variation of the positional angle of fore and hind wings during flight of Schistocerca gregaria. A variety of experimental studies has enabled a better understanding of the nature of wing articulation by insects in hover and forward flight [4], [11], [13]. Moreover, several modeling and control laws are also proposed by researchers [2], [3], [6], [7], [10].

In this research, we investigate the altitude control of flapping MAV. Although some researchers have been investigating this problems with application of various control theories, most of the results are not implementable in the current days. To implement those control law we requires powerful onboard computer and various sensors. However, these equipments are hardly possible to carry onboard, due to the limitation of size and weight. Consequently, we don’t intend to propose a new or fascinating control law in this paper. Instead, we focus on the practical problems about how to implement the control law and realize autonomous flight of the flapping MAV under current technology level.

In detail, taking the Golden Snitch as the example, we derive the equations of motion and obtain those dynamical coefficients with experimental data. To solve the weight problem, we select a commercial IR transmission module, which unfortunately constrains our control ability. We also propose an non-intrusive method, using stereo-vision, to obtain the position and attitude of the flapping-wing vehicle. Based on the current hardware that we can construct, a P-control with modified output feedback algorithm is proposed, and numerical simulations and flight tests are provided to demonstrate the robustness of our algorithm.

II. DYNAMICS MODEL

The flapping-wing MAV developed by the MEMS Laboratory in the Tamkang University, the Golden Snitch, is shown in Fig. 1, and its full nonlinear dynamics of this vehicle has been discussed in Ref. [15].

In this paper, however, we only consider the dynamics in the vertical motion with focus on the implementation of altitude control law. Although realizing autonomous flight is the ultimate goal, in the current stage we are under the constraint of mechanisms, leading to the limited operation capability in flapping frequency and null ability in direction and attitude controls. As a consequence, this paper mainly discuss the control law and implementation in vertical motion, and put the direction and attitude controls in future schedule.

According to the Newton’s second law, the dynamics of the vertical motion can be formulated as

\[ \sum F_z = m\ddot{z} \]

where forces include the weight, \( mg \), the averaged lift force over one flapping period generated by the main wing, \( F_w \),
and the lift force generated by the tail, $F_t$. Hence we expand the equation of motion as

$$F_w + F_t - mg = m\ddot{z}$$  \hspace{1cm} (1)$$

According to Ref. [5] and [15], the lift forces of the main wing and the tail are functions of several parameters, given by

$$F_w = \frac{1}{2} \rho U^2 S_w C_{L_w}$$ \hspace{1cm} (2)$$

$$F_t = \frac{1}{2} \rho U^2 S_t C_{L_t},$$ \hspace{1cm} (3)$$

respectively, where $\rho$ denotes the air density, $U$ the incoming wind speed, $S$ the area of the wing, and $C_L$ the lift coefficient. The subscript $w$ and $t$ denote the main wing and the tail, respectively. For simplicity, we would like to let $K_w = \rho S_w/2$ and $K_t = \rho S_t/2$, and replace those coefficients with $K_w$ and $K_t$ in future derivations.

The averaged lift coefficient over one flapping period generated by the main wing is a function of the advance ratio $J$ [15], given by

$$C_{L_w} = \zeta e^{-\eta J} + \xi$$ \hspace{1cm} (4)$$

where the constants $\zeta$, $\eta$, and $\xi$ are functions of set angle. No theory so far is mature enough to predict those constants and the experimental values for the Golden Snitch are listed in Table I [5]. In addition, the advanced ratio, $J$, is defined as

$$J = \frac{U}{2b f \Phi}$$ \hspace{1cm} (5)$$

where $\Phi$, $f$, and $b$ are the stroke angle, flapping frequency, and semi-wingspan, respectively. The experimental values of the tail lift coefficient as a function of angle of attack are provided in Ref. [15] and listed in Table II.

### III. HARDWARE

Aside from the dynamics of the Golden Snitch, we now turn to the hardware implemented for the control of the flapping-wing MAV. Different from a regular unmanned aerial vehicle, on which most control laws can be easily implemented, an MAV is greatly constrained by its size and weight. Consequently, implementation of control laws and realization of autonomous flight must take consideration of these two factors.

#### A. Hardware Architecture

Figure 2 provides the traditional hardware architecture for the autonomous flight of unmanned aerial vehicles. Traditionally the aircraft is equipped with an onboard computer (OBC), in charge of communicating with ground station, receiving flight data from sensors/inertia measurement unit (IMU), and calculating control signals based on designed control law. Sensors or an IMU is also equipped onboard in order to collect flight data; antennas and encoder/decoder are also necessary for the purpose of communication. Trajectory is usually pre-assigned or controlled by the ground station via the communication module.

In the regime of MAV, however, the traditional architecture may not be suitable due to the limitation of size and weight. Take the Golden Snitch for example. We install a commercial infrared (IR) communication module (less than 1 grams) onboard, and keep the total weight of the vehicle under 10 grams. If the traditional architecture were to applied, the lightest RF communication module, including encoder and decoder, is $\sim 3$ g, and the MEMS gyro is $\sim 30$ g. These are definitely too heavy to apply, much less onboard computer or control chips.

A modified architecture is designed in this research, depicted in Fig. 3. As mentioned previously, the traditional RF communication module is replaced by a 1-gram IR module, and we developed an alternative navigation methodology using stereo vision. The guidance and control signal are accomplished and in the ground station, instead of onboard computer. As a result, we don’t equip any onboard control unit in this research. This structure indeed confines the flight capability and applicability of the flapping-wing MAV, but
Fig. 3. The modified hardware architecture for the autonomous flight of our flapping-wing MAV.

### TABLE III

<table>
<thead>
<tr>
<th>Thrust Level (No)</th>
<th>Flapping Rate (Hz)</th>
<th>Thrust Level (No)</th>
<th>Flapping Rate (Hz)</th>
<th>Thrust Level (No)</th>
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<td>6</td>
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<td>7</td>
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<tr>
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<td>11.76</td>
<td>9</td>
<td>12.35</td>
<td>14</td>
<td>12.8</td>
</tr>
</tbody>
</table>

Fig. 4. Geometries of the stereo camera. (Left) Crossing method and (Right) parallel method

Fig. 5. Definition of “camera coordinate system”

Fig. 6. (Left) The view of the left camera; (Right) The view of the right camera

C. Stereo-Vision Based Navigation

1) Position Acquisition Using Stereo Vision: There are two types of geometry to obtain stereo vision: the crossing method and the parallel method, as shown in Fig. 4. The cross method resembles human eyes more. In this project, however, we would select the parallel method.

Consider an experiment setting shown in Fig. 5. Let point $P$ be the target to observe. Then Fig. 6 shows the view of the left and the right camera, respectively. Define $P_{\text{max}}$ as the largest pixel numbers counted from the central line, and $\theta_{\text{max}}$ as the field of view, as shown in Fig. 7 (a). Then the $x$ coordinate of the target is given by [1]

$$x = C \left( \frac{1}{2} + \frac{1}{\rho - 1} \right)$$

(6)

where $C$ is the dispersion of the two camera, $P_1$ and $P_2$ are the pixels away from the center seen by the left and the right camera, respectively, and $\rho = P_1/P_2$. By defining $\gamma = P_2/P_{\text{max}}$ we can further obtain the depth $y = L$ by

$$L = \frac{C}{\gamma (\rho - 1) \tan \theta_{\text{max}}}$$

(7)

The height of the target can be computed in the similar way. we define $\phi_{\text{max}}$ the field of view in the vertical direction. Definitions of $q$ and $q_{\text{max}}$ are shown in Fig. 7 (b). Then [1],

$$z = \mu \tan \phi_{\text{max}}$$

(8)

where $\mu = q/q_{\text{max}}$.

it is the easiest and the only way to realize the autonomous flight nowadays.

B. The Communication Module of Golden Snitch

As mentioned earlier the limitation of carry-on weight makes us select a specific commercial infrared communication module, which transmits signals in three channels, potentially applicable to control of the thrust, the direction, and the mode.

In channel one, 14 levels of signals are designated and applied to thrust control, corresponding to 14 spin rates of the driving motor, which drives the main wing through a four-bar linkage [5]. As a result, the 14 levels of signals generate thrusts through modifying the flapping frequencies. The relation between the thrust level and the flapping frequency is given in Table III. On the other hand, 6 levels of signals are designated and reserved for future direction control.

3 levels of command can be assigned in each direction (right or left). Finally, one of three modes can be selected so that signals will not interfere with each other if more than one bird are flying.

Due to the constraint of the communication module, a regular state feedback or output feedback is not feasible since the control signal cannot be simply proportional to the state or output. A different control law must be investigated.
Fig. 7. a) Definition of largest pixel in horizontal direction and the half field of view; b) Definition of pixels in vertical direction

![Diagram](image)

Fig. 8. Definition of z-axis in "camera coordinate system"

### IV. CONTROL LAW DESIGN

As stated previously this research mainly focuses on how the control law can be implemented in our robotic bird. Accordingly, a complicated control law is unrealistic since we only have limited choices in control signals. Therefore, the traditional P-control is investigated and shown robustness in practical implementation.

#### A. Linearized Dynamics

The nonlinear dynamics describing the altitude of the vehicle is given in Eq. (1). Assume that the vehicle is original in the cruise condition: \( z = z_0, \dot{z} = 0, U = U_0, \) and \( f = f_0. \) Moreover, assume that the pitch angle of the fuselage is \( \Theta, \) as shown in Fig. 9(a). When in cruise condition, \( \Theta = \Theta_0. \) We conclude that the set angle for the main wing and angle of attack the tail are approximately \( \alpha_w = \Theta_0 + C_w \) and \( \alpha_t = \Theta_0 + C_t, \) respectively. Here, \( C_w \) and \( C_t \) are certain constant, depending on the installation angle of the wings. Moreover, if the vehicle moves upward with a vertical speed \( \dot{z}, \) the angle of attack decreases by

\[
\Delta \Theta = \tan^{-1} \left| \frac{-\dot{z}}{U} \right|
\]

as shown in Fig. 9(b).

To apply the P-control, we first linearize the dynamics about the cruise condition, given by

\[
\delta F_w + \delta F_t = m \delta \dot{z}
\]

Consider the perturbation of the force generated by the main wing. Eqs. (2) and (4) indicate that the force is a function of \( U, \zeta, \eta, \xi, \) and \( J. \) Additionally, \( J \) itself is a function of flapping frequency \( f, \) and \( \{ \zeta, \eta, \xi \} \) are functions of set angle \( \Theta. \) Therefore, we can write

\[
F_w = F_w(U, \zeta, \eta, \xi, f)
\]

The perturbation is then given by

\[
\delta F_w = \frac{\partial F_w}{\partial U} \delta U + \frac{\partial F_w}{\partial \zeta} \delta \zeta + \frac{\partial F_w}{\partial \eta} \delta \eta + \frac{\partial F_w}{\partial \xi} \delta \xi + \frac{\partial F_w}{\partial J} \delta J
\]

Assume the vehicle is suffered from vertical position and speed perturbations, \( \delta z \) and \( \delta \dot{z}, \) respectively. Then we conclude

\[
\delta U = \frac{\partial U}{\partial \dot{z}} \delta \dot{z} = \frac{\partial}{\partial \dot{z}} \sqrt{U_0^2 + \dot{z}^2} \delta \dot{z} = 0
\]

\[
\delta \zeta = \frac{\partial \zeta}{\partial \Theta} \delta \Theta = \frac{\partial \zeta}{\partial \Theta} \delta \dot{z}
\]

\[
\delta \eta = \frac{\partial \eta}{\partial \Theta} \delta \Theta = \frac{\partial \eta}{\partial \Theta} \delta \dot{z}
\]

\[
\delta \xi = \frac{\partial \xi}{\partial \Theta} \delta \Theta = \frac{\partial \xi}{\partial \Theta} \delta \dot{z}
\]

As shown in Fig. 9(b), with an upward speed perturbation the influence to the fuselage angle of attack is given by

\[
\frac{\partial \Theta}{\partial \dot{z}} = \frac{\partial}{\partial \dot{z}} \left( \tan^{-1} \left( \frac{\dot{z}}{U} \right) \right) \bigg|_{U = U_0, \dot{z} = 0} = -\frac{1}{U_0}
\]

On the other hand, the perturbation of \( J \) in terms of the change of control frequency \( \delta f \) can be written as

\[
\delta J = \frac{\partial J}{\partial \delta f} = -\frac{U_0}{2 \delta f \delta J} \delta f
\]

As a result, the perturbation of \( F_w \) can be expanded as

\[
\delta F_w = -\frac{1}{U_0} \left( \frac{\partial F_w}{\partial \zeta} \frac{\partial \zeta}{\partial \Theta} + \frac{\partial F_w}{\partial \eta} \frac{\partial \eta}{\partial \Theta} + \frac{\partial F_w}{\partial \xi} \frac{\partial \xi}{\partial \Theta} \right) \delta \dot{z}
\]

\[
+ \frac{U_0}{2 \delta f \delta J} \delta f
\]

\[
= -K_w U_0 \left( e^{-\eta J} \frac{\partial \zeta}{\partial \Theta} - J \zeta e^{-\eta J} \frac{\partial \eta}{\partial \Theta} + \frac{\partial \xi}{\partial \Theta} \right) \delta \dot{z}
\]

\[
+ \frac{U_0}{2 \delta f \delta J} \delta f
\]

Similarly, the perturbation of \( F_t \) can be expanded as

\[
\delta F_t = \frac{\partial F_t}{\partial \Theta} \delta \Theta
\]

\[
= K_t U_0^2 \frac{\partial C_{L_t}}{\partial \Theta} \frac{\partial \Theta}{\partial \dot{z}} \delta \dot{z}
\]

\[
= -K_t U_0 \frac{\partial C_{L_t}}{\partial \Theta} \delta \dot{z}
\]
For simplicity, we drop the sign “δ” and denote the partial derivative of any parameter \(X\) with respect to \(\Theta\) as \(X_{,\Theta}\) in the future derivation. The linearized equation of motion can then be written as

\[
m\ddot{z} + B\dot{z} = Rf
\]

where

\[
B = K_u U_0 \left( e^{-nJ\zeta_{,\Theta}} - J\zeta e^{-nJ\eta_{,\Theta}} + \xi_{,\Theta} \right) + K_u U_0 C_{L_{z,\Theta}}
\]

\[
R = K_u U_0^2 \eta e^{-nJ} \frac{U_0}{2\Phi f_0}
\]

B. Control Law Design

The linearized equation of motion for the Golden Snitch is given in Eq. (20). As we can see, this is a very neat feedback. The transfer function from the control adjustment equation and can be controlled by using the traditional output feedback signal. The transfer function to the vertical perturbation can be expressed as

\[
\frac{z(s)}{f(s)} = \frac{R}{ms^2 + Bs}
\]

As a result, if a position feedback \(f = -Kz\) is applied, this system will be stable anyway regardless of the value of \(K\).

In our system, we have to consider the discontinuity of the control signal. As stated in the preceding sections, the nominal control is a set of 14 levels, denoting as \(\{f_0, f_0, \cdots f_{0,14}\}\). Consider the cruise condition under input \(f_0\). By taking the difference, we obtain the sequence \(f = \{\Delta f_k\}\), where \(\Delta f_k = f_0 - f_{0,k} k = 1, \cdots, 14\).

As a result, if we consider the position feedback \(f = -Kz\) with a pre-designed feedback gain \(K\) and round it to the nearest element in the control sequence \(\Delta f_k\), then we can view the system as a position feedback with gain \(K' = \Delta f_k/z\). Since this system is stabilizable regardless of the value of the feedback gain, we conclude that this system will converge to the nominal state eventually.

V. Numerical Simulations

The physical parameters of the Golden Snitch are provided in Table IV. The unit for length is meter; for mass is gram; for frequency is Hz; for angle is rad; and for the derivatives is l/rad. Based on those parameters we obtain \(R = 2.0541\) and \(B = 102.0409\). From the root locus analysis, the damping ratio is about 0.7 provided \(K = 300\).

<table>
<thead>
<tr>
<th>Item</th>
<th>Value</th>
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<td>(S_r)</td>
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<td>(S_t)</td>
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<tr>
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<td>(\eta)</td>
<td>4.174</td>
<td>(\xi)</td>
<td>1.181</td>
</tr>
<tr>
<td>(C_{L_{z,\Theta}})</td>
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<td>(\eta_{,\Theta})</td>
<td>1.1007</td>
<td>(\xi_{,\Theta})</td>
<td>2.3945</td>
</tr>
</tbody>
</table>

Figure 10 provides a simulation of this control law, rounding the feedback signal \(f = -Kz\) to the nearest element in the control sequence \(\Delta f_k\). At beginning we assume that the vehicle suffered from a vertical position offset by +10 cm. Then, as shown in Fig. 10(a) the control law starts to work and brings this vehicle back to the nominal height. Figure 10(b) demonstrates the control history. The solid line denotes the history by continuous control while the dashed line denotes the history by our current control module.

We can see that our current control module is less efficient because of the limitation in control capability. Therefore, originally they differ very much. Even so, the current control module, collaborated with the designated control law, still stabilizes the altitude.

VI. Flight Tests

Two types of flight tests are arranged in this research. The first type of flight tests is accomplished inside the laboratory and designed to measure the quantitative properties, while the second type is performed in an indoor open space to verify the qualitative performance of the whole loop. Examples of these experiments are shown in Figs. 11 and 12, whereas a video showing the experiments is provided at [16].

Constrained by the limited space in the lab, the MAV is hung to the ceiling with a string of negligible mass, and this forces the MAV to fly within a certain range. Moreover, the string is long enough not to affect its vertical motion. The system loop is constructed as in Fig. 3, where the altitude of the MAV is measured by the computer stereo-vision system. However, as shown in Fig. 11, we still set up several blue stripes on the wall, denoting 1.8 m, 1.6 m, 1.4 m, and 1.2 m, respectively, from top to bottom, to provide visual references for the human operator.

The experiment results are given in Figs. 13 (a) and (b), where the nominal altitudes are set as 1.5 m and 1.2 m, respectively. Although the measured height seems noisy, it is apparent that the flapping-wing MAV always remains around the vicinity of assigned altitudes. One the other hand, from Fig. 11 where the nominal altitude is set as 1.5 m, we also can see that the altitude of the MAV remains fairly close to the designed 1.5 m. Consequently, we conclude that the severe oscillation of the observed data comes from the measurement noise by the stereo-vision system, and this would be the task to be improved in the future. However, these flight tests still quantitatively verify the design of the control law.

The flight tests in an indoor open space is shown in Fig. 12. Since there is no other method to cross check the
measurements by the stereo-vision in open space, we can only qualitatively verify the structure of the system loop. It is obvious that the robotic bird flies at the assigned height, under the control of the feedback system. Unfortunately, it doesn’t function as well as that inside the lab, due to the complicated background which causes much noise when doing image processing.

![Flight tests in the laboratory.](image1)

![Flight tests in the indoor open space.](image2)

![Nominal altitude is set as a) 1.5 m; b) 1.2 m](image3)

**VII. CONCLUSION**

In this paper we investigate a control law to stabilize the vertical motion of the particular flapping-wing MAV developed in Tamkang University, and develop a system architecture that is potentially helpful to realize autonomous flight of a very-light flapping-wing MAV. At beginning we briefly review the dynamics for the altitude control. Experiments including wind tunnel tests are also done to obtain dynamical and aerodynamical parameters. Considering the limited payload-carrying capability, we modify the control architecture so that automatic control of flight altitude of a very-light flapping-wing MAV is possible under current technology. Taking the hardware constraint into account, we analytically prove that the modified P-control should have worked very well in this case. Numerical simulations and flight tests are also provided to demonstrate the robustness of our control law and the system architecture. We are confident that this should be the first flapping-wing MAV under 10 grams capable of autopilot.

**VIII. ACKNOWLEDGEMENT**

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[16] http://www.youtube.com/watch?v=bexOl4YNnd0