Packet-based robust MPC for Wireless Networked Control using co-design

Jian Chen, George W. Irwin and Adrian McKernan

Abstract—This paper presents a packet-based robust Model Predictive Control (MPC) approach in a co-design framework for Wireless Networked Control Systems (WNCS). Lyapunov stability is guaranteed if the optimization problem is feasible. An Inverse Gaussian model which represents the statistical distribution of the round-trip delay (RTD) in a IEEE802.11 WNCS was applied to support a simulation study. Monte-Carlo simulation results on a cart-mounted inverted pendulum are presented to confirm the performance advantages of packet-based robust MPC scheme.

I. INTRODUCTION

Networked Control Systems (NCS) have received much attention in recent years [1]. The focus of this paper is the use of wireless channels in feedback control. This presents considerable challenges for both control design and implementation as the performance and even the stability of the closed-loop system are significantly more likely to deteriorate than with a wired NCS [2]. The reason is that wireless channels are inherently prone to transmission errors caused by channel outages and/or interference, in the form of multipath reflections or packet collisions from other stations. Such errors will lead to packet loss without retransmissions which in turn cause a growth in random communication delays [3] adding to those produced by channel contention for multiple control loops. The common assumption of delays smaller than one sample period made in much of the NCS literature to date is therefore invalid for Wireless Networked Control Systems (WNCS) where large delays of many sample periods are common.

In recent papers on robust model predictive control (MPC), notably reference [4], instead of parameterizing the input as a single linear state-feedback law for the entire infinite horizon, \( N \geq 1 \), free moves were added. In [5], the calculation of the end-point state-weighting matrix was incorporated into the on-line control optimization problem. The advantage of this approach is that a natural and automatic trade-off between feasibility and optimality is obtained. Motivated by [5] and [6], reference [7] adopted the rationale of the first technique to improve the feasibility and optimality of the second one. Independently, in [8], the authors extended the algorithms in [7] from a delay-free system to a delayed one with \( N > 1 \) control moves. However, they did not utilize the free control moves to compensate for the delay in the forward path but rather only implemented the first control move. The technique of splitting the infinite horizon for a delayed system into a set of control moves with an infinity control horizon in the last part [8] is used here for realizing robust packet-based MPC for a wireless NCS.

The major problem with most of the existing literature is that, by ignoring the interdependence between the control system and the communication network, sub-optimal results are produced. Co-design [9], which merges control theory with communications technology, offers a unified approach to improve both the control and communication performances within WNCS. Figure 1 illustrates the principle of co-design and its essential elements: control algorithm, statistical analysis of the network and Monte-Carlo simulation. In this paper, the Quality-of-Service (QoS) measurement of round-trip delay (RTD) in a IEEE802.11 WNCS is integrated with the proposed robust MPC design as explained in section IV-B.

The main original contribution in this paper arises from the rigorous co-design framework proposed for testing and analysing a new packet-based robust MPC scheme for WNCS under worst-case multipath channel conditions in the reverberation chamber. The state model is reformulated and the controller redesigned to make the algorithm applicable to WNCS and capable of tolerating delays in both forward and feedback paths with certain characteristics. Extensive Monte-Carlo simulation results are provided and analysed to confirm the effectiveness of the proposed control algorithm in improving the performance of the wireless network control system.

This paper is organized as follows. Section II presents a statistical analysis of experimentally recorded RTD model. Section III derives the proposed packet-based robust MPC algorithm for state delays to be applied in WNCS. An application, in the form of wireless control of a cart-mounted inverted pendulum, and simulation results for the closed-loop control system are presented in section IV. The paper ends with Conclusions.

II. STATISTICAL ANALYSIS OF ROUND TRIP DELAY (RTD)

Wireless channel conditions are time varying and unrepeateable in a real-time laboratory experiment. The repetition of the same channel conditions for comparison of different control algorithms is difficult and time consuming. Therefore, it’s necessary to know the statistics of the delays caused by the wireless channel to characterize the efficacy of each control algorithm under the varying conditions which can be expected in a realistic WNCS. The model presented in this paper was obtained from experiments carried out in
a reverberation chamber [10] which provides a controlled environment isolated from external interferences and also facilitates difficult multipath conditions. This generates a range of worst-case conditions to which a wireless node could be subjected to.

The round-trip delay (RTD) distribution in a WLAN is usually unimodal and asymmetric (i.e. it is skewed) [11]. The experimentally recorded RTD delay measured from a real-time simulation of the cart-mounted inverted pendulum model with an IEEE802.11b WiFi card in a reverberation chamber had a similar characteristic due to the action of the back-off algorithm. This skewed distribution is caused by having a small percentage of long delays. Caused by a collision occurs when the contentention window doubles each time leading to longer data queues which in turn causes the RTD to increase exponentially.

For statistical modelling of the measured RTD, a Gamma distribution suggested in [12] and an Inverse Gaussian distribution were both fitted to the experimental data. The latter gave the best fit as shown in the Cumulative Distribution Function (CDF) of Figure 3. The resultant Inverse Gaussian CDF model of the measured delay in the WNCS is then given by:

\[
P(\tau) = \Phi\left(\sqrt{\frac{\lambda}{\tau}} \sqrt{\frac{\tau}{\mu}} - 1\right) + \exp\left(\frac{\lambda}{\mu}\right)\Phi\left(-\sqrt{\frac{\lambda}{\tau}} \sqrt{\frac{\tau}{\mu}} + 1\right)
\]

(1)

Here \(\Phi(\cdot)\) is the normal (Gaussian) distribution CDF. In (1), \(\tau\) represents the RTD, the shape parameter \(\lambda = 1.79659\), the mean \(\mu = 8.23547\) and the variance can be calculated as \(\frac{\mu^2}{\lambda} = 310.897\). The model in (1) was used to generate the random RTD for use in the simulation studies in section IV.

### III. Packet-based Robust Model Predictive Control for State Delays

The structure of the proposed packet-based robust MPC for state delays in a wireless network control system is shown in Figure 2. The controller and the plant are connected through a wireless network as shown, thus delays arising from the network will exist in both the forward and feedback paths. Eqn (2) models an open-loop system with state delays. Here, \(d\) is an unknown integer representing the number of delay units in the states, generated from the statistical delay distribution (1). This can accurately represent the delay characteristic in a real wireless channel, such that \(0 \leq d \leq d^*\) where \(d^*\) represents the bounded maximum delay. In this work, the time delay is approximated as an integer multiple of the sampling interval \(\tau \approx mT\), where \(m\) is non-negative integer. If the state vector is augmented with the delayed states, then (2) reduces to the regular state model used for control design. In system (2), the delay component for the control input is not included. This forward path delay will be handled using a set of predicted control signals (\(\tilde{u}(k)\) in (5)) which are transmitted over the network and selected appropriately according to a timestamp, as explained in the section IV-C.

\[
x(k + 1) = A(k)x(k) + A_d(k)x(k - d) + B(k)u(k),
\]

(2)

with

\[
[A(k) \ A_d(k) \ B(k)] \in \Omega
\]

Here \(A(k), \ A_d(k)\) and \(B(k)\) are given by:

\[
A = e^{A_c T}, \quad A_d = \frac{T}{2} A B_c F(k)
\]

(3)

\[
B = \int_0^T e^{A_s d} ds B_c
\]

In (3) \(A_c, B_c\) are the plant and input matrices for the continuous-time system, \(T\) is the sampling interval and \(F(k)\) is the state feedback gain, updated on-line.

Further, \(u(k) \in \mathbb{R}^{nu}\) is the control input, \(x(k) \in \mathbb{R}^{nx}\) are the measurable states of the plant and \(\Omega\) is some prespecified set.

For polytopic systems, the set \(\Omega\) is the polytope

\[
\Omega = \text{Co}([A_1 \ A_{d1} \ B_1], [A_2 \ A_{d2} \ B_2], \ldots, [A_L \ A_{dL} \ B_L])
\]

(4)

where \(\text{Co}\) denotes a convex hull. In other words, if \([A(k) \ A_d(k) \ B(k)] \in \Omega\) then, for some non-negative scaler parameters \(\lambda_1, \lambda_2, \ldots, \lambda_L\) summing to one, it follows that

\[
[A(k) \ A_d(k) \ B(k)] = \sum_{i=1}^L \lambda_i [A_i \ A_{d1} \ B_i],
\]

The case of \(L = 1\) corresponds to the nominal LTI system. This robust MPC algorithm is aimed at designing a predictive controller that brings system (2)-(4) to steady state, at each time \(k\) to realize the cost:

\[
\min_{\tilde{u}(k)} \max_{i \geq 0} \sum_{i=0}^{\infty} J_{\infty} = \sum_{i=0}^{\infty} [x(k+i|k)Q_1 x(k+i|k) + u(k+i|k)R u(k+i|k)]
\]

(5)

where \(Q_1\) and \(R\) are positive-definite weighting matrices and \(\tilde{u}(k) = [u(k|k)^T, u(k + 1|k)^T, \ldots, u(k + N|k)^T]^T\) are the predicted control variables. The optimization (5) is repeated at each sampling time.

Minimizing (5) without imposing other restrictions is not attractive since it requires an infinite number of degrees of freedom \(u(k+i|k) \forall i \geq 0\). Therefore, parametrization of the input \(\tilde{u}(k)\) is divided into two parts by a switching horizon \(N\), as is the cost function (5):

\[
\min_{u(k+i|k), 0 \leq i < N} \max_{i \geq 0} \sum_{i=0}^{N-1} [x(k+i|k)Q_1 x(k+i|k) + u(k+i|k)R u(k+i|k)]
\]

(6)
and
\[
\min_{u(k+i), \ i \geq N} \max_{\{A(k+i), A_d(k+i), B(k+i)\} \in \Omega, \ i \geq N} J_2(k) = (7)
\]

\[
\infty \sum_{i=N}^{\infty} [x(k + i|k)^T Q_1 x(k + i|k) + u(k + i|k)^T R u(k + i|k)]
\]

Eqn.(6) constitutes a finite horizon 'min-max' optimization problem, and (7) is an infinite horizon one. Linear state feedback is introduced for solving (7):

\[
u(k+i|k), \ i \geq N
\]

\[
\max_{\{A(k+i), A_d(k+i), B(k+i)\} \in \Omega, \ i \geq N} J_2(k) = (7)
\]

\[
\infty \sum_{i=N}^{\infty} [x(k + i|k)^T Q_1 x(k + i|k) + u(k + i|k)^T R u(k + i|k)]
\]

In order to predict the N steps control moves, prediction of the states \(x(k+i|k), \ i > 0\) is necessary. Although it is not possible to make deterministic state predictions for (2), it is possible to define a set which the future states will lie on [4]. The following lemmas which are based on the treatment in [8] show how the tightest of these sets can be calculated.

**Lemma 3.1**: Define the set \(\varsigma(k+i|k) = Co\{x^{l_1-1, \ldots, l_1, 0}(k+i|k), 1 \leq l_j \leq L, 0 \leq j \leq i-1\}\). Assuming that \(x(k+i|k) \in \varsigma(k+i|k)\), then \(\varsigma(k+i+1|k)\) is the tightest set that contains all possible future states \(x(k+i+1|k)\) of the system (2) if \(x^{l_1-1, \ldots, l_1, 0}(k+i+1|k, l_0, l_1, \ldots, l_i \in \{1, 2, \ldots, L\}\) satisfies

\[
x^{l_1-1, \ldots, l_1, 0}(k+i+1|k) = A^{l_1}(k+i|k)x^{l_1-1, \ldots, l_1, 0}(k+i|k) + A_d^{l_1}(k+i|k)x^{l_1-1, \ldots, l_1, 0}(k+i-d(k)) + B^{l_1}(k+i|k)u^{l_1, \ldots, l_1, 0}(k+i|k)
\]

for all \(l_0, l_1, \ldots, l_i \in \{1, 2, \ldots, L\}\), where

\[
u^{l_1, \ldots, l_1, 0}(k+i|k) = F(k+i|k)x^{l_1-1, \ldots, l_1, 0}(k+i|k) + c(k+i|k)
\]

and

\[
\infty \sum_{i=N}^{\infty} [x(k + i|k)^T Q_1 x(k + i|k) + u(k + i|k)^T R u(k + i|k)]
\]

A Lyapunov-Krasovskii function for the delay system is then defined as

\[
V(x(k|k)) = x(k|k)^T P x(k|k)
\]

where

\[
P = P^T > 0, \ S = S^T > 0
\]

The robust stability constraint is then given by

\[
V(x(k+i+1|k)) - V(x(k+i|k)) \leq -[x(k+i|k)^T Q_1 x(k+i|k) + u(k+i|k)^T R u(k+i|k)]
\]

The sufficient conditions for robust stability are that, if and only if, there exist the variables \(\gamma > 0, Q = \gamma P^{-1} > 0, W = \gamma S^{-1} > 0\) which satisfy:

\[
\begin{bmatrix}
Q & * & * & * & * \\
A_d Q + B_d Y & Q & * & * & * \\
Q^{1/2} Q & 0 & \gamma I & * & * \\
R^{1/2} Y & 0 & 0 & \gamma I & * \\
Q & 0 & 0 & 0 & W \\
0 & 0 & 0 & 0 & W^T A_d^T W
\end{bmatrix} \geq 0,
\]

with \(Y = FQ, \ i \geq N\). The state feedback controller \(u(k+i|k) = F(k)x(k+i|k), \ i \geq N\) then exponentially stabilizes the system for any

\[
\chi(k+N) \in \varepsilon(\Lambda) = \{\chi \in \mathbb{R}^{n(d^*+1)}|\chi^T \Lambda^{-1} \chi \leq 1\},
\]

For \(i \geq N\), \(\chi(k+i)\) always remains in the region \(\varepsilon(\Lambda)\). By considering lemma 3.2, the terminal constraint (13) can be transformed to (14) by applying Schur complement:

\[
\begin{bmatrix}
1 & \chi^{l_1-1, \ldots, l_1, 0}(k+N|k) & * \\
\chi^{l_1-1, \ldots, l_1, 0}(k+N|k)^T & \Lambda
\end{bmatrix} \geq 0,
\]

1 \(\leq l_j \leq L, 0 \leq j \leq N - 1\)

Summing (11) from \(i = N\) to \(\infty\) yields

\[
\max_{\{A(k+i), A_d(k+i), B(k+i)\} \in \Omega, \ i \geq N} J_2 = \infty \sum_{i=N}^{\infty} [x(k+i|k)^T Q_1 x(k+i|k) + u(k+i|k)^T R u(k+i|k)]
\]

For \(i \geq N\) a state-feedback control law \(u(k+i|k) = F(k)x(k+i|k)\) is applied such that:

\[
x(k+i+1) = A(k+i)x(k+i) + A_d(k+i)x(k+i-d) + B(k+i)u(k+i)
\]

\[
= (A(k+i) + B(k+i)F(k|k))x(k+i) + A_d(k+i)x(k+i-d)
\]

(9)
According to (11), \( V(\chi(k+N|k)) = \chi(k+N|k)^T \Lambda_p \chi(k+N|k) \) is defined where \( \Lambda_p = \text{diag}\{P, S, \ldots, S\} \).

\( P = \gamma Q^{-1} > 0, \ S = \gamma W^{-1} > 0 \)

Therefore, combining (6) and (15) the 'min-max' optimization (5) becomes

\[
\mathcal{J}(k) = \sum_{i=0}^{N-1} [x(k+i|k)^T Q_1 x(k+i|k) + u(k+i|k)^T Ru(k+i|k)] + \chi(k+N|k)^T \Lambda_p \chi(k+N|k)
\]

Now define

\[
x(k+i|k)^T Q_1 x(k+i|k) + u(k+i|k)^T Ru(k+i|k) \leq \gamma_i \quad i \in \{0, 1, \ldots, N-1\}
\]

\[
\chi(k+N|k)^T \Lambda_p \chi(k+N|k) \leq \gamma 
\]

The constraint (18) is equivalent to (13), while (17) can be transformed to

\[
\begin{bmatrix}
F(k+i|k)x^{i-1}_{l_0}(k+i|k) + c(k+i|k) & R^{-1} & 0 \\
x^{i-1}_{l_0}(k+i|k) & 0 & Q_1^{-1}
\end{bmatrix} \geq 0, \\
1 \leq l_i \leq L, \ 0 \leq i \leq N-1
\]

Thus, the control optimization problem at each time \( k \) can be formulated as:

\[
\min_{\gamma_1, \gamma_i, c(k), F(k), Q, W} \gamma + \sum_{i=0}^{N-1} \gamma_i \quad \text{s.t.} (12), (14) \text{ and } (19)
\]

**A. Stability analysis**

**Theorem 2.** Suppose (20) is feasible at the initial time \( k = 0 \). The receding horizon implementation of \( u(k) = F(k) x(k|k) + c(k|k) \) can then exponentially stabilize the closed-loop system.

Proof: Assume there exists a feasible solution at time \( k \) by obtaining \( c^*(k|k), F^*(k|k), \ldots, F^*(k+N-1|k) \) and \( \Lambda^*_p \).

Then, at time \( k+1 \), the following is feasible according to (12):

\[
u(k+i+1|k+1) = F^*(k+i+1|k) x^*(k+i+1|k) + c^*(k+i+1|k)
\]

\[
i = 0, \ldots, N-2,
\]

\[
u(k+i+1|k+1) = F^*(k)x^*(k+i+N|k)
\]

\[
i \geq 0, \ \Lambda_p(k+1) = \Lambda^*_p(k)
\]

Applying (16), (21) and (22), produces:

\[
\mathcal{J}(k+1) = \sum_{i=0}^{N-1} [x(k+i+1|k+1)^T Q_1 x(k+i+1|k+1) + u(k+i+1|k+1)^T Ru(k+i+1|k+1)] + \chi(k+N|k)^T \Lambda_p \chi(k+N+1|k)
\]

\[
= \sum_{i=1}^{N} [x^*(k+i|k)^T Q_1 x^*(k+i|k) + u^*(k+i|k)^T Ru^*(k+i|k)] + \chi^*(k+N+1|k)^T \Lambda_p^* \chi^*(k+N+1|k)
\]

According to (11):

\[
\chi^*(k+N+1|k)^T \Lambda_p^* \chi^*(k+N+1|k) \leq \chi^*(k+N|k)^T \Lambda_p^* \chi^*(k+N|k)
\]

\[
- [x^*(k+N+1|k)^T Q_1 x^*(k+N|k)] + u^*(k+N|k)^T Ru^*(k+N|k)
\]

Substituting (25) into (24)

\[
\mathcal{J}(k+1) = \sum_{i=1}^{N-1} [x^*(k+i|k)^T Q_1 x^*(k+i|k) + u^*(k+i|k)^T Ru^*(k+i|k)] + \chi^*(k+N|k)^T \Lambda_p^* \chi^*(k+N|k) \leq \mathcal{J}^*(k)
\]

After \( \mathcal{J}(k+1) \) has been optimized at time \( k+1 \), the optimum \( J^*(k+1) \leq \mathcal{J}(k+1) \). Hence, \( J^*(k) \) is a Lyapunov function and \( x(k) \rightarrow 0 \), as \( k \rightarrow \infty \). This completes the proof.

**IV. MONTE-CARLO SIMULATIONS**

**A. Application study**

The application comprised wireless control of a cart-mounted inverted pendulum [13]. This constitutes a 4th order, single-input, two-output plant. The objective here is to control the cart displacement \( x \), using the force \( u \) applied to the cart, while regulating the pendulum angle \( \theta \) to zero. Here the continuous-time plant is described in state-space form as:

\[
\begin{bmatrix}
\dot{x} \\
\dot{x} \\
\dot{\theta} \\
\dot{\theta}
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 \\
-(1+m^2) & 0 & 0 & 0 \\
0 & 0 & -m & 0 \\
0 & 0 & 0 & -\frac{m^2 g}{p}
\end{bmatrix}
\begin{bmatrix}
x \\
x \\
\theta \\
\theta
\end{bmatrix} +
\begin{bmatrix}
0 \\
\frac{m}{p} \\
\frac{m}{p} \\
\frac{m}{p}
\end{bmatrix} u
\]

(27)
The physical parameters $b, m, l, g, I$ and $M$ in (27) as defined in Table 1 and $p = I(M + m) + Mml^2$. This is an open-loop unstable system and therefore constitutes a challenging real-time WNCs application with the added advantage that it has been widely used in the wireless control literature [14], thereby providing a useful basis for comparison.

B. Controller setting

In the simulation, the sampling period was $T = 10ms$ and the weighting matrices in (5) were set to be $Q_1 = \text{diag}[200 \ 1 \ 50 \ 1]$ and $R = 1$. The maximum delay $d^*$ of concern here was $10T = 100ms$ and the prediction horizon $N$ was chosen as $10T = 100ms$ as well. In section II, the CDF distribution showed that $100ms$ covers more than 95% of the measured delay values encountered, although in the worst case this grew to $400ms$. Therefore, the selected value for the prediction horizon $N$ and the maximum delay $d^*$ were considered sufficient for the controller design.

C. Simulation results

The simulation was started from the plant side with all the initial states at 0. Whenever the states arrive at the controller, the measured states update the past state sequence. This enables a feasible solution to be found and then generates the new feedback gain $F(k)$ (for $i \geq N$) and the future control moves (for $i \in 0, 1, \ldots, N - 1$). If the feedback states are delayed and cannot arrive on time, the previous state values are used again, repeating the optimization process at the current sampling time $k$. This procedure can avoid the need for synchronization between the controller and the sensor. The exact delay that the feedback states experience does not need to be known for updating the control signals.

For forward path delay compensation, as shown in Figure 2, a buffer node is placed at the actuator to store the received control sequence and then to select the necessary predicted control signal. Here, the controller and the actuator are assumed to be synchronized, a timestamp is attached to each generated control sequence. To compensate for random delays the buffer at the actuator node only takes the most recent generated control sequence from the controller. It then selects the appropriate control signal to use by comparing the timestamp to the local time at the actuator. As shown in (28), $p(k_{min}\{\tau_1, \tau_2, \ldots, \tau_N\})$ is the latest predicted control sequence, and $\hat{u}(k) = \hat{u}(k - min\{\tau_i\})$ is the optimal predicted control value for time $k$. If the buffer is not refreshed at the next sampling time, the next predicted control signal in the previous control sequence received is applied by the actuator. Thus:

$$p(k_{\tau_i}) = \begin{cases} u(k - \tau_i + j|k - \tau_i) \\ \text{for } i \in 1, k; \quad (28) \\ j \in 0, \ldots, N - 1 \end{cases}$$

If the optimization problem defined in (20) subject to (12), (14) and (19) is feasible with a known upperbound delay (the measured maximum delay) at initial time $k$, the packet-based robust MPC for state delays is guaranteed to robustly stabilize the closed-loop system.

The closed-loop step-responses for pendulum angle of the cart-mounted inverted pendulum shown in Figure 4 was derived from 30 Monte-Carlo simulation trials using the random RTD generated by (1). In each case the corresponding hard-wired responses are included for comparison purposes. Most of the controlled step responses are close to the hard-wired ones and meet the system performance criteria in Table II. The few outliers, which obviously have slightly larger peak-overshoots and longer settling times, are caused by long delays in the wireless channel, but even so still will strictly meet the the design specifications (where the peak overshoot did not exceed $12\%$ to ensure the pendulum did not fall over). Figure 5 shows the averaged step-responses from the Monte-Carlo trials. The error bars indicate the maximum and minimum responses at those particular sample instants. In this case, all the responses were very close to the desired one and the variation in response were very small. This clearly demonstrates that the controlled step-responses can meet the design criteria listed in Table II.

These results suggest that the proposed packet-based robust MPC for state delays algorithm can effectively handle the random transmission delays in a wireless network control system.

V. CONCLUSIONS

Based on the co-design concept which integrates the theoretical controller design with wireless network analysis, a coherent study of a packet-based robust MPC algorithm suitable for WNCs were demonstrated. Monte-Carlo simulation results on a cart-mounted inverted pendulum confirmed the effectiveness and advantages of the proposed algorithm for handling the random delays in a WNCs.

VI. ACKNOWLEDGMENTS

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Fig. 2. Structure of wireless networked packet-based robust MPC for state delays

Fig. 3. Measured CDF of delay with fitted Gamma and Inverse Gaussian distributions.

Fig. 4. Variation in pendulum angle step responses for 30 Monte-Carlo simulations for a wireless channel incorporating an Inverse Gaussian model of round-trip delay with PB-RMPC

Fig. 5. Variation in averaged pendulum angle step responses from 30 Monte-Carlo simulations for a wireless channel incorporating an Inverse Gaussian model of round-trip delay with PB-RMPC

TABLE I
PARAMETERS AND VALUES

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<thead>
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<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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<tr>
<td>M</td>
<td>Mass of cart</td>
<td>0.5 kg</td>
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<tr>
<td>m</td>
<td>Mass of pendulum</td>
<td>0.2 kg</td>
</tr>
<tr>
<td>b</td>
<td>Friction of cart</td>
<td>0.1 Nms⁻¹</td>
</tr>
<tr>
<td>l</td>
<td>Length of pendulum</td>
<td>0.3 m</td>
</tr>
<tr>
<td>I</td>
<td>Inertia of pendulum</td>
<td>0.006 kgm²</td>
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<tr>
<td>g</td>
<td>Gravity force</td>
<td>9.8 ms⁻²</td>
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TABLE II
DESIGN CRITERIA FOR CART-MOUNTED INVERTED PENDULUM.

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<th>Symbol</th>
<th>Criterion</th>
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<td>x</td>
<td>Settling Time</td>
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<tr>
<td>x</td>
<td>Rise Time</td>
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<td>θ</td>
<td>Overshoot stable</td>
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<td>x and θ</td>
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REFERENCES


