Optimal Emergency Maneuvers on Highways for Passenger Vehicles with Two- and Four-Wheel Active Steering

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Abstract—This paper presents and utilizes an optimal-control based method for quantifying the maneuverability of actively-controlled passenger vehicles during emergency highway-speed situations. The research is motivated by the desire to better understand the performance benefits of additional actuated degrees of freedom when compared to a typical vehicle. Specifically, the performance benefits of front-steer vehicles are compared to front- and rear-steer vehicles. In both cases, the vehicle is assumed to be capable of applying independent accelerating/braking torques to the front and rear wheels. The emergency maneuvers considered are obstacle avoidance, barrier avoidance, lane changes, and recovery from an arbitrary state. Necessary conditions for optimality and optimal control laws are found for each case. Solutions are found using a lumped-parameter planar single-track vehicle model and nonlinear tire dynamics are assumed. Front- and rear-steering vehicles are shown to have better maneuverability during all emergency maneuvers. Based on the optimal vehicle trajectories, the benefits are quantified using appropriate performance indices.

I. INTRODUCTION

A critical safety metric in the design of automobiles is maneuverability. In particular, this ability for vehicles to quickly change speed and direction is important for lightweight vehicles since they are most susceptible to damage should a collision occur. The primary motivation for this research is to investigate and quantify the potential performance benefits of augmenting a typical front-steer vehicle with rear-steering as well as independent accelerating/braking torques for each.

Interest in optimal emergency maneuvers has intensified over the past two decades. Research can generally be categorized into open-loop methods aimed at finding optimal or near-optimal vehicle trajectories for various objectives, and closed-loop methods for following a desired trajectory. Early methods for finding open-loop lane change maneuvers for front-wheel steering vehicles were proposed by Chee et al. [1] and Hatipoglu et al. [2]. The lane change maneuver was later formulated as an optimization problem by Shiller et al. [3] and approximately solved as a parameter optimization problem for front-wheel actively-steered vehicles. In [4], the general problem of finding optimal trajectories for changing heading given acceleration constraints is discussed. A lane-change trajectory assuming simplified vehicle dynamics is discussed along with closed-loop implementations in [5]. Closed-loop methods for executing lane change maneuvers has also been the subject of intense study [6], [7]. With recent technological advancements in automation and actuation, four-wheel active steering and braking has become a feasible technology within the automotive industry. Some of the advantages of active steering were outlined by Ackermann et al. [8]. Closed-loop utilizations of active steering in four-wheel vehicles are studied by Higuchi et al. [9] and Cho et al. [10]. It appears that the study of optimal avoidance maneuvers for vehicles with multiple actuated degrees of freedom has scarcely been studied to date.

The purpose of this paper is to present an optimal control based method that can be used to quantify the performance of a vehicle during various highway-speed maneuvers, and to utilize the method to quantify the advantages of vehicles with various actuated degrees of freedom. A planar single-track bicycle model is assumed; pitch and roll dynamics are therefore neglected. Nonlinear tire dynamics are assumed and serve as the critical constraint bounding vehicle performance. Pontryagin’s Minimum Principle is utilized to find the optimal control laws, and the optimal open-loop control inputs and trajectories are presented. The tradeoffs and benefits of the different vehicles are discussed by direct comparison of the results. Since the trajectories are optimal and generally maximize the tire tractive force at all wheels, the maneuvers are not robust and therefore impractical to implement directly. Extensions to closed loop control are certainly possible, but outside the scope of this paper.

II. SYSTEM MODELING

The selection of a system model requires making tradeoffs between conflicting goals. Given the goal of finding optimal control trajectories in continuous time, a simple model is desired. On the other hand, relevant vehicle dynamics must be captured by the model for the results to be useful. The model presented in this section is chosen as a compromise. The chosen system model contains three components: the chassis, front wheel model, and rear wheel model. The model, as mentioned in the introduction, is planar. The dynamics of the left and right wheels are lumped together into single front- and rear-wheel tire models.
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TABLE I: Vehicle parameters used for numerical optimization

A. Chassis Model

The chassis is modeled as a planar rigid-body with input forces at the locations of the lumped front and rear wheels, as shown in Figure 1. Such a model without pitch and roll dynamics provides simplicity for the purpose of solving optimal control problems, but at the cost of omitting certain dynamic behaviors of the vehicle. In particular, the neglected pitch and roll dynamics are increasingly important for vehicles with a high center of mass. The equations governing the dynamics of the chassis are given by Equations (1) through (5).

$$F_{l,f} + F_{l,r} = m_v (\dot{v}_l - v_l \dot{\theta})$$  \hspace{1cm} (1)

$$F_{s,f} + F_{s,r} = m_v (\dot{v}_s + v_s \dot{\theta})$$  \hspace{1cm} (2)

$$F_{z,f} + F_{z,r} = m_v g$$  \hspace{1cm} (3)

$$F_{s,f} l_f - F_{z,f} l_r = 0$$  \hspace{1cm} (4)

$$F_{s,f} l_f - F_{z,r} l_r = l_z \ddot{\xi}$$  \hspace{1cm} (5)

In these equations, $F$ refers to the forces acting upon the tire; the index $f$ refers to the front wheel, $r$ to the rear wheel, $l$ to the longitudinal direction, $s$ to the lateral direction, and $z$ to the vertical (normal force) direction.

B. Vehicle Parameters

To obtain solutions, numerical values are necessary for all vehicle and environmental parameters; optimization and simulation parameters are given in Table I. The chassis parameters were estimated to represent as closely as possible a Volkswagen Golf MK2, and the wheel parameters were obtained from [11]. This vehicle was chosen since it exemplifies the mass and tractive properties of a typical lightweight vehicle, by which this research was motivated.

C. Road/Tire Interaction Model

The purpose of the road/tire interaction model is to map a wheel’s steering angle $\theta$ and rotational velocity $\omega_a$ to longitudinal and lateral forces $F_l$ and $F_s$ acting on the chassis. A simplified variant of the “magic tyre formula” proposed by Pacejka [11] is assumed and governed by Equations (6) through (9). For simplicity, all equations are given the subscript $f$ and refer to the equations that govern the front wheel; with appropriate changes, they are also valid for the rear wheel. This model assumes the magnitude of the tire force $F_f$ to be a nonlinear function of the tire slip magnitude and independent of the slip direction. Isotropic tire properties are thus implicitly assumed.

$$f_{t_a}(\sigma_f) = \frac{\alpha_3}{a_2} \arctan \left( a_2 (1 - a_3) \sigma_f + \frac{a_5}{a_2} \arctan (a_2 \sigma_f) \right)$$  \hspace{1cm} (6)

$$F_f = \mu F_{z,f} f_{t_a}(\sigma_f)$$  \hspace{1cm} (7)

$$F_{l,f} = -\frac{\sigma_{l,f}}{\sigma_f} F_f$$  \hspace{1cm} (8)

The parameters $a_2$, $a_3$, and $a_5$ are tire-dependent constants, and $\mu$ is a “grip coefficient” that is equal to unity under nominal conditions and may change, for example, with the moisture content of the road. The longitudinal and lateral tire slip quantities $\sigma_l$ and $\sigma_s$ are the ratios of the contact patch speed to the tire speed, as defined in [11].

The direction, $\beta$, is applied in the opposite direction of the velocity of the tire’s contact patch. It is given for the front wheel by

$$\beta_f = \theta_f - \arctan \left( \frac{v_{s,f}}{v_{l,f} - \omega_a l_f \sigma_f} \right)$$  \hspace{1cm} (10)

where $\omega_a$ represents rotational wheel speed about the axle and $r$ represents the wheel’s radius.

Since the wheel speed $\omega_a$ is assumed to be completely controllable, the force direction given by Equation (10) is also controllable. This is evident by the observation that the $\arctan$ function is surjective over a full range of steering angles $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$. Further, the tire force $F_f$, as given by Equations (6) and (7), can be shown to be bijective and therefore invertible over a restricted domain and range (ref. Figure 2):

$$F_f : \left[ -\frac{\pi}{2}, \frac{\pi}{2} \right] \times \left[ -\frac{\pi}{2}, \frac{\pi}{2} \right] \rightarrow \left[ -\mu F_{z,f}, \mu F_{z,f} \right]$$  \hspace{1cm} (11)

Therefore, within this domain it follows that a one-to-one relationship exists between the wheel force components $[F_{l,f}, F_{s,f}]$ and the physical inputs: the wheel speed $\omega_a$ and steering angle $\theta$. The domain is enforced with a constraint

$$\sqrt{F_{l,f}^2 + F_{s,f}^2} \leq \mu F_{z,f}$$  \hspace{1cm} (12)

which limits the magnitude of the forces. Given this constraint and the bijective relationship between the force components and wheel angle and speed, it is thus possible to neglect the road/tire interaction model completely when the steering angle and wheel speeds are unconstrained. In such a case the force components can be directly used as inputs. When the wheel speed is controllable but not the steering angle, forces cannot be used directly as inputs.

D. Input-Output Models

The chassis and road/tire interaction models can be combined to form input-output models for the vehicles of interest. For all vehicles, the state vector $x$ is first defined. The

Fig. 2: 2D representation of the slip-based tire tractive force. When the domain (slip $\sigma$) is restricted to the bold region, the function is bijective.

$$F_{s,f} = -\frac{\sigma_{s,f}}{\sigma_f} F_f$$  \hspace{1cm} (9)
states consist of the position, orientation, velocity, and rotational velocity of the vehicle. For simplicity of the equations of motion, velocity components \( v_l \) and \( v_s \) are expressed in the vehicle's reference frame.

\[
x \triangleq \begin{bmatrix} x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6 \end{bmatrix} \quad \Delta \quad \begin{bmatrix} x \\
y \\
\zeta \\
v_l \\
v_s \\
\zeta \end{bmatrix}
\]  

(13)

For the dual-steer dual-brake vehicle, the control vector \( u \) is defined as the force components on the front and rear wheels:

\[
u \triangleq \begin{bmatrix} u_1 \\
u_2 \\
u_3 \\
u_4 \end{bmatrix} \quad \Delta \quad \begin{bmatrix} F_{l,f} \\
F_{s,f} \\
F_{l,r} \\
F_{s,r} \end{bmatrix}
\]  

(14)

For the front-steer front-brake vehicle, the control vector simply omits \( u_3 \) and \( u_4 \). For the front-steer dual-brake vehicle, the control vector is:

\[
u \triangleq \begin{bmatrix} u_1 \\
u_2 \\
u_3 \\
u_4 \end{bmatrix} \quad \Delta \quad \begin{bmatrix} F_{l,f} \\
F_{s,f} \\
\omega_{u,l} \end{bmatrix}
\]  

(15)

where the wheel speed \( \omega_{u,l} \) feeds into the road/tire interaction model – along with the vehicle states – to yield the force components \( F_{l,f} \) and \( F_{s,f} \).

Given the state vector and control input vectors, the input-output equations of motion can now be stated for each vehicle. The differential equations governing the dual-steer dual-brake vehicle are:

\[
\dot{x} = \begin{bmatrix} x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6 \end{bmatrix} = \begin{bmatrix} x_4 \cos x_3 - x_5 \sin x_3 \\
x_4 \sin x_3 + x_5 \cos x_3 \\
x_6 \\
x_4 + u_1 + u_2 + u_3 + u_4 \right) \\
x_4 \sin x_3 + x_5 \cos x_3 \end{bmatrix} + \begin{bmatrix} x_4x_6 \\
x_5x_6 \end{bmatrix} \right) \\
\right) \\
\right) \\
\right) \\
\right)
\]  

(16)

The differential equations governing the vehicles with fewer control inputs are derived in a similar manner, but are more complex since the rear tire forces are found using the nonlinear road/tire interaction model, rather than being used directly as inputs.

III. OPTIMAL CONTROL FORMULATION

The purpose of this section is to formalize optimal control problems corresponding to avoidance maneuvers in such a way that the performance of each maneuver is measurable. Four general scenarios are considered: A pure obstacle avoidance maneuver, a barrier avoidance maneuver, a recovery maneuver, and a double lane change maneuver.

A. “Pure Obstacle Avoidance” Maneuver

The goal of the “pure obstacle avoidance” optimal control problem is to quantify a vehicle’s ability to avoid an obstacle of known width, as depicted in Figure 3(a). The vehicle is assumed to be moving straight forward along the highway at a steady state. Beginning at some initial time, the vehicle must attempt to minimize the distance traveled along the highway until the vehicle translates laterally by a fixed distance \( y_f \), which corresponds to the lateral displacement needed to avoid the obstacle. This longitudinal distance traveled is a direct quantification of the performance of a vehicle executing the maneuver. The problem is formalized as

\[
\min_{u(t) \in S} x_1(t_f)
\]  

(17)

subject to

\[
x(t) = f(x(t), u(t))
\]  

(18)

\[
x(0) = \begin{bmatrix} 0 & 0 & 0 & v_{l,i} & 0 & 0 \end{bmatrix}^T
\]  

(19)

\[
x(t_f) \in \mathbb{R}^6: x_2(t_f) = y_f
\]  

(20)

where \( S \) represents the set of allowable control inputs, \( f(\cdot) \) represents the equations of motion of the given vehicle, and \( v_{l,i} \) represents the initial longitudinal velocity.

B. “Barrier Avoidance” Maneuver

The purpose of the “barrier avoidance” maneuver is to reduce to zero the velocity of the vehicle in the direction of the barrier, as depicted in Figure 3(b). An arbitrary initial state is assumed which could, for example, be the final state achieved by the “pure obstacle avoidance” maneuver. The performance of such a maneuver can be quantified as the total distance traveled in the direction of the barrier. The objective is only to minimize this distance; additional end constraints such as yaw angle and rate are therefore not specified. Since no control effort is used to achieve a nominal final vehicle state, the vehicle can terminate this maneuver in an extremely dynamic state which might include large tire slip magnitudes and angles. The problem is formalized as

\[
\min_{u(t) \in S} x_2(t_f)
\]  

subject to (18) and

\[
x(0) = \begin{bmatrix} x_i & y_i & \zeta_i & v_{l,i} & v_{s,i} & \dot{\zeta}_i \end{bmatrix}^T
\]  

(22)

\[
x(t_f) \in \mathbb{R}^6: x_5(t_f) = -x_4(t_f)\tan(x_3(t_f))
\]  

(23)

C. “Recovery” Maneuver

The purpose of the “recovery” maneuver is to bring the vehicle from an arbitrary initial state to a nominal final state, as depicted in Figure 3(c). The initial state could, for example, be the final state achieved by the “pure obstacle avoidance” or “barrier avoidance” maneuvers. Conflicting objectives prevent a simple quantification for vehicle performance during this maneuver. While one objective is to return to a nominal state in a short amount of time, another objective is to prevent a trajectory that leaves the width of the highway. The second objective could be obtained with state constraints. However, such a method significantly increases the complexity of the
The purpose of the “double lane change” maneuver is to execute a double lane change such that the vehicle begins and ends in a nominal state, as depicted in Figure 4. The performance of such a maneuver is again quantified by the longitudinal distance to complete the maneuver. The problem is formalized as

$$\min_{u(t) \in S} \int_0^{t_f} (x^2(t) - y_f)^2 \, dt$$

subject to (18) and

$$x(0) = \begin{bmatrix} x_i & y_i & \zeta_i & v_{li} & v_{si} & \dot{\zeta}_i \end{bmatrix}^T$$

$$x(t_f) \in \begin{bmatrix} \mathbb{R} & y_f & 0 & \mathbb{R} & 0 & 0 \end{bmatrix}^T$$

where $y_f$ represents the desired final lateral position on the highway.

D. “Double Lane Change” Maneuver

The purpose of the “double lane change” maneuver is to execute a double lane change such that the vehicle begins and ends in a nominal state, as depicted in Figure 4. The performance of such a maneuver is again quantified by the longitudinal distance to complete the maneuver. The problem is formalized as

$$\min_{u(t) \in S} x_1(t_f)$$

subject to (18) and

$$x(0) = \begin{bmatrix} 0 & 0 & 0 & v_{li} & 0 & 0 \end{bmatrix}^T$$

$$x(t_f) \in \begin{bmatrix} \mathbb{R} & y_f & 0 & \mathbb{R} & 0 & 0 \end{bmatrix}^T.$$
The Hamiltonian is clearly affine with respect to \( u \). Given the set of allowable control inputs – see Equation (12) – it is also convex, i.e.
\[
S = \left\{ u \in \mathbb{R}^4 \mid \sqrt{u_1^2 + u_2^2} \leq \mu F_{z,f}, \sqrt{u_3^2 + u_4^2} \leq \mu F_{z,r} \right\},
\]
the optimal control law can be solved analytically using the Karush-Kuhn-Tucker conditions [14]. The resulting optimal control input force vectors are best expressed in polar coordinates:
\[
\hat{F}_f = \sqrt{u_1^2 + u_2^2} = \mu F_{z,f} = \frac{\mu l_f m_v g}{l_f + l_r} \tag{36}
\]
\[
\hat{\beta}_f = \arctan \left( \frac{\dot{u}_2}{\dot{u}_1} \right) = \arctan \left( \frac{l_f \hat{\lambda}_5 + l_f m_v \hat{\lambda}_6}{l_r \hat{\lambda}_4} \right) \tag{37}
\]
\[
\hat{F}_r = \sqrt{u_3^2 + u_4^2} = \mu F_{z,r} = \frac{\mu l_f m_v g}{l_f + l_r} \tag{38}
\]
\[
\hat{\beta}_r = \arctan \left( \frac{\dot{u}_4}{\dot{u}_3} \right) = \arctan \left( \frac{l_f \hat{\lambda}_5 - l_r m_v \hat{\lambda}_6}{l_r \hat{\lambda}_4} \right) \tag{39}
\]
Equations (36) and (38) demonstrate that it is optimal to utilize the maximum possible tire force at all times. The controller is thus a bang-bang controller. The force directions, however, given by Equations (37) and (39), vary as a function of the optimal costate values.

In the cases of the front-steer, dual-brake vehicle, the Hamiltonian is complicated by the inclusion of the road/tire force dynamics in the equations of motion. The Hamiltonian is in fact nonlinear with respect to the rear wheel control input. A “typical” plot of the Hamiltonian as a function of the rear wheel contact patch speed \( v_{c,r} \triangleq u_4 r_c \) is given in Figure 5. Due to the nature of the chosen road/tire interaction model, global minimization of the Hamiltonian using the calculus of variations is not possible. At any given time \( t \) an iterative approach is instead used. First, the Hamiltonian is searched for a minimum over a discretized input space of the control input. The discretization granularity and limits are chosen heuristically to ensure that a point in the convex neighborhood of a global minimum is found. From this point, a Newton descent method can be used to find a local minimum within a desired tolerance. Given that the Hamiltonian is continuous and that appropriate granularity and search limits of the input space are chosen, this technique will yield a global minima [14].

V. RESULTS

The results of this study were obtained by utilizing the methods outlined in the previous section. The methods were applied using each of the three different vehicles outlined in Section II and the four different maneuvers outlined in Section III. The purpose of this section is to present the most interesting and informative optimal control maneuvers and to quantify the advantages and disadvantages of each vehicle during each maneuver. For each maneuver, the optimal trajectories are first presented along with the quantified performance measurements. Optimal control inputs are then presented and interesting observations are discussed.

A. Pure Obstacle Avoidance Manuever

The optimal trajectories corresponding to each vehicle during the “pure obstacle avoidance” maneuver are given in Figure 6. The performance of each vehicle is quantified by the final longitudinal position. For the front-steer dual-brake (FSDB) vehicle, the maneuver is completed in 27.74 meters. When rear-wheel steering is added to the vehicle (DSDB), the maneuver is completed in 26.49 meters. The results therefore show that for a typical avoidance maneuver, the addition of active rear-wheel steering can reduce the longitudinal distance required to avoid an obstacle by approximately 1.25 meters, or 4.5%.

The optimal control strategies of the vehicles are best interpreted by examining the optimal wheel force vectors in polar coordinates, as in Figure 7. In the case of the DSDB vehicle, the control strategy is straightforward: the force is always maximized, and the force direction is constant and in the direction orthogonal to the final velocity of the vehicle. In the case of the FSDB vehicle, the rear wheel’s force vector cannot be fully controlled. The force is again always maximized for the front wheel, but the force direction varies slightly from that of the DSDB vehicle, especially early in the maneuver when the front wheel is able to initiate an ideal...
sideslip magnitude for the rear wheel. Large braking torques are applied by the rear wheel early in the maneuver when the wheel has little sideslip magnitude to generate lateral forces.

B. Barrier Avoidance Maneuver

The optimal trajectories corresponding to each vehicle during the “barrier avoidance” maneuver are shown in Figure 8. During this maneuver, all vehicles begin from a common initial state for comparison purposes. The performance of each vehicle is quantified by the distance traveled in the direction of the barrier. For the front-steer dual-brake (FSDB) vehicle, the distance traveled towards the barrier is 4.99 meters. When rear-wheel braking is added to the vehicle (DSDB), the distance is 3.75 meters. The results therefore show that for this particular “barrier avoidance” maneuver, the addition of active rear-wheel steering can reduce the distance traveled towards the barrier by 1.24 meters, or 24.9%.

The optimal control strategies of the vehicles are again interpreted by examining the optimal wheel force vectors in polar coordinates, shown in Figure 9. Once again, the force magnitude of the front wheel is always maximized for all vehicles since the front wheel is fully controllable. The rear wheel force magnitude is also constant for the DSDB vehicle and generally utilized as much as possible given the slip magnitude for the other vehicles. The force directions are also noteworthy. Not surprisingly, the DSDB vehicle applies its front and rear wheel forces in a direction away from the barrier (−90°). The FSDB vehicle utilizes its rear-wheel braking capability in an interesting way during the early part of the maneuver. During this time, the vehicle’s rear wheel has a large lateral slip magnitude in the direction away from the barrier. Without braking capability, this slip magnitude would result in a force towards the barrier. The vehicle therefore locks its rear wheels (or in fact spins them in reverse at the largest feasible velocity) resulting in the force direction being applied longitudinally with little or no lateral force. This behavior is seen whenever the FSDB vehicle encounters a situation with an undesired lateral slip direction.

C. Recovery Maneuver

The “recovery” maneuver demonstrates the ability for a vehicle to return to a nominal driving condition when starting at an arbitrary state. As discussed in Section II, there is no single measurable quantity that serves as a good performance metric. Instead, an objective was chosen such that the vehicle returns to a nominal state in a reasonable amount of time while heavily penalizing any motion towards the highway barriers. Figure 10 presents an optimal trajectory of the FSFB recovering from an extreme initial state, which is in fact the final state from the “pure obstacle avoidance” and “barrier avoidance” maneuvers being executed in sequence. The maneuver is completed in a reasonable distance with no overshoot towards either barrier. Since this vehicle contains the fewest number of controllable degrees of freedom, this example suffices to show that an acceptable recovery maneuver is possible for all the vehicles considered.

D. Phased-Approach Maneuver

The “phased approach” maneuver consists of the previous three maneuvers being executed in sequence. The optimal trajectory from such an approach represents a vehicle’s ability to avoid an obstacle of a known width as sharply as possible, then to eliminate any velocity towards the barrier, and finally to return to a nominal state. Since quantification of the “recovery” phase of this maneuver is not of interest, only the first two phases are measured. Figure 11 depicts the optimal trajectories of the vehicles performing the first two phases of this approach. The resulting quantifications and optimal control inputs are similar to the “barrier avoidance” results, and therefore not presented.
E. Double Lane Change Maneuver

The optimal trajectories corresponding to all vehicles during the “double lane change” maneuver are given in Figure 12. The performance of each vehicle is quantified by the longitudinal distance traveled. For the front-steer front-brake (FSFB) vehicle, the distance traveled towards the barrier is 54.17 meters. When rear-wheel steering and braking is added to the vehicle (DSDB), the distance decreases to 46.12 meters. A solution for the front-steer dual-brake (FSDB) vehicle was not found for this maneuver, likely related to the tendency for the controller to heavily utilize the rear brakes, thereby bringing the vehicle to a stop before the requisite end conditions can be obtained. Although these results suggest that adding rear-wheel control fields a significant improvement, such improvement is not apparent midway through the maneuver. In fact, as pinpointed with triangles in Figure 12, both the rear-wheel control fields a significant improvement, such improvement is not apparent midway through the maneuver. In fact, as pinpointed with triangles in Figure 12, both the rear-wheel control fields a significant improvement, such improvement is not apparent midway through the maneuver. In fact, as pinpointed with triangles in Figure 12, both the rear-wheel control fields a significant improvement, such improvement is not apparent midway through the maneuver.

F. Discussion

The “double lane change” and “phased” approaches lead to differing conclusions about the maneuverability of the vehicles. For comparison purposes, the optimal trajectories of the FSFB and DSDB vehicles performing each maneuver are overlaid in Figures 13b and 13a. On one hand, the phased approach indicates that when an optimal “pure obstacle avoidance” maneuver is performed, it is difficult for the front-steering vehicles to avoid hitting the highway barrier. On the other hand, the trajectory of the “double lane change” approach is essentially identical to the phased approach for the first half of the maneuver. This plot indicates that for the FSFB vehicle, the “double lane change” maneuver may in fact be a much better approach than the “phased approach”. The performance benefit of the “phased approach” is essentially nonexistent for obstacles passed during the first half of the maneuver.

The difference in the control strategies is most visible when considering the slip angle trajectories, as depicted in Figure 14. The “double lane change” approach sacrifices some of its front wheel control effort to maintain ideal rear wheel behavior. On the other hand, the “phased approach” uses its control inputs only in the interest of accomplishing the short-term goals of each particular phase. Interestingly, it only takes a small rolloff in the front wheel slip angle towards the end of the first phase of the maneuver to keep the rear wheel from breaking loose into a rally-car maneuver.

Contrary to the observations for the FSFB vehicle, the DSDB vehicle does much better at avoiding obstacles when using the “phased approach” than when using the “double lane change approach”. This behavior occurs because the DSDB vehicle is in full control of the slip angle of both wheels at all times. It is not necessary to “sacrifice” some control
However, for more complex situations or with many control inputs, the maneuvers are often non-obvious. For example, a typical FSDB vehicle realizes a very significant performance benefit during a barrier avoidance maneuver by locking its rear wheel at certain points during the maneuver. Unfortunately, performance benefits appear to be the least significant during a pure obstacle avoidance maneuver. Instead, benefits are significant only when attempting to return to a nominal operating condition.

Due to the broad nature of this study, many extensions are possible. Pitch and roll dynamics could be added to the model in addition to independent control of all four wheels. Such additions would yield a comprehensive optimal control strategy but would be substantially more difficult to solve using the same optimal control methods. This research has shown that all optimal wheel force magnitudes are “bang-bang” in nature; therefore another extension is to research closed-loop controllers that maintain peak traction in the force directions such that the optimal behavior is still realized.

The main conclusion of this research is that active control of lighter, more agile vehicles with many controllable degrees of freedom is an interesting and potentially useful possibility for future vehicles.

### Table II: Initial states and associated optimal costates for selected optimal solutions

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<th>FSDB</th>
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<td>1.0000</td>
<td>1.0000</td>
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<td>$x_4(0)$</td>
<td>30 m/s</td>
<td>$l_4(0)$</td>
<td>0.9634</td>
<td>0.9795</td>
<td>0.9251</td>
</tr>
<tr>
<td>$x_5(0)$</td>
<td>0 m/s</td>
<td>$l_5(0)$</td>
<td>-2.7911</td>
<td>-2.6185</td>
<td>-2.9161</td>
</tr>
<tr>
<td>$x_6(0)$</td>
<td>0 rad/s</td>
<td>$l_6(0)$</td>
<td>-3.5193</td>
<td>-2.7202</td>
<td>-0.0000</td>
</tr>
</tbody>
</table>

#### VI. CONCLUSION

For a typical lightweight passenger vehicle traveling at highway-speed, this study has compared emergency maneuvers given varying objectives and controllable degrees of freedom. In all maneuvers, a vehicle with independently-controllable front and rear wheels including braking torques to each wheel was shown to perform better than the same vehicle without rear-wheel steering. Performance further decreases when braking capability is removed from the rear wheel. The performance of the vehicle under each maneuver was quantified using objective functions corresponding to physical distances. The performance benefit of active rear-wheel steering varies greatly depending on the initial vehicle state and maneuver objective. For the “pure obstacle avoidance” maneuver, the dual-steer dual-brake (DSDB) vehicle performs only slightly better than the front-steer vehicles. On the other hand, the DSDB vehicle performs much better given the “barrier avoidance” maneuver. The front-steer dual-brake (FSDB) vehicle also performs much better than the front-steer front-brake (FSFB) vehicle on the same maneuver.

For front-wheel steering vehicles, the “phased” approach of performing a “pure obstacle avoidance” maneuver and a “barrier avoidance” maneuver in sequence is shown not to offer a significant performance benefit over the “double lane change” approach. Therefore, the optimal “double lane change” maneuver seems to be an appropriate strategy for avoiding obstacles. However, for dual-steering vehicles, the “phased” approach offers significant advantages and is thus the more appropriate avoidance strategy.

At a high level, this research demonstrates that significant performance benefits can be realized by independent and active control of steering angles and wheel torques of both the front and rear wheels. In the most basic emergency maneuvers, the optimal control strategy is fairly intuitive.