Abstract—This paper presents vision-assisted cyclic pursuit for nonholonomic vehicles without direct communication among the agents. The requirement on communication among agents is relaxed by vision-assisted estimation via a local onboard camera for each agent. A fast estimator is utilized for estimation of the other agent’s unknown time-varying velocity. The effectiveness of the proposed technique is demonstrated by numerical simulation examples.

Key Words: Cyclic pursuit, vision-assisted estimation, fast estimator.

I. INTRODUCTION

Coordination of robotic networks has received significant attention in recent years due to its potential impact in civilian, homeland security, and military applications. Consensus algorithms, i.e., the interaction rule that specifies the information exchange between an agent and some (or all) of its neighbors on the network, have been studied for single-integrator kinematics and double-integrator dynamics, under various communications scenarios (such as fixed or time-varying network topology), with uncertainties or delays. Among these, the cyclic pursuit strategies, where each agent (or robot) pursues only one other vehicle with the network topology forming a unidirectional ring, are particularly simple in that the $n$ robots are ordered such that the robot $i$ pursues the robot $i + 1$, modulo $n$.

Problems based on the notion of pursuit have appealed to the curiosity of mathematicians and scientists over centuries [1]. It was shown in [1], [2] that equilibrium formations for multiple unicycle systems (wheeled vehicles subject to single nonholonomic constraint) in cyclic pursuit include (locally stable) rendezvous at a point, evenly-spaced circular formation, and evenly-spaced logarithmic spirals, depending on the ratio of some controller gains. Reference [3] reported that these regular formations are not the only stable behaviors. Instead, the agents might “weave” in and out, while the formation as a whole moves along a linear trajectory. An alternative cyclic pursuit strategy was given in [3], where each agent pursues the leading neighbor along the line of sight rotated by a common offset angle $\vartheta \in [-\pi/n, \pi/n]$, where $n$ is the number of agents. The above-mentioned three formations can be achieved depending on the value of $\vartheta$.

These results were extended in [4] from 2D to 3D and to the general network topology.

In this paper, vehicle formation control is studied from the perspective that direct communication among agents is unavailable and the agents need to obtain estimates of their neighbors using local sensors. From this viewpoint, the cyclic pursuit is an attractive approach since it requires the minimum number of communication links ($n$ links for $n$ agents) to achieve a formation [3]. The $n$ communication links for the $n$ agents can be readily replaced by installing camera(s) on the agents where agent $i$ only needs to sense agent $i + 1$. In this way, the image processing task on each agent can be affordable.

More specifically, each agent is equipped with a single calibrated camera. A conventional pin-hole camera is used and the camera is calibrated beforehand. The visual information collected by the camera is processed by a fast estimator that provides information (i.e., the other agent’s unknown velocity) needed for the cyclic pursuit algorithm. For feasibility, it is further assumed that the sizes of the agents are known such that estimation of the other agent’s time-varying velocity (for example, in the circular formation) can be obtained. Each agent is a nonholonomic robot modeled by double integrators.

It has been noticed that multi-robot coordination was implemented in [5] to verify the pursuit formation scheme in [1], [2], where each agent is equipped with a stereo-vision system. Vision-assisted satellite formation control and localization were addressed in [6], [7], respectively, where the estimation was obtained via Kalman filter. Single-view depth estimation based formation control of robotic swarms was discussed in [8], [9], where the estimation was performed by direct computation. These results are different from this work in either the vision system setup or the estimation scheme.

The paper is organized as follows. Section II reviews the cyclic pursuit strategy for both single and double integrators. Estimation of the other agent’s unknown velocity is presented in Sec. III. Simulation results are shown in Sec. IV. Finally, Section V concludes the paper.

II. REVIEW OF CYCLIC PURSUIT

Consider $n$ mobile robots (uniquely labeled by an integer $i \in 1, 2, \ldots, n$) in the plane, where agent $i$ pursues the next $i + 1$, modulo $n$. Let $r_i(t) = [x_i(t), y_i(t)]^T \in \mathbb{R}^2$ be the position of the agent $i$ at time $t \geq 0$. If the kinematics of
each agent is described by a simple integrator
\[ \dot{r}_i(t) = u_i(t), \] 
(1)
the following control input [3]
\[ u_i(t) = K \left( r_{i+1}(t) - r_i(t) \right), \quad K \in \mathbb{R}^+ \]
(2)
constructs the classic cyclic pursuit strategy, where agent \( i \) pursues its leading neighbor, agent \( i + 1 \), along the line of sight with a velocity proportional to the vector from agent \( i \) to its leading neighbor. In the scenario where each agent pursues the leading neighbor along the line of sight rotated by a common offset angle \( \vartheta \in [-\pi, \pi] \), the following control input for the agent \( i \)
\[ u_i(t) = K R(\vartheta) \left( r_{i+1}(t) - r_i(t) \right) \]
(3)
with
\[ R(\vartheta) = \begin{bmatrix} \cos \vartheta & \sin \vartheta \\ -\sin \vartheta & \cos \vartheta \end{bmatrix} \]
(4)
achieves rendezvous to a point \( (\vartheta | \in \pi/n) \), evenly-spaced circular formation \( (\vartheta = \pi/n) \), and evenly-spaced logarithmic spiral formation \( (\vartheta \in (\pi/n, 2\pi/n)) \), depending on the value of the common offset angle \( \vartheta \) [3]. For example, when \( n = 2 \), the above three formations can be achieved for \( \vartheta \in (-\pi/2, \pi/2), \vartheta = \pm \pi/2, \text{ and } |\vartheta| \in (\pi/2, \pi) \), respectively. When \( n = 3 \), these three formations can be achieved for \( \vartheta \in (-\pi/3, \pi/3), \vartheta = \pm \pi/3, \text{ and } |\vartheta| \in (\pi/3, 2\pi/3) \).

Hereafter, the subscript \( i + 1 \) is used to denote the leading neighbor (agent \( i + 1 \)) of agent \( i \).

Consider a nonholonomic case where each agent is described by the following nonlinear state-space model:
\[
\begin{bmatrix}
\dot{x}_i(t) \\
\dot{y}_i(t) \\
\dot{\theta}_i(t) \\
\dot{\omega}_i(t)
\end{bmatrix} =
\begin{bmatrix}
v_i(t) \cos(\theta_i(t)) \\
v_i(t) \sin(\theta_i(t)) \\
\omega_i(t) \\
0
\end{bmatrix} +
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 1/m & 0 \\
0 & 0 & 0 & 1/J
\end{bmatrix}
\begin{bmatrix}
F_i(t) \\
\tau_i(t)
\end{bmatrix},
\] 
(5)
where \( r_i(t) = [x_i(t), y_i(t)]^T \) is the inertial position of the robot \( i \), \( \theta_i(t) \) is the orientation, \( v_i(t) \) is the linear velocity, \( \omega_i(t) \) is the angular velocity, \( m \) is the mass, \( J \) is the moment of inertia, \( F_i(t) \) is the force input, and \( \tau_i(t) \) is the torque input. Let \( u_i(t) = [F_i(t), \tau_i(t)]^T \). Following [3], [10], define the “hand” position of the agent to be the point \( h_i(t) = [h_x(t), h_y(t)]^T \), which is on distance \( l \neq 0 \) along the line that is normal to the wheel axis and intersects the wheel axis at the center point (Fig. 1). The objective is to maintain in formation the “hand” position of all agents.

The hand position dynamics are given by \( \dot{h}_i(t) = \nu_i(t) \) and the output feedback linearizing controller is given by [3], [10]:
\[
\begin{bmatrix}
v_i(t) \\
\nu_i(t)
\end{bmatrix} =
\begin{bmatrix}
\frac{1}{m} \cos(\theta_i(t)) & -\frac{1}{m} \sin(\theta_i(t)) \\
\frac{1}{J} \cos(\theta_i(t)) & \frac{1}{J} \sin(\theta_i(t))
\end{bmatrix}^{-1}
\begin{bmatrix}
\dot{x}_i(t) - \dot{y}_i(t) \cos(\theta_i(t)) - l \omega_i^2(t) \cos(\theta_i(t)) \\
\dot{y}_i(t) + \dot{x}_i(t) \sin(\theta_i(t)) - l \omega_i^2(t) \sin(\theta_i(t))
\end{bmatrix}.
\] 
(6)

The following control law
\[
u_i(t) = K \left( R(\vartheta)(h_{i+1}(t) - h_i(t)) - \dot{h}_i(t) \right) + R(\vartheta) \left( \dot{h}_{i+1}(t) - \dot{h}_i(t) \right), \quad K \in \mathbb{R}^+
\] 
(7)
asymptotically provides the desired hand velocity [3]. For simplicity, \( K \) is selected to be 1 without loss of generality.

That is, the control laws (6) and (7) together guarantee globally stable rendezvous to a point, globally stable evenly-spaced circle formation, and globally stable evenly-spaced logarithmic spirals, dependent on the value of the common offset angle \( \vartheta \).

The above control policy requires that each agent knows its orientation \( \theta_i(t) \), its velocity \( \dot{h}_i(t) \), the relative position \( h_{i+1}(t) - h_i(t) \), the relative velocity \( \dot{h}_{i+1}(t) - \dot{h}_i(t) \), and the total number of agents \( n \). As compared to most of the work on coordinated control and cyclic pursuit that require explicit communication among agents [1]–[4], [10]–[13], the problem considered here is to achieve pursuit formation without communication among the agents. Computation of the control law is based solely on sensing and data processing carried out locally. This is achieved by i) equipping each agent with a single camera and ii) assuming that each agent knows the size of the agent to pursue. The second assumption is imposed because the agents’ velocities are typically time-varying during cyclic pursuit and limited results exist so far for estimation of time-varying signals. Fortunately, the second assumption is reasonable when a group of agents work collaboratively. For simplicity, assume that each agent is of the same size, denoted by \( L \). Visual measurements of the leading agent’s center are processed by an estimator that calculates the time variation of the lead agent’s linear velocity, orientation, and angular velocity that are needed to obtain estimates of the relative position \( h_{i+1}(t) \) and relative velocity \( \dot{h}_{i+1}(t) \) in the control law (7).

III. VISION-BASED ESTIMATION

The cyclic pursuit algorithm in (6) and (7) for the nonholonomic system (5) requires knowledge of the relative position \( h_{i+1}(t) - h_i(t) \) and the relative velocity \( \dot{h}_{i+1}(t) - \dot{h}_i(t) \).
When this information is unknown, vision sensors can be used to obtain the estimates. In this section, vision-based estimation problem is formulated in a 2D setting, as shown in Fig. 2. It is assumed that the camera is calibrated beforehand and the agent \( i + 1 \) is within the sensing range of agent \( i \). It is also assumed that some image processing algorithm is available to extract the subtended angle \( \alpha_i(t) \) and the bearing angle \( \beta_i(t) \) for the agent \( i \). Further, each agent is able to recognize and track its leading neighbor when multiple objects are in the scene [5]. For example, each agent can be labeled by a distinguished color or pattern to facilitate pattern recognition [5], [15]. The objective for the agent \( i \) is to estimate the relative position \( \hat{h}_{i+1}(t) - \hat{h}_i(t) \) and the relative velocity \( \dot{\hat{h}}_{i+1}(t) - \hat{h}_i(t) \). For the estimation task, the available information and measurements are listed below:

1) The size \( L \) of the agents.
2) The bearing angle \( \beta_i(t) \) and the subtended angle \( \alpha_i(t) \), as shown in Fig. 2.
3) Each agent’s information from onboard sensors, such as the linear velocity \( v_i(t) \) and orientation \( \theta_i(t) \).

Consider the control law in (7). When information of the agent \( i + 1 \) is available to the agent \( i \) (for example, via direct communication), the position \( \hat{h}_{i+1}(t) \) can be computed as

\[
\hat{h}_{i+1}(t) = r_{i+1}(t) + l \begin{bmatrix} \cos(\theta_{i+1}(t)) \\ \sin(\theta_{i+1}(t)) \end{bmatrix},
\]

where \( l \) is the distance from the “hand” position to the vehicle center. Notice that \( l \) defines a different quantity than \( L \), where \( L \) denotes the size of the vehicle. For simplicity, all agents are assumed to have the same \( l \) and \( L \). Differentiating (8) with respect to time gives [10]

\[
\dot{\hat{h}}_{i+1}(t) = \begin{bmatrix} \cos(\theta_{i+1}(t)) & -l \sin(\theta_{i+1}(t)) \\ \sin(\theta_{i+1}(t)) & l \cos(\theta_{i+1}(t)) \end{bmatrix} \begin{bmatrix} v_{i+1}(t) \\ \omega_{i+1}(t) \end{bmatrix}.
\]

It can be noticed from equations (7) and (9) that the linear velocity \( v_{i+1}(t) \), the orientation \( \theta_{i+1}(t) \), and the angular velocity \( \omega_{i+1}(t) \) of agent \( i + 1 \) are required for implementation of the control law in (7). Estimation of these quantities are presented in the next.

Consider the motion of agent \( i \) and agent \( i + 1 \) in a 2D Cartesian space \((X,Y)\). Let \( z_i(t) = [x_{i,x}(t), z_{i,y}(t)]^T \) be the vector of relative distance between the center of agent \( i \) and the center of agent \( i + 1 \). Notice that \( z_i(t) \) can be computed from onboard sensors and visual measurements of the leading agent as

\[
z_i(t) = \begin{bmatrix} z_{i,x}(t) \\ z_{i,y}(t) \end{bmatrix} = d_i(t) \begin{bmatrix} \cos(\theta_i(t) - \beta_i(t)) \\ \sin(\theta_i(t) - \beta_i(t)) \end{bmatrix},
\]

where \( d_i(t) \) denotes the range between the two agents

\[
d_i(t) = \frac{L}{2 \tan(\alpha_i(t)/2)}.
\]

In the kinematic setting, the relative dynamics can be described by

\[
\begin{bmatrix} \dot{z}_{i,x}(t) \\ \dot{z}_{i,y}(t) \end{bmatrix} = -v_i(t) \begin{bmatrix} \cos(\theta_i(t)) \\ \sin(\theta_i(t)) \end{bmatrix} + v_{i+1}(t) \begin{bmatrix} \cos(\theta_{i+1}(t)) \\ \sin(\theta_{i+1}(t)) \end{bmatrix}.
\]

Let

\[
\eta_i(t) = v_{i+1}(t) \begin{bmatrix} \cos(\theta_{i+1}(t)) \\ \sin(\theta_{i+1}(t)) \end{bmatrix}, \quad \eta_i(0) = \eta_{i0}.
\]

Since the moving ground target is a mechanical system, subject to Newton’s second law, its velocity and acceleration are bounded. Therefore there exist constants \( \mu_\eta \) and \( d_\eta \) such that

\[
\|\dot{\eta}_i(t)\| \leq \mu_\eta < \infty, \quad \forall \ t \geq 0,
\]

\[
\|\ddot{\eta}_i(t)\| \leq d_\eta < \infty, \quad \forall \ t \geq 0.
\]

The estimates of leading agent’s linear velocity \( \dot{v}_{i+1}(t) \), the orientation \( \dot{\theta}_{i+1}(t) \), and the angular velocity \( \dot{\omega}_{i+1}(t) \) (denoted by \( \dot{\eta}_{i+1}(t) \), \( \dot{\theta}_{i+1}(t) \), and \( \dot{\omega}_{i+1}(t) \), respectively) can be obtained through the following steps [16]–[19]:

1) **State Predictor:**

\[
\dot{\zeta}_i(t) = A_m \zeta_i(t) - v_i(t) \begin{bmatrix} \cos(\theta_i(t)) \\ \sin(\theta_i(t)) \end{bmatrix} + \hat{\eta}_i(t),
\]

\[
\zeta_i(t) = \hat{z}_i(t) - \eta_i(t), \quad \zeta_i(0) = z_{i0},
\]

where \( A_m \) is a known Hurwitz matrix.

2) **Update Law:**

\[
\dot{\hat{\eta}}_i(t) = \Gamma_c \text{Proj}(\hat{\eta}_i(t), -P\zeta_i(t)), \quad \hat{\eta}_i(0) = \eta_{i0},
\]

where \( \Gamma_c \in \mathbb{R}^+ \) determines the adaptation gain, chosen sufficiently large to ensure fast convergence, and \( P \) is the solution of the algebraic equation

\[
A_m^T P + PA_m = -Q
\]

for some choice of matrix \( Q > 0 \).

3) **Low-Pass Filter:** Let

\[
\eta_{ie}(s) = C(s) \hat{\eta}_i(s), \quad \eta_{ie}(0) = \hat{\eta}_{i0},
\]
where $C(s)$ is a diagonal matrix, whose $i$th diagonal element $C_i(s)$ is a strictly proper, stable transfer function with low-pass gain $C_i(0) = 1$ for $i = 1, 2$, with $s$ being the Laplace variable. Let

$$C_i(s) = \frac{c}{s + c}, \quad i = 1, 2, \quad c > 0. \quad (18)$$

4) **Extraction of $\hat{v}_{i+1}(t)$ and $\hat{\theta}_{i+1}(t)$ from $\eta_{ie}(t)$:**

$$\begin{align*}
\hat{v}_{i+1}(t) &= \sqrt{\eta_{ie,1}^2(t) + \eta_{ie,2}^2(t)}, \\
\hat{\theta}_{i+1}(t) &= \tan^{-1}\left(\frac{\eta_{ie,2}(t)}{\eta_{ie,1}(t)}\right),
\end{align*} \quad (19)$$

where $\eta_{ie,1}(t)$ and $\eta_{ie,2}(t)$ are the two components of $\eta_{ie}(t)$ represented by $\eta_{ie}(t) = [\eta_{ie,1}(t), \eta_{ie,2}(t)]^T$.

5) **Estimation of Angular Velocity:** To estimate the angular velocity of the leading agent, consider the “hand” position of agent $i + 1$. Let $\hat{z}_i(t) = [\hat{z}_{i,x}(t), \hat{z}_{i,y}(t)]^T$ be the vector of relative distance between the center of agent $i$ and the “hand” position of agent $i + 1$. One can have

$$\hat{z}_i(t) = z_i(t) + \left[\begin{array}{c}
cos(\theta_{i+1}(t)) \\
\sin(\theta_{i+1}(t))
\end{array}\right]. \quad (20)$$

Differentiating (20) with respect to time gives

$$\begin{align*}
\hat{z}_{i,x}(t) &= -v_i(t) \left[\begin{array}{c}
\cos(\theta_i(t)) \\
\sin(\theta_i(t))
\end{array}\right] + v_{i+1}(t) \left[\begin{array}{c}
\cos(\theta_{i+1}(t)) \\
\sin(\theta_{i+1}(t))
\end{array}\right] \\
&\quad + \omega_i(t) \left[\begin{array}{c}
-\sin(\theta_i(t)) \\
\cos(\theta_i(t))
\end{array}\right],
\end{align*} \quad (21)$$

Replacing $\theta_{i+1}(t)$ by $\hat{\theta}_{i+1}(t)$ and $v_{i+1}(t)$ by $\hat{v}_{i+1}(t)$, the above system also exhibits the structure to which the fast estimator can be applied to obtain the estimate of $\omega_{i+1}(t)$. Details are omitted from here due to the similarity of those procedures taken when estimating the linear velocity.

6) **Estimation of Position $\hat{h}_{i+1}(t)$:** Estimate of the position $h_{i+1}(t)$, denoted by $\hat{h}_{i+1}(t)$, can be obtained by

$$\hat{h}_{i+1}(t) = \left[\begin{array}{c}
x_i(t) + d_i(t) \cos(\theta_i(t) - \beta_i(t)) \\
y_i(t) + d_i(t) \sin(\theta_i(t) - \beta_i(t))
\end{array}\right] \\
+ L \left[\begin{array}{c}
\cos(\hat{\theta}_{i+1}(t)) \\
\sin(\hat{\theta}_{i+1}(t))
\end{array}\right], \quad (22)$$

with $d_i(t)$ and $\hat{\theta}_{i+1}(t)$ given in (11) and (19), respectively.

7) **Estimation of Velocity $\hat{h}_{i+1}(t)$:** Estimate of $\hat{h}_{i+1}(t)$, denoted by $\hat{h}_{i+1}(t)$, can be computed as:

$$\hat{h}_{i+1}(t) = \left[\begin{array}{c}
\cos(\hat{\theta}_{i+1}(t)) - l\sin(\hat{\theta}_{i+1}(t)) \\
\sin(\hat{\theta}_{i+1}(t)) + l\cos(\hat{\theta}_{i+1}(t))
\end{array}\right] \left[\begin{array}{c}
\hat{v}_{i+1}(t) \\
\hat{\omega}_{i+1}(t)
\end{array}\right], \quad (23)$$

utilizing the estimated linear velocity, orientation, and angular velocity.

With the estimated quantities, equation (7) is redefined by replacing the unknown position and unknown velocity of agent $i + 1$ with their estimates $\hat{h}_{i+1}(t)$ and $\hat{\dot{h}}_{i+1}(t)$, given by

$$\nu_i(t) = K \left(R(\vartheta)(\hat{h}_{i+1}(t) - h_i(t)) - \hat{h}_i(t)\right) + R(\vartheta) \left(\hat{h}_{i+1}(t) - \hat{h}_i(t)\right), \quad (24)$$

with $\hat{h}_{i+1}(t)$ and $\hat{\dot{h}}_{i+1}(t)$ given in (22) and (23). By doing so, the need of direct communication among agents is relaxed by vision-assisted estimation.

**IV. SIMULATION RESULTS**

Cyclic pursuit with vision-based estimation is implemented in Matlab. The following settings are used in the simulations:

1) Formation setup: $n = 6$, $\vartheta = \left\{ \frac{\pi}{2n}, \frac{\pi}{n}, \frac{1.2\pi}{n} \right\}$.
2) Robot parameters: $m = 1$, $J = 1$, $l = 0.1$, $L = 1$.
3) Robot initial conditions: let $r_i(t)$ be the center of the agent $i$ at time instance $t$ and choose

$$\begin{align*}
r_1(0) &= \left[\begin{array}{c}
8.84 \\
14.90
\end{array}\right], \quad r_2(0) = \left[\begin{array}{c}
-21.92 \\
-0.93
\end{array}\right], \\
r_3(0) &= \left[\begin{array}{c}
-7.00 \\
11.90
\end{array}\right], \quad r_4(0) = \left[\begin{array}{c}
-8.01 \\
-6.96
\end{array}\right], \\
r_5(0) &= \left[\begin{array}{c}
-10.95 \\
-0.04
\end{array}\right], \quad r_6(0) = \left[\begin{array}{c}
-1.94 \\
-7.88
\end{array}\right].
\end{align*}$$

4) Fast estimator parameters:

$$A_m = -I_2, P = I_2/2, \Gamma_c = 10^6, c = 5.$$
\(h_{i+1}(t)\) and \(\dot{h}_{i+1}(t)\), along with their estimates \(\hat{h}_{i+1}(t)\) and \(\hat{\dot{h}}_{i+1}(t)\), are shown in Fig. 7.

V. CONCLUSION

This paper considers cyclic pursuit with vision-based estimation for nonholonomic robots equipped with a single camera. A fast estimator is used to estimate the velocity of the leading neighbor. The effectiveness of the proposed technique is demonstrated by numerical simulation examples.

REFERENCES

Fig. 5. Evenly-spaced logarithmic spiral formation.


Fig. 6. Estimation of the linear velocity and orientation of agent 2 by agent 1 in the circular formation.

Fig. 7. The quantities $h_2(t), \dot{h}_2(t)$ and their estimates $\hat{h}_2(t), \dot{\hat{h}}_2(t)$ obtained by agent 1 in the circular formation.