Adaptive Controller with Delay Compensation for Air-Fuel Ratio Regulation in SI Engines

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Abstract—Stringent requirements to maintain low emission levels over the vehicle lifetime constrain the combustion Air-Fuel Ratio (AFR) within a narrow band around the stoichiometric value and hence pose a challenging AFR control problem. In order to match the desired AFR level Universal Exhaust Gas Oxygen (UEGO) sensors have been recently employed within control loops involving nonlinear engine dynamics. Sensor measurement uncertainties and inevitable effects of aging of these sensors motivate the use of online tuning algorithms for the controller gains. In this paper we develop an adaptive control structure which consists of an adaptive PI controller and an adaptive Smith predictor for time-delay systems with unknown plant parameters. We implement our control design using a plant model representing simplified engine dynamics with a constant UEGO sensor delay based on the assumption that the overall time constant of the plant is unknown. The performance of the adaptive controller is demonstrated through simulation scenarios where additional benefits of our design approach can be observed.

I. INTRODUCTION

RECENT developments in Spark Ignition (SI) engine controls have been aligned with the advances in AFR control design approaches [1]–[6]. Maintaining AFR at the stoichiometric value is necessary to ensure maximum efficiency of Three-Way Catalysts (TWCs) which allow for simultaneous reduction of NO\textsubscript{x}, and oxidation of CO\textsubscript{2} and unburnt HC\textsubscript{s}.

The introduction of the catalytic converter to reduce emissions of the internal combustion engines was over 30 years ago. Today new control technologies are being developed in order to lower emissions. UEGO sensors have recently become more popular in use for control purposes replacing Heated Exhaust Gas Oxygen (HEGO) sensors [7]. The UEGO sensor has its own dynamics and associated response time which act as limiting factors on the closed-loop system performance.

Robustness to parametric uncertainties in the presence of time-delay is an additional concern for control design purposes. Parameter identification for time-delay systems is known to be a complex problem on which only a few results have been reported [8]–[11]. In order to design gain-scheduled AFR controllers an online identification method for time-delay in the fuel path of an SI engine is provided in [12]. An adaptive postacast controller is proposed in [13]. An iterative adaptive AFR control methodology is offered in [14]. Instead of updating a number of controller parameters as in [13], in this paper we only estimate the time constant such as the case in [14]. Another similarity with [14] is that we rely on a gradient algorithm to tune this parameter. On the other hand, in contrast to [14], we set up the adaptive controller to run continuously and adapt a single parameter based on the tracking error between the plant output and the output of the ideal closed-loop reference model. Hence, the proposed adaptive controller runs well even when the initial guess of the unknown parameter otherwise leads to an unstable closed-loop system, as illustrated by our simulations.

The Smith predictor, invented by Otto Smith in 1957, is used to assist the outer control loop constructed for a time-delay system by introducing a predictive inner loop estimating the output of the plant which is yet unobservable due to time-delay. Researchers later introduced several versions of the Smith predictor proving useful within a range of applications involving systems with time-delays. Existing literature on Smith predictor applications includes various examples as listed in [15]–[18].

We consider engine dynamics and sensor measurement uncertainties, and model a time-delay system where we implement a Smith predictor in addition to a properly designed PI controller. Using our adaptive version of the basic Smith predictor we address uncertainties in model parameters. A reference model is developed for the nominal system in Section II. An adaptive controller is designed in Section III where the parameters of both the PI controller and the Smith predictor are updated online using a gradient-based estimation algorithm for the unknown time constant. Simulation results demonstrating the system performance improvement are provided in Section IV. Finally, our conclusions appear in Section V.

II. REFERENCE MODEL FOR NOMINAL SYSTEM

In an SI engine the AFR path from the fuel injectors to the AFR sensor can be approximated by a series connection of a first-order lag and a time-delay [19]. If we describe a first-order plant with time-delay, \( T_d \) using

\[
P(s) = \frac{\alpha}{s + \alpha} e^{-Ts_d} \tag{1}
\]
If we represent the estimates of $\alpha$ and $T_c$ by $\hat{\alpha}$ and $\hat{T_c}$, we can construct an adaptive Smith predictor using either

\begin{equation}
SP(s) = \frac{\hat{\alpha}}{\hat{\alpha} s + 1}(1 - e^{-T_d s})
\end{equation}

In this paper we assume that the time-delay of the plant is accurately known. The time constant of the plant in Equation (1) can be derived in terms of the plant parameter $\alpha$ using the following relation

\begin{equation}
T_c = 1/\alpha
\end{equation}

Hence, the transfer functions in Equation (1) and Equation (2) can be equivalently written as

\begin{align}
P(s) &= \frac{1}{T_c s + 1} e^{-T_d s} \\
SP(s) &= \frac{1}{T_c s + 1}(1 - e^{-T_d s})
\end{align}

A first-order model has been verified through experiments run at 1000 RPM, 0.16 load in closed-loop with a PI controller. A set of five experimental data is obtained as shown in Figure 1. The simulation model and the engine controller use the same PI gains. The AFR disturbance is generated by a step change in the amount of injected fuel by 27%. The delay and the time constant of the model are 0.31 sec and 0.27 sec, respectively.

For any controller $C(s)$ the transfer function of the closed-loop system from the reference signal, $r$ to the system response, $y$ then becomes

\begin{equation}
y = \frac{P(s)C(s)}{1 + P(s)C(s)SP(s)} r
\end{equation}

The above relationship can be equivalently written as

\begin{equation}
y = \frac{P(s)C(s)}{1 + C(s)SP(s) + P(s)C(s)} r
\end{equation}

where we also assume that any changes in plant parameters are slow in time. The nominal control structure is displayed as a block diagram in Figure 2.

The controller $C(s)$ is assumed to be PI in our application, and it can be written in its general form as

\begin{equation}
C(s) = k_p + \frac{k_i}{s}
\end{equation}

where $k_p$ and $k_i$ are the two controller gains which are yet to be tuned. Choosing a relationship between the proportional and integral gains such that

\begin{equation}
k_p = \frac{k_i}{\alpha}
\end{equation}

would allow a left-hand pole-zero cancellation in the closed-loop and result in the PI controller of the form

\begin{equation}
C(s) = \frac{k_i s + \alpha}{s} = k_i \frac{T_c s + 1}{s}
\end{equation}

The plant in Equation (4), the Smith predictor in Equation (5), and the controller in Equation (10) would then result in the following closed-loop response

\begin{equation}
y_m = \frac{k_i s + \alpha}{s} e^{-T_d s} r
\end{equation}

The ideal closed-loop response, $y_m$ in Equation (11) can then be treated as the reference model for the adaptive system to be designed. If the two main parameters of the system, namely the plant time constant, $T_c$ and the time-delay, $T_d$ are assumed to be accurately known, the system response would perfectly follow the desired reference model response. To address the case where the plant time constant is unknown, we propose an adaptive algorithm to be combined with the PI controller and the Smith predictor, as described in the next section.

III. Adaptive Control Design

If the time constant, $T_c$ is assumed to be unknown, the plant dynamics would be given as

\begin{equation}
P(s) = \frac{\alpha}{s + \alpha} e^{-T_d s} = \frac{1}{T_c s + 1} e^{-T_d s}
\end{equation}

If we represent the estimates of $\alpha$ and $T_c$ by $\hat{\alpha}$ and $\hat{T_c}$, we can construct an adaptive Smith predictor using either
one of these estimates such that
\[ SP(s) = \hat{\alpha} \frac{s}{s + \hat{\alpha}} (1 - e^{-T_ds}) = \frac{1}{T_cs + 1} (1 - e^{-T_ds}) \]  
(13)

The adaptive PI controller can also be represented by
\[ \hat{C}(s) = \hat{k}_p + \frac{k_i}{s} \]  
(14)
where the relation \( \hat{k}_p = \frac{k_i}{\hat{\alpha}} \) can be used to write
\[ \hat{C}(s) = \frac{k_i}{\hat{\alpha}} s + \frac{1}{\hat{\alpha}} = k_i \frac{T_cs + 1}{s} \]  
(15)

The system output, \( y \), can then be written as a function of the reference signal
\[ y = \frac{P(s)\hat{C}(s)}{1 + \hat{C}(s)SP(s) + P(s)\hat{C}(s)} r \]  
(16)
where we again use the assumption that the plant parameters change slowly in time. We rewrite Equation (16) as
\[ y = \frac{\frac{1}{T_cs + 1} e^{-T_ds} k_i}{1 + k_i \frac{T_cs + 1}{s} (1 - e^{-T_ds}) + \frac{1}{s} e^{-T_ds} k_i \frac{T_cs + 1}{s}} \frac{T_cs + 1}{s} \]  
(17)

whereas the reference model output would still be equal to \( y_m \) in Equation (11) since it is derived using the nominal system parameters in the first place.

Using an instantaneous cost function as in [20]
\[ J(\hat{T}_c) = \frac{e^2}{2} \]  
(18)
we construct our adaptive law based on gradient algorithm
\[ \dot{\hat{T}}_c = -\gamma \nabla J(\hat{T}_c) = -\gamma e \frac{\partial e(\hat{T}_c)}{\partial \hat{T}_c} \]  
(19)
where \( \gamma > 0 \) is a design parameter which can also be referred to as the adaptation gain, and \( e \) is the estimation error which can be defined as
\[ e = y - y_m \]  
(20)

Since \( y_m \) is independent of \( \hat{T}_c \) one can write
\[ \frac{\partial e}{\partial \hat{T}_c} = \frac{\partial y}{\partial \hat{T}_c} \]  
(21)
and the adaptive law in Equation (19) takes the form
\[ \dot{\hat{T}}_c = -\gamma e \frac{\partial y}{\partial \hat{T}_c} \]  
(22)

We can implement the reference signal, \( y_m \), and measure the output signal, \( y \), so that the estimation error, \( e \) can also be measured directly. To be able to implement the adaptive law in Equation (22), we only need to derive \( \frac{\partial y}{\partial \hat{T}_c} \).

The output signal, \( y \), is rewritten in terms of \( \hat{T}_c \) which is the estimate of the time constant, \( T_c \) and \( \hat{T}_c \) itself following some algebraic manipulations as
\[ y = \frac{k_i \frac{T_cs + 1}{s} e^{-T_ds}}{1 + k_i \frac{T_cs + 1}{s} [1 - (1 - \frac{T_cs + 1}{T_c s + 1}) e^{-T_ds}]} r \]  
(23)

For simplification purposes we define a variable \( m \) as a function of \( T_c \) and \( \hat{T}_c \) as follows
\[ m = \frac{T_cs + 1}{T_c s + 1} \]  
(24)
whose partial derivative with respect to \( \hat{T}_c \) would be
\[ \frac{\partial m}{\partial \hat{T}_c} = \frac{s}{T_c s + 1} \]  
(25)

The system output, \( y \) in Equation (23) is then written in terms of the parameter \( m \) such that
\[ y = \frac{k_i me^{-T_ds}}{s + k_i[1 - (1 - m)e^{-T_ds}]} r \]  
(26)
whereas the derivative of \( y \) would be given by
\[ \frac{\partial y}{\partial \hat{T}_c} = \frac{\partial m}{\partial \hat{T}_c} \frac{\partial y}{\partial m} \]  
(27)

If the second partial derivative in Equation (27), \( \frac{\partial y}{\partial m} \) is

![Fig. 3. Adaptive system structure where the parameters of the controller and the Smith predictor are updated online using the same adaptive law](image)
calculated, the result can possibly be written as
\[
\frac{\partial y}{\partial m} = \frac{k_i e^{-T_{ds}} [s + k_i (1 - e^{-T_{ds}})]}{(s + k_i [1 - (1 - m) e^{-T_{ds}}])^2} \tag{28}
\]

Finally, the partial derivative of \( y \) with respect to \( \hat{T}_c \) can be obtained through Equation (27), using Equation (25) and Equation (28)
\[
\frac{\partial y}{\partial \hat{T}_c} = \frac{s}{T_c s + 1} \frac{k_i e^{-T_{ds}} [s + k_i (1 - e^{-T_{ds}})]}{(s + k_i [1 - (1 - m) e^{-T_{ds}}])^2} \tag{29}
\]
\[
= \frac{s}{s + k_i (1 - e^{-T_{ds}})} \frac{s}{s + k_i [1 - (1 - m) e^{-T_{ds}}]} \frac{1}{T_c s + 1} \tag{30}
\]

We cannot compute the sensitivity as typical in gradient-based adaptive laws because \( m \) itself contains an unknown parameter, namely \( \hat{T}_c \). Instead we can use the following approximation for \( m \) which is defined by Equation (24)
\[
m \approx 1 \tag{31}
\]
in order to rewrite Equation (29) which results in
\[
\frac{\partial y}{\partial \hat{T}_c} \approx \frac{s}{s + k_i (1 - e^{-T_{ds}})} \frac{s}{s + k_i} \frac{1}{T_c s + 1} \tag{32}
\]

We can complete our derivation of the adaptive law by returning back to Equation (19) such that
\[
\hat{T}_c = -\gamma \epsilon \frac{s}{s + k_i} \frac{1}{T_{c s} + 1} \frac{s}{s + k_i} \tag{33}
\]
\[
\approx \gamma \epsilon \frac{s}{s + k_i} \frac{1}{T_{c s} + 1} y \tag{34}
\]
The adaptive control structure is displayed in Figure 3.

IV. SIMULATIONS

A. Nominal System Performance

The nominal system parameters are fixed and chosen as \( T_c = 0.5 \text{ sec}, T_d = 0.5 \text{ sec}, k_i = 3 \). The nominal system structure in Figure 2 is implemented where the time constant, \( T_c \) is assumed to be known. The external reference signal, \( r \) is a square wave. Note that the square wave signal might come from cycling the pre-catalyst AFR across the stoichiometric value based on the relay-type feedback from the post-catalyst HEGO sensor [6].

As a result the control input provided in Figure 4 is applied, and the performance of the nominal system closely matches the response of the reference model represented by \( y_m \) as can be observed from Figure 5.
Fig. 8. Part B: Time constant, $T_c$ estimation performed over time, initialized from $T_{c0} = 0.1\, sec$

Fig. 12. Part C: Time constant, $T_c$ estimation performed over time, initialized from $T_{c0} = 1.2\, sec$

Fig. 9. Part C: Input disturbance and reference centered at 0, illustrating relatively small SNR

Fig. 13. Part D: Input disturbance and reference centered at 0, first 30 sec performance

Fig. 10. Part C: Adaptive system performance in the presence of input disturbance

Fig. 14. Part D: Adaptive system performance in comparison to the fixed gain controller

Fig. 11. Part C: Adaptive control input for output regulation in the presence of input disturbance and system delay

Fig. 15. Part D: Oscillatory system response to the fixed gain controller
B. Unknown Time Constant and Adaptive Control

The adaptive control structure in Figure 3 is implemented. The time constant, $T_c = 0.5 \text{ sec}$, is assumed to be unknown for control design purposes. The adaptive gain is $\gamma = 20$, the proportional gain, $k_p$, is updated online using the adaptive law, and the integral gain is kept at its constant value of $k_i = 3$ for the rest of the simulations. The initial estimate is set to $T_{o0} = 0.1 \text{ sec}$. The system response and the control input are displayed in Figure 6 and Figure 7, respectively. The time constant estimation history is shown in Figure 8. The estimate converges to its real value.

C. Adaptive Control Performance in the Presence of Control Input Disturbance

The adaptive controller is now initiated using an estimate of $T_{o0} = 1.2 \text{ sec}$. The adaptive gain is again chosen to be $\gamma = 20$. The input disturbance and the reference which is shifted to 0 shown in Figure 9 illustrate how unfavorable the Signal-to-Noise Ratio (SNR) is in this case. The output signal can be observed from Figure 10, and the control signal is provided in Figure 11. One can conclude from Figure 12 that the estimated time constant converges closely to the real value of the unknown time constant, $T_c$.

D. Adaptive Control Performance Comparison to a Fixed Gain Controller

Here we repeat the simulations in the previous subsection by focusing on the first $30 \text{ sec}$. Figure 13 zooms in on the disturbance and the shifted reference signals during the first $30 \text{ sec}$ of the run shown in Figure 9. The system response is displayed in Figure 14 where the initially large oscillations can be observed to be suppressed as the estimate of the time constant converges to its real value in time. If a fixed gain controller is designed assuming $T_c = 1.2 \text{ sec}$ and applied with no adaptation to a system with $T_c = 0.5 \text{ sec}$, the closed-loop system response goes unstable and diverges to infinity. If we also clip the control input between 0 and 2, the fixed gain controller produces the oscillatory system response shown in Figure 15.

V. CONCLUSIONS

The first-order plant model with time-delay, the reference signal, and the input disturbance used in this paper are motivated from the AFR control problem in SI engines. We propose an adaptive control structure including an adaptive PI controller and an adaptive Smith predictor which can be used for delay compensation in the presence of parametric uncertainties. The adaptive algorithm is used to estimate only a single parameter which is the time constant of the plant. The estimation is used to update both the controller and the Smith predictor parameters online. Simulation results demonstrate the effectiveness of our design for various conditions. Potentially unstable dynamics due to unknown time constant and input disturbances can be stabilized using the proposed adaptive controller with delay compensation.

REFERENCES