Control of
Two Contact Point Sheet Registration Devices
for Xerographic Printers

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Abstract—Xerographic printers printing on pre-cut sheets need to register (align) each sheet with its corresponding image before image transfer. There exists a variety of sheet registration devices that perform this function, utilizing different actuator and sensor configurations. A majority of these devices have three or more degrees of freedom and contact the sheet at two points during registration. A control algorithm for this class of devices has been developed. The main contribution of the proposed control approach is that it avoids explicitly dealing with the non-holonomic constraints of the combined sheet and registration device system. This simplifies the control design compared to other approaches [1] [2]. The proposed control algorithm comprises of two parts: a generic part for two contact point devices and a registration device specific part which depends on the actuator configuration and kinematics of the registration device. It can also incorporate active control of the buckle that may form in the sheet portion between the two contact points during registration, hereby minimizing the possibility of sheet damage and jams.

I. INTRODUCTION TO SHEET REGISTRATION DEVICES

Xerographic printers and copiers printing on pre-cut sheets need to register each sheet, i.e. align each sheet with its corresponding image, before transferring the image onto the sheet. This is required in order to ensure that each image is printed in the desired location on each sheet. In most applications, the image location is fixed on the image-carrying device, a photo-sensitive drum or belt, and only the option of registering the sheet to the image is available. The sheet has to be registered along all of its 3 degrees of freedom in the plane (along which it travels in a printer media path): process direction (direction of travel), cross-process direction (perpendicular to the direction of travel) and skew.

Most registration devices are mechatronic and utilize a combination of DC and stepper motors to drive the sheet and various sheet sensors to sense its position. These sensors are often optical point sensors, which detect the presence or non-presence of a sheet at the sensor location, or CCD array sensors which detect the position at which a sheet edge covers the sensor.

An example registration device is B [3] in figure 2 which is used in the Xerox Nuvera Digital Production System printers depicted in figure 1. It utilizes a splitnip shaft where each drive roller is driven by a separate motor. The whole shaft assembly can move in the cross-process direction. This gives the registration device 3 degrees of freedom.

Another example is the registration device A used in the Xerox iGen3 Digital Production Press, see [4] and [5]. It utilizes a split nipple similar to [3]. There is however no cross-process direction movement possible. With only 2 degrees of freedom this registration system is underactuated and non-holonomic and requires a different control approach than outlined in this paper. It necessitates the use of a non-holonomic trajectory planner to generate the reference trajectory for the sheet and the reference velocities for each drive roller in order to control a sheet along its 3 degrees of freedom.

Another interesting registration device is C [6], utilizing a pair of spherical balls and backer balls at each contact point. The device utilizes four identical motors to drive two spherical balls.

Another device with 4 degrees of freedom is the Steerable Nips device D [7], which has been studied in the previous control approaches [1] [2].

Interestingly enough, despite such a variety of sheet
registration devices, a majority of them contact the sheet at two points and have 3 or more degrees of freedom. The proposed control algorithm is applicable to and has been derived for this class of devices.

II. MOTIVATION

Many previous approaches to sheet registration device control have been complex [1] [2]. They model, analyze and perform control design for the combined sheet and registration device system. This system has 6 or more degrees of freedom and non-holonomic constraints due to the no-slip condition at the contact points between the registration device nip rollers and the sheet. These non-holonomic constraints capture the fact that it is not possible to have sheet motion at the contact points perpendicular to the driving direction of the nip rollers (assuming no-slip conditions at the contact points). For the steerable nips registration device considered in [1] these non-holonomic constraints are outlined in detail in [2].

This paper presents a simpler approach to the control design for sheet registration devices with 3 or more degrees of freedom. In industrial applications, often the most simple control design approach which yields the required performance is preferred, and this is the motivating factor for the proposed control approach.

The difference between this and the previous approaches can be illustrated using an example kinematic system of a coin rolling on a plane. This system has non-holonomic constraints between the coin rolling rotation angle and the distance travelled on the plane. However, if the intended application is not concerned with what the coin rolling rotation angle is, but only concerns itself with the coin’s position and the angle of the coin’s velocity vector along the plane, one can choose to ignore the non-holonomic constraint.

For sheet registration applications, what is important is the sheet’s position, orientation and velocity. The fact that there are non-holonomic constraints between the sheet trajectory and the registration device nip rolling angles is of minor importance. As long as the sheet gets to the desired final position with the required orientation and at the desired velocities, the control has been successful. What the nip rolling angles are at that point is irrelevant.

Another example to illustrate the concept is the non-holonomic constraint between the angular position of the pedals on a bicycle and the trajectory driven by the bike. As long as we can control the bike to drive from the starting position to any desired final position, we are not overly concerned with the specific angles of the pedals during the trip and with their final values.

III. CONTROL APPROACH

In order to devise a sheet registration control algorithm, the system is split into two parts, sheet and registration device, which are first considered separately. The reason for this is that this allows the non-holonomic constraints of the combined sheet and registration device system to be ignored.

Since the sheet registration device has three or more degrees of freedom, it will be able to impart (within the capability of the actuation constraints and dynamics of the registration device) any desired velocity onto the sheet since the sheet only has three degrees of freedom. Given this premise, first a controller is designed for the sheet as if its velocity was directly controllable. No non-holonomic constraints need to be considered for this system.

Second, a controller for the registration device is designed that controls its actuators such that the two contact point velocity vectors are driven at some desired reference velocity vectors. There are no non-holonomic constraints to be considered for this system either.

These two controllers are then connected together in a cascade control configuration using a block derived in section V that maps the desired sheet velocity to the two desired contact point velocities, please see figure 3 for a detailed overview of the control architecture.

The fact that the combined sheet registration device and the sheet system exhibits various non-holonomic constraints is of minor importance since from the sheet’s perspective it is irrelevant that certain velocities, positions or angles in the sheet registration device are constrained to move only along certain trajectories. As long as the sheet can be steered along any trajectory along its 3 degrees of freedom, the control is successful.

In order to elaborate on the proposed control algorithm, the four sub-tasks of the overall control algorithm and the full closed-loop controlled sheet registration system are now described in more detail:

1) The Sheet Controller determines the desired sheet velocity necessary to track the reference trajectory based on the current estimated sheet position and its intended trajectory.
2) The General Registration Controller converts this velocity to the desired velocities of the sheet at the two contact points A and B.
3) The Registration Device Inverse Kinematics determines the actual internal device velocities and states required to achieve the desired velocities at contact points A and B.
4) The Registration Device Controller controls the actuators of the registration device to achieve the desired device velocities and states.
5) The Sheet Observer uses the two contact point velocities, \( v_{Sh,A} \) and \( v_{Sh,B} \), and signals from sheet sensors placed in and around the registration device to estimate and update the current sheet position.

This approach circumvents the need to explicitly have to deal with the non-holonomic constraints that exist between the states of the combined registration system (sheet and registration device together). Additionally, the division of
the controller into two sub-controllers, one for the sheet, a second one for the registration device, enables us to use a sheet controller where each degree of freedom is independently controlled from each other which simplifies sheet controller design and tuning substantially.

An overview of a sheet being driven by a three degree of freedom, two contact point registration device is shown in figure 4.

IV. SHEET KINEMATICS

For sub-task 2) of the proposed control algorithm, the relationship between the two contact point velocities, \(v_{\text{Sh}, A}\) and \(v_{\text{Sh}, B}\), and the sheet velocity at its center of mass (CM), \(v\) and the angular velocity \(\omega\) needs to be derived.

Assuming the sheet is a rigid body, the velocities of the two contact points A & B of the sheet in contact with the registration device, at positions \(r_A\) and \(r_B\) relative to CM respectively, are governed by rigid body kinematics:

\[
\begin{align*}
    v_{\text{Sh}, A} &= v + \omega \times r_A \\
    v_{\text{Sh}, B} &= v + \omega \times r_B
\end{align*}
\]
To calculate the relationship between the sheet velocities at contact points A & B, \( v_{Sh,A} \) and \( v_{Sh,B} \), and the sheet angular velocity \( \omega \), subtract the two contact point velocities:

\[
v_{Sh,A} - v_{Sh,B} = \omega \times (r_A - r_B)
\]

\[
= \omega \times [(x_A - x) - (x_B - x)]
\]

\[
= \omega \times (x_A - x_B) = \omega \times BA
\]

(3)

On component form, the above can be written as

\[
\begin{bmatrix}
v_{Sh,A,x} - v_{Sh,B,x} \\
v_{Sh,A,y} - v_{Sh,B,y} \\
0
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
\omega
\end{bmatrix} \times \begin{bmatrix}
BA_x \\
BA_y \\
0
\end{bmatrix}
\]

\[
\begin{bmatrix}
-\omega BA_y \\
\omega BA_x
\end{bmatrix}
\]

(4)

The sheet angular velocity can be calculated using one of the following formulae. The formula that avoids dividing by zero is selected, depending on the location of the contact points:

\[
\omega = \begin{cases} 
\frac{\frac{v_{Sh,A,x} - v_{Sh,B,x}}{BA_x} - \frac{v_{Sh,A,y} - v_{Sh,B,y}}{BA_y}}{BA} & \text{if } BA_x \neq 0 \\
\frac{v_{Sh,A,y} - v_{Sh,B,y}}{BA_y} & \text{if } BA_y \neq 0
\end{cases}
\]

(5)

To determine the relationship between the sheet velocities at contact points A & B, \( v_{Sh,A} \) and \( v_{Sh,B} \), and the sheet velocity \( v \), add the two contact point velocities:

\[
v_{Sh,A} + v_{Sh,B} = v + \omega \times r_A + v + \omega \times r_B = 2v + \omega \times (r_A + r_B)
\]

(6)

This can now be solved for the sheet velocity \( v \):

\[
v = \frac{1}{2} [v_{Sh,A} + v_{Sh,B} - \omega \times (r_A + r_B)]
\]

(7)

V. SHEET CONTROL

In order to take the proposed control approach and implement the General Registration Controller part, the inverse of the sheet kinematic map (see section IV) from the combined desired sheet velocities \( [v_x \ v_y \ \omega]^T \) to the desired contact point velocities \( [v_{Sh,A,x} \ v_{Sh,A,y} \ v_{Sh,B,x} \ v_{Sh,B,y}]^T \) needs to be determined.

The simplest approach is to utilize the rigid body dynamics equations (1) and (2). If the registration device is successful in applying those desired contact point velocities \( v_{Sh,A,d} \) and \( v_{Sh,B,d} \) to the sheet, they will result in the desired \( v \) and \( \omega \) sheet velocities, with no buckling/stretching of the sheet.

However, with sheet registration devices with four or more degrees of freedom, the additional degrees of freedom over the sheet’s three can be used to control the sheet buckle in between the two contact points.

Additionally, in a real sheet registration device, due to various disturbances in the system, the actual contact point velocities of the registration device will be different from the desired. By keeping track of the buckle in this case and modifying the no buckling/stretching constraint, one can design a control law that can control buckle. E.g. one choice could be to keep the buckle at 0 mm, another to keep it a 1 mm (sheet slightly buckled throughout registration), a third to keep it at -1 mm (sheet slightly stretched throughout registration), a fourth would be to keep the buckling velocity at -1 mm/s (sheet is slightly stretched continuously throughout registration). The last option is a good choice to handle disturbances, measurement noise etc. and still ensure that the sheet is not excessively stretched while assuring there is less chance of buckling that could jam the sheet during registration.

When controlling the real registration device and sheet, an observer which uses the nip velocity measurements is used to estimate the sheet position and buckle.

To introduce the ability to inject any arbitrary buckling velocity into the control algorithm, the no buckling/stretching constraint is modified slightly and the map from sheet velocities to contact point velocities is re-derived using linear matrix equations that incorporate the buckling control input.

Introduce the following definition for the sum of the relative position of points A and B:

\[
D = r_A + r_B = D_x e_x + D_y e_y
\]

(8)

The matrix representation of this map can now be derived from equation (7) as follows:

\[
v_x = v \cdot e_x = \frac{1}{2} [v_{Sh,A} + v_{Sh,B} - \omega \times (r_A + r_B)] \cdot e_x = \frac{1}{2} [v_{Sh,A,x} + v_{Sh,B,x} - \omega \cdot D \cdot e_x]
\]

(9)

\[
v_y = v \cdot e_y = \frac{1}{2} [v_{Sh,A} + v_{Sh,B} - \omega \times D] \cdot e_y = \frac{1}{2} [v_{Sh,A,y} + v_{Sh,B,y} - \omega \cdot D \cdot e_y]
\]

(10)

Evaluating \( \omega \times D \) yields:

\[
\omega \times D = \omega e_x \times D_x + \omega e_y \times D_y = \omega D_x (e_x \times e_y) + \omega D_y (e_y \times e_y) = \omega D_x e_x + \omega D_y e_y
\]

(11)

On component form, \( v_x \) and \( v_y \) can now be expressed as:

\[
v_x = \frac{1}{2} v_{Sh,A,x} + \frac{1}{2} v_{Sh,B,x} - \frac{1}{2} \omega D_y e_x + \omega D_x e_y = \frac{1}{2} v_{Sh,A,x} + \frac{1}{2} v_{Sh,B,x} + \frac{1}{2} \omega D_y
\]

(12)

\[
v_y = \frac{1}{2} v_{Sh,A,y} + \frac{1}{2} v_{Sh,B,y} - \frac{1}{2} \omega D_y e_x + \omega D_x e_y = \frac{1}{2} v_{Sh,A,y} + \frac{1}{2} v_{Sh,B,y} - \frac{1}{2} \omega D_x
\]

(13)
Re-arranging the above in preparation for matrix form equations yields:

\[ v_x - \frac{1}{2} D_y \omega = \frac{1}{2} v_{Sh,A,x} + \frac{1}{2} v_{Sh,B,y} \]  
(14)

\[ v_y + \frac{1}{2} D_x \omega = \frac{1}{2} v_{Sh,A,y} + \frac{1}{2} v_{Sh,B,y} \]  
(15)

The no buckling/stretching of the sheet constraint can be expressed as:

\[ v_{Sh,A} \cdot BA = v_{Sh,B} \cdot BA \iff (v_{Sh,A} - v_{Sh,B}) \cdot BA = 0 \]  
(16)

To introduce the buckling velocity control input, modify the above constraint to include the buckling velocity in the direction of \( BA \), \( \delta \):

\[ (v_{Sh,A} - v_{Sh,B}) \cdot BA = \dot{\delta} \]  
(17)

Using the definition of \( BA \):

\[ BA = BA_x e_x + BA_y e_y \]  
(18)

the constraint can be re-written as

\[ [BA_x BA_y - BA_x BA_y] \begin{bmatrix} v_{Sh,A,x} \\ v_{Sh,A,y} \\ v_{Sh,B,x} \\ v_{Sh,B,y} \end{bmatrix} = \delta \]  
(19)

The above equations (5), (14), (15) and (19) can now be written on matrix form:

\[ \begin{bmatrix} 1 & 0 & -D_y/2 & 0 \\ 0 & 1 & +D_x/2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ \omega \\ \delta \end{bmatrix} = \begin{bmatrix} v_{Sh,A,x} \\ v_{Sh,A,y} \\ v_{Sh,B,x} \\ v_{Sh,B,y} \end{bmatrix} \]  
(20)

By introducing the following nomenclature:

\[ Q = \begin{bmatrix} 1 & 0 & -D_y/2 & 0 \\ 0 & 1 & +D_x/2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]  
(21)

and

\[ P = \begin{bmatrix} 1 & 0 & 1/2 & 0 \\ 0 & 1 & 0 & 1/2 \\ -1/BA_x & 0 & 1/BA_y & 0 \\ BA_x & BA_y & -BA_x & -BA_y \end{bmatrix} \]  
(22)

The matrix equation (20) can now be written as

\[ \begin{bmatrix} v_{Sh,A,x} \\ v_{Sh,A,y} \\ v_{Sh,B,x} \\ v_{Sh,B,y} \end{bmatrix} = P^{-1} Q \begin{bmatrix} v_x \\ v_y \\ \omega \\ \delta \end{bmatrix} \]  
(23)

It is now a map from sheet velocities to registration device contact point velocities that includes the sheet buckling velocity control input and is the main content of the General Registration Controller in figure 3.

In order to perform closed loop sheet control, a Sheet Controller block is utilized that receives the desired sheet reference trajectory from the Reference Generator and the current estimated sheet position from a Sheet Observer and outputs the required sheet velocities that will drive the sheet to track its reference trajectory. Note that each degree of freedom of the sheet is decoupled and the sheet controller can simply be designed as three separate SISO controllers, one for each degree of freedom of the sheet, which simplifies controller design and tuning.

**VI. SIMULATION RESULTS**

In the simulations, the sheet enters the registration device with the following position errors: process direction error, 30 mm late, cross-process direction, 8 mm up and with a skew of 25 mrad.

The reference position trajectory is a ramp in the \( x \)-direction and \( y \) and \( \theta \) equal to 0.

The sheet position during registration is shown in figure 5. The position of the sheet center of mass (CM) is shown with a blue dot and the location of the two contact points A and B are shown with red dots.

The main sheet states during registration are shown in figure 6. The left column shows the sheet position \( x, y \) and \( \theta \). The middle column shows the sheet position errors \( e_x = x_d - x, e_y = y_d - y \) and \( e_\theta = \theta - \theta_d \). Finally, the right column shows the sheet velocities, \( v_x, v_y \) and \( \omega \).

**VII. CONCLUSIONS**

The main contribution of this paper is a novel and simpler approach to the control of 3 or more degrees-of-freedom, two contact point sheet registration devices that circumvents
dealing with the non-holonomic constraints of the combined sheet and registration device system.

The proposed approach divides the sheet and sheet registration device into two separate systems and designs controllers for each separately. This simplifies the control design considerably since no non-holonomic constraints need to be considered. The sheet and registration device controllers are then combined using a cascade control approach. The non-holonomic constraints present in the combined system are of no importance to the sheet registration function. They do constrain the registration device state trajectories, but not the sheet state trajectories. Since the primary function of sheet registration control is to control the sheet states, the fact that the registration device states are constrained is of no importance.

An added benefit of this approach is that the design and tuning of the sheet registration controller and the registration device controller are de-coupled. Also, the control of each degree of freedom of the sheet is also decoupled. Hence one can more easily design and tune the responses of the two controllers separately and also have the possibility to switch between several sheet registration devices and retain the overall sheet registration dynamic response.

REFERENCES


VIII. LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>[m]</td>
<td>— Sheet position in the process direction (at center of mass – CM)</td>
</tr>
<tr>
<td>( y )</td>
<td>[m]</td>
<td>— Sheet position (at CM) in the cross-process direction</td>
</tr>
<tr>
<td>( \theta )</td>
<td>[rad]</td>
<td>— Sheet angular position</td>
</tr>
<tr>
<td>( v_x = \dot{x} )</td>
<td>[m/s]</td>
<td>— Sheet velocity (at CM) in the process direction</td>
</tr>
<tr>
<td>( v_y = \dot{y} )</td>
<td>[m/s]</td>
<td>— Sheet velocity (at CM) in the process direction</td>
</tr>
<tr>
<td>( \omega = \dot{\theta} )</td>
<td>[rad/s]</td>
<td>— Sheet angular velocity, counter-clockwise</td>
</tr>
<tr>
<td>( x = \begin{bmatrix} x \ y \ 0 \end{bmatrix} )</td>
<td>[m]</td>
<td>— Sheet position vector</td>
</tr>
<tr>
<td>( \omega = \begin{bmatrix} 0 \ 0 \ \omega \end{bmatrix} )</td>
<td>[rad/s]</td>
<td>— Sheet angular velocity vector</td>
</tr>
<tr>
<td>( \mathbf{r}_A )</td>
<td>[m]</td>
<td>— Position of contact point A relative to the sheet CM</td>
</tr>
<tr>
<td>( \mathbf{r}_B )</td>
<td>[m]</td>
<td>— Position of contact point B relative to the sheet CM</td>
</tr>
<tr>
<td>( \mathbf{x}_A )</td>
<td>[m]</td>
<td>— Absolute position of contact point A</td>
</tr>
<tr>
<td>( \mathbf{x}_B )</td>
<td>[m]</td>
<td>— Absolute position of contact point B</td>
</tr>
<tr>
<td>( \mathbf{v}_A )</td>
<td>[m/s]</td>
<td>— Surface velocity of contact point A</td>
</tr>
<tr>
<td>( \mathbf{v}_B )</td>
<td>[m/s]</td>
<td>— Surface velocity of contact point B</td>
</tr>
<tr>
<td>( \mathbf{v}_{Sh,A} = \mathbf{v}_A )</td>
<td>[m/s]</td>
<td>— Velocity of the sheet at contact point A</td>
</tr>
<tr>
<td>( \mathbf{e}_x )</td>
<td>[]</td>
<td>— Unit vector in the process direction</td>
</tr>
<tr>
<td>( \mathbf{e}_y )</td>
<td>[]</td>
<td>— Unit vector in the cross-process direction</td>
</tr>
<tr>
<td>( \mathbf{e}_z )</td>
<td>[]</td>
<td>— Unit vector in the direction perpendicular to the paper plane</td>
</tr>
<tr>
<td>( \mathbf{B}_A )</td>
<td>[m]</td>
<td>— Vector from contact point B to contact point A</td>
</tr>
<tr>
<td>( \mathbf{D} )</td>
<td>[m]</td>
<td>— Sum of relative positions of points A and B</td>
</tr>
</tbody>
</table>

\[
\mathbf{D} = \mathbf{r}_A + \mathbf{r}_B = D_x \mathbf{e}_x + D_y \mathbf{e}_y
\]