Iterative Learning Control of the Redundant Upper Limb for Rehabilitation

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Abstract—A non-linear iterative learning control approach is developed for application to stroke rehabilitation. The subject is seated in a robotic workstation and electrical stimulation is applied to their triceps muscle to assist the tracking of trajectories in a horizontal plane. In addition to rotation about vertical axes through the shoulder and elbow joints, the forearm is also permitted to elevate in order to provide full arm extension. A dynamic model of the human arm is first developed, and then constraints are introduced to overcome kinematic redundancy. The expressions necessary to implement the control law are derived, and experimental results confirm its ability to achieve a high level of performance in practice.

I. INTRODUCTION

During the last decade there has been growing evidence for the effectiveness of technologies, such as rehabilitation robots and functional electrical stimulation (FES), in recovery of movement post-stroke. Studies have shown that when FES is associated with a voluntary attempt to move the limb, improvement is enhanced, and motivate its application to precisely assist the patient during trajectory tracking tasks. Whilst research into FES for rehabilitation is very active, there exist few model-based control schemes which have been applied to the upper extremity, and fewer still which have transferred into clinical practice. Challenges include highly nonlinear behaviour and model inaccuracy due to the presence of time-varying properties (e.g. spasticity, fatigue).

An experimental test facility incorporating a five-link planar robotic arm and an overhead trajectory projection system has been developed to provide a controlled environment in which to apply FES to stroke patients [1]. During treatment the subject is seated with their arm strapped to the robot, as shown in Fig. 1, and their task is to repeatedly track a number of reaching trajectories using a combination of voluntary effort and surface FES applied to their triceps. The subject’s arm is returned to the start position after each trial, and, following a short rest period, the task is repeated. The FES is mediated using iterative learning control (ILC), a technique that is applicable to systems operating in such a cyclical mode [2]. This must operate in the presence of the patient’s remaining voluntary effort, and the robot is used to provide additional assistance (whilst allowing FES drive the task completion). Clinical trials with 5 patients led to statistically significant improvement in unassisted tracking performance and isometric strength [3].

In previous work using the workstation, the patient’s forearm was constrained to lie in a horizontal plane. However, to maximise the treatment’s potential for rehabilitation, it is necessary to use a wider range of more functional movements [3]. This paper therefore extends the previous model of the arm to include the kinematically redundant case in which the forearm is raised to permit full arm extension. Whilst significantly complicating the control problem, the presence of such redundancy in many biomechanical models gives the proposed control approach scope for wide applicability.

II. STIMULATED HUMAN ARM MODEL

Figure 2 shows the assumed human arm model, whose three degrees of freedom comprise \( \theta_u \) (upper arm rotation), \( \theta_f \) (forearm rotation), and \( \delta \) (forearm elevation). The point, \( Q_i \), is where the subject’s hand grasps the robot, and components of the forces applied in the \( x \) and \( y \) directions are denoted by \( F_x \) and \( F_y \) respectively. Motivated by clinical need, stimulation is applied to the triceps, which has been modelled as supplying a torque, \( T_\beta \geq 0 \), acting about an axis orthogonal to both the upper arm and forearm. The subject’s shoulder joint position is fixed throughout all tests, and the robotic arm imposes the constraint \( \delta \geq 0 \). This also means that the upper arm elevation angle, \( \gamma \), then satisfies \( l_3 s_f + l_1 s_\delta = l_3 s_\gamma \), where \( \gamma = \bar{\gamma} \) in the case of no lift (\( \delta = 0 \)).

The dynamic model of the arm can then be expressed as

\[
\mathbf{B}(\dot{q}) \dot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{F}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{\tau} - \mathbf{J}_u(\mathbf{q})\mathbf{h}
\]

where \( \mathbf{q} = [\theta_u, \theta_f, \delta]^T, \mathbf{h} = [F_x, F_y]^T, \mathbf{J}(\mathbf{q}) \) is the Jacobian,

\[
\mathbf{\tau} = \frac{T_\beta}{\sqrt{1 - c_f^2 c_\delta^2 - s_f^2 c_\delta^2 - 2 c_f c_\gamma s_\gamma s_\delta c_\delta}} \begin{bmatrix} 0 \\ -s_f c_\gamma c_\delta \\ s_f c_\delta - c_f c_\gamma s_\delta \end{bmatrix} \begin{bmatrix} c_\delta = \cos(\delta), c_\gamma = \cos(\gamma), c_u = \cos(\theta_u), c_f = \cos(\theta_f), \text{ and likewise for \sin(\cdot)} \end{bmatrix}
\]

In addition, \( \mathbf{B}(\mathbf{q}), \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) \) are of the form

\[
\begin{bmatrix} \mathbf{b}_1 \mathbf{b}_2 \mathbf{b}_3 \\ \mathbf{b}_2 \mathbf{b}_3 \mathbf{b}_4 \\ \mathbf{b}_3 \mathbf{b}_4 \mathbf{b}_5 \end{bmatrix} = \begin{bmatrix} -2c_1 \dot{\theta}_f - c_1 \dot{\theta}_f - 2c_2 \dot{\theta}_u - c_3 \dot{\theta}_f + c_4 \dot{\delta} \\ c_1 \dot{\theta}_u 0 2c_5 \dot{\theta}_u - 2c_6 \dot{\theta}_f + (c_4 + c_1) \dot{\delta} \\ -c_2 \dot{\theta}_u - 2c_5 \dot{\theta}_f - c_5 \dot{\theta}_f 2c_7 \dot{\theta}_f + c_8 \dot{\delta} \end{bmatrix}
\]
respectively, with the parameters bi, ci, i = 1, 2, ..., functions of \( \vartheta_u, \vartheta_f, \) and \( \delta. \) The form of the friction term considered is

\[
F(q, q, \dot{q}) = \left[ F_u(\vartheta_u, \dot{\vartheta}_u), F_f(\vartheta_f, \dot{\vartheta}_f), R_\delta(q, q, \dot{q}) \right]^T
\]

where the last component includes the effect of gravity and the reaction of the robot and is given by

\[
R_\delta(q, q, \dot{q}) = \left( m_l f_1 + m_u \frac{l_1 f_1}{l_2} \right) c_\delta + M_\delta(q, q, \dot{q}) + F_\delta(\delta, \dot{\delta}).
\]

Here \( F_u(\cdot), F_f(\cdot) \) and \( F_\delta(\cdot) \) are piecewise linear functions, and \( M_\delta(q, q, \dot{q}) \) is the reaction torque of the arm holder to impose the constraint \( \delta \geq 0 \) (full details appear in [4]).

A widely assumed model of the torque, \( T_\beta, \) generated by electrically stimulated muscle acting about a single joint is

\[
T_\beta(\beta, \dot{\beta}, u, t) = h(u, t) \times F_m(\beta, \dot{\beta}) + F_s(\dot{\beta}, \dot{\beta})
\]

where \( u \) denotes the stimulation pulse width applied, and \( \beta \) is the joint angle [5]. \( h(u, t) \) is a Hammerstein structure incorporating a static non-linearity, \( h_{IRC}(u), \) representing the isometric recruitment curve, cascaded with linear activation dynamics, \( H_{LAD}(s). \) \( F_m(\beta, \dot{\beta}) \) describes the multiplicative effect of the joint angle and angular velocity on the torque developed, and \( F_s(\dot{\beta}, \dot{\beta}) \) accounts for the passive properties of the joint. It can also be shown that \( \beta = \arccos(-c_1 c_2 c_3 - s_2 s_3) \), so that \( \beta \) is a function of \( \dot{\vartheta}_f, \delta, \dot{\delta}, \) and \( \dot{\delta}. \)

### III. ROBOTIC ASSISTANCE

A form of impedance control is used to control the robotic arm, in order to guarantee safe interaction with an unknown environment [6]. This results in the relationship

\[
-h = K_{KJ} \ddot{x} - K_{B} \dot{x} - K_{M} x
\]

at \( Q, \) where \( \ddot{x} = \dot{x} - x = k(q), \dot{x} = J(q) q \) and \( x = J(q) q \) or \( x = J_1(q) q + J_2(q, \dot{q}) \dot{q}. \) Here \( x = k(q) \) is the direct kinematics equation for the human arm system. The trajectories used during the treatment consist of constant velocity, elliptical reaching tasks for the subject’s impaired arm. Whilst FES applied to the triceps assists the task completion, it cannot guarantee perfect tracking. The robot will therefore supply further assistance to allow patients to complete tracking tasks in a manner that is governed by the FES (which hence promotes rehabilitation). To find the necessary robotic assistance, (1) and (3) are combined to give

\[
B(q) \dot{q} + C(q, \dot{q}) \dot{q} + F(q, \dot{q}, \ddot{q}) = J^T(q) (K_{KJ} \ddot{x} - K_{B} \dot{x} - K_{M} x) + \tau
\]

However since \( J^T(q) \) is not square, arbitrary dynamics cannot be imposed by the robot about all joints of the human arm. Since this is critical to permit effective assistance, and regulate dynamics for patient safety and treatment variation, the conditions necessary to remove dependence of one of the joint angles will be examined.

From (1), the condition for planar movement (with no forearm lift, \( \delta = 0 \)), is

\[
T_\beta \leq \sqrt{1 - c_1^2 c_2^2} \left( -m_l f_1 f_2 \left( \dot{\vartheta}_u + \dot{\vartheta}_f \right) s_f + \left( \dot{\vartheta}_u + \dot{\vartheta}_f \right)^2 c_f \right)
\]

\[
- \left( m_l f_1 f_2 s_f + m_u \frac{l_1 f_1}{l_2} s_f + L \frac{l_1 f_1}{l_2} c_f \right) \dot{\beta}^2 + l f_2 c_u F_x + l f_2 s_u F_y + F_\delta(0,0) \left( m_l f_2 + m_u \frac{l_1 f_1}{l_2} \right) g \right) / s_f
\]

where \( F_\delta \) and \( F_f \) are governed by the robotic controller. If instead sufficient torque is supplied to fully extend the forearm, producing \( \vartheta_f = 0, \) the forearm dynamics are

\[
\left( m_l f_1 f_2 c_\delta + I f_\delta \right) \left( \ddot{\vartheta}_u + \ddot{\vartheta}_f \right) + m_l f_1 f_2 c_\delta \dot{\vartheta}_u + 2m_l f_1 f_2 c_\delta \dot{\vartheta}_u \dot{\beta} - \left( 2m_l f_1 f_2 c_\delta + I f_\delta \right) \left( \dot{\vartheta}_u + \dot{\vartheta}_f \right) \dot{\beta}
\]

\[
= l f_2 c_u F_x - I f_\delta c_u F_y - F_f(0, 0, \dot{\vartheta}_f)
\]

to maintain \( \vartheta_f = 0. \) At times when the system satisfies either (5) or (6), the joint angle which is zero may be removed from the system model (1). If trajectories can be found which can be tracked whilst satisfying either condition at all times, then it will be shown that redundancy is avoided and full control over the remaining system dynamics is possible.

In the analysis which follows it will be shown that this is possible using trajectories which comprise the components

\[
\vartheta_u^*(t) = \begin{cases} \Psi_1(t) & t \in [0, T_1] \\
\Psi_2(t) & t \in [T_1, T_2] \end{cases}, \hspace{1cm} \vartheta_f^*(t) = \begin{cases} 0 & t \in [0, T_1] \\
\lambda(t) & t \in [T_1, T_2] \end{cases}, \hspace{1cm} \delta^*(t) = \begin{cases} \Psi_1(T_1) = \Psi_2(T_1) & \Psi_1(T_1) = \Psi_2(T_1) \end{cases}
\]

at \( Q, \) where \( \Psi_1(\cdot), \Psi_2(\cdot), \lambda(\cdot) \) and \( \lambda(\cdot) \) are monotone increasing. These split the task into two components: over \( t \in [0, T_1] \) planar movement alone (\( \delta = 0 \)) is required, and over \( t \in [T_1, T_2] \) elbow lift with \( \vartheta_f = 0 \) is necessary.

### IV. PLANAR MOVEMENT CONTROLLERS

First consider the case of planar arm movement (\( \delta = 0 \)). Whilst satisfying (5), over \( t \in [0, T_1] \), the system (1) becomes
The validity of assuming that the forearm does not lift and dynamics into (5), to give

\[ \hat{\beta} = -c_{q_f} s_{q_f} \frac{1 - c_{q_f}}{s_{q_f}} \hat{\vartheta}_f = f(\hat{\vartheta}_f, \hat{\vartheta}_f) \]

\( F_1(\beta, \hat{\beta}) \) in (2) is now accounted for in \( F_1(\hat{\vartheta}_f, \hat{\vartheta}_f) \) and can be neglected. In [7] a Newton-method based ILC algorithm was derived to control the stimulation applied to the muscular system so that \( \hat{\vartheta}_f \) accurately tracked the reference \( \hat{\vartheta}_f^* \). This scheme used the system (7) with robotic controller (8), and comfortably satisfied (10) throughout.

V. ELBOW LIFT ROBOTIC CONTROLLER

Having adopted the ILC scheme of [7], full forearm rotation may be achieved (\( \hat{\vartheta}_f = 0 \)). Further increasing the level of FES then will result in the forearm lifting since (10) will no longer be satisfied. Over \( t \in [T_1, T_2] \), providing (6) is satisfied, the previous system (7) is therefore replaced by

\[
\begin{bmatrix}
    b_{1,l} & 0 \\
    0 & b_{b,l}
\end{bmatrix}
\begin{bmatrix}
    \hat{\vartheta}_u \\
    \delta
\end{bmatrix}
+ 
\begin{bmatrix}
    0 & 2c_{2,l} \dot{\vartheta}_u \\
    -c_{2,l} \dot{\vartheta}_u & c_{8,l} \delta
\end{bmatrix}
\begin{bmatrix}
    \dot{\vartheta}_u \\
    \delta
\end{bmatrix}
+ 
\begin{bmatrix}
    F_{\delta,l}(\hat{\vartheta}_u, \hat{\vartheta}_f) \\
    R_{\delta,l}(\hat{\vartheta}_u, \hat{\vartheta}_f)
\end{bmatrix}
= \tau_l - J_l^T(\hat{\vartheta}_f)h
\]

(11)

where

\[ b_{1,l} = m_f(l_a c_{\gamma} + l_f c_{\gamma})^2 + m_a l_a^2 c_{\gamma} + I_u c_{\gamma} + I_f c_{\gamma} \]

\[ b_{b,l} = m_f l_f (l_f - 2f l_f c_{\gamma} - \frac{m_f l_f^2}{l_a^2} + \frac{I_f}{l_a^2}) \frac{c_{\gamma}^2}{c_{\gamma}} + I_d \]

\[ c_{2,l} = m_f (l_d s_{\gamma} c_{\gamma} - l_d s_{\gamma} c_{\gamma} - \frac{m_f}{l_a} \frac{c_{\gamma}}{c_{\gamma}}) + m_a \frac{l_a}{l_a} \frac{s_{\gamma}}{c_{\gamma}} + I_{\delta} \frac{c_{\gamma}}{c_{\gamma}} \]

\[ c_{8,l} = m_f l_f (l_f s_{\gamma} \frac{c_{\gamma}}{c_{\gamma}} - 2c_{\gamma} s_{\gamma} - 2c_{\gamma} s_{\gamma}) \]

\[ \tau_l = [0, T_{\delta,l}]^T \]. The action of the robot is to again provide assistance in directions that FES cannot, yet still allow FES to drive the movement. As in Section III, the robot will therefore impose arbitrary second order dynamics about the shoulder. Damping and inertia must also be prescribed about the elevation angle, \( \delta \), to result in safe, effective treatment. The fixed forearm extension means that the robotic assistance scheme (4) is now replaced by

\[
\begin{bmatrix}
    B_l(\hat{\vartheta}_u) \hat{q}_l \\
    C_l(\hat{\vartheta}_u, \hat{\vartheta}_f) \hat{q}_l + F_l(\hat{\vartheta}_u, \hat{q}_l)
\end{bmatrix}
= J_l^T(\hat{\vartheta}_f) \left(K_{K_l, \hat{x}} - K_{B_l, \hat{x}} - K_{M_l, \hat{x}}\right) + \tau_l
\]

(12)

To impose dynamics about the human arm, set

\[
K_{K_l, \hat{x}} - K_{B_l, \hat{x}} - K_{M_l, \hat{x}} = J_l^T(\hat{q}_l) \left(K_{K_l, \hat{q}_l} - K_{B_l, \hat{q}_l} - K_{M_l, \hat{q}_l}\right)
\]

(13)

where \( \hat{q}_l = \hat{q}_l - q_l \) and \( \hat{q}_l = k^{-1}_l(\hat{x}) \). Let the gains be given by \( K_{K_l} = \text{diag} \{ K_{K_l, \hat{x}} \} \), \( K_{B_l} = \text{diag} \{ K_{B_l, \hat{x}} \} \), \( K_{M_l} = \text{diag} \{ K_{M_l, \hat{x}} \} \), with the reference \( \hat{q}_l = [\hat{\vartheta}_u, \delta]^T \). This produces the expression

\[
\begin{bmatrix}
    K_{K_l} \ddot{\vartheta}_u - K_{B_l} \dot{\vartheta}_u - K_{M_l} \dot{\vartheta}_u \\
    -K_{B_l} \ddot{\delta} - K_{M_l} \ddot{\delta}
\end{bmatrix}
+ \tau_l
\]

(14)
for the r.h.s of (12), which provides the necessary dynamic relationship for both components of the torque. Components of (13) are then compared to yield the relationship
\[ K_{K_{i}}(\dot{x} - x) = J_{T}^{T}(\dot{q}_{i}) \left[ K_{K_{i}} \left( \hat{\theta}_{u} - \vartheta_{u} \right) \right] = \frac{K_{K_{i}}(\hat{\theta}_{u} - \vartheta_{u})}{l_{u}c_{\gamma} + l_{f}c_{\delta}} \left[ -s_{u} \right] \]
This leads to a solution for the robotic scheme of
\[ K_{K_{i}} = \frac{K_{K_{i}}(\hat{\theta}_{u} - \vartheta_{u})}{\dot{x} - x} \left( l_{u}c_{\gamma} + l_{f}c_{\delta} \right) I, \quad \dot{x} = x + \dot{x} - x \left[ -s_{u} \right] \]
so that \( \dot{x} \) is a point passing through \( x \) orthogonal to the upper arm and forearm. This is shown in Fig. 3b) in which the horizontal length of the arm is given by \( l(\delta) = l_{u}c_{\gamma} + l_{f}c_{\delta} \). The remaining robotic controller matrices may be chosen to satisfy
\[
\begin{align*}
K_{B_{i}} &= J_{T}^{T}(\dot{q}_{i})K_{B_{i}}J_{T}^{-1}(\dot{q}_{i}) \\
K_{M_{i}} &= J_{T}^{T}(\dot{q}_{i})J_{T}^{-1}(\dot{q}_{i}) = J_{T}^{T}(\dot{q}_{i})K_{M_{i}} \dot{q}_{i}
\end{align*}
\]
The reference point can now be defined as \( \dot{x} := k_{l} \left( \Psi_{2}(\Lambda^{-1}(\delta)) \right) \left[ \Psi_{2}(\Lambda^{-1}(\delta)) \right] \)
where \( \lambda_{l} \) is a scalar. The gain (15) can then be written explicitly as
\[ K_{K_{i}}(x, \Psi_{2}(\Lambda^{-1}(\cdot))) = \frac{K_{K_{i}}(\Psi_{2}(\Lambda^{-1}(\delta)) - \vartheta_{u})}{\lambda_{l}(l_{u}c_{\gamma} + l_{f}c_{\delta})} I \]
The validity of assuming that the forearm angle will remain at zero can now be checked by substituting the assistance dynamics into condition (6) to give
\[
(\frac{m_{f}l_{f}^{2}c_{\delta}^{2}}{c_{\gamma}} + l_{f}c_{\delta} + m_{f}l_{f}c_{f_{\delta}^{2}}) \frac{\hat{\theta}_{u} + (2m_{f}l_{f}c_{f_{\delta}^{2}} - 2m_{f}l_{f}c_{\delta}c_{\delta}) \frac{\hat{\theta}_{u}}{\dot{l}_{u}c_{\gamma} + l_{f}c_{\delta}} \hat{\theta}_{u} - F_{\delta}(\dot{\delta})}{F_{\delta}} \]
If it is assumed that the assistance results in perfect tracking of \( \hat{\theta}_{u} \) by \( \hat{\theta}_{u} \), and the derivatives of \( \hat{\theta}_{u} \) and \( \Lambda(\cdot) \) are small, then this reduces to \( F_{\delta}(0, 0) = 0 \). In addition, the bottom row of (11) with the dynamics (14) produces
\[
T_{\delta} = (h_{bF}(\delta - K_{M_{2}}) - K_{M_{2}}) \frac{\hat{\theta}_{u} + (c_{\delta}L(\delta) - K_{M_{2}}) \hat{\theta}_{u}}{\dot{l}_{u}l_{f}c_{\delta}} + M_{\delta}(\dot{\delta}, \dot{\delta}) + (m_{f}l_{f}^{2}c_{\delta} + m_{f}l_{f}c_{\delta}) c_{\delta} g
\]
where the arm holder torque constraining \( \delta \geq 0 \) is
\[
M_{\delta}(\dot{\delta}, \dot{\delta}, \ddot{\delta}) = \begin{cases} -\min \left\{ T_{\delta} - K_{B_{i}} - \frac{\Lambda_{l}}{l_{u}c_{\gamma} + l_{f}c_{\delta}} \right\} & \text{if } \delta = 0 \\ 0 & \text{otherwise} \end{cases}
\]
The combined muscle and arm system is shown schematically in Fig. 4. The Newton ILC scheme will again be adopted in order to control the stimulation pulse width, \( u \), applied to produce accurate tracking of the elevation angle. The necessary equations are derived in the next section. Since \( \beta = \dot{\delta} - \dot{\gamma} = f(\delta, \dot{\delta}) \), \( F_{\delta}(\dot{\delta}, \dot{\delta}) \) is accounted for in \( F_{\delta}(\dot{\delta}, \dot{\delta}) \) and can again be neglected from the muscle model.

VI. ELBOW LIFT STIMULATION CONTROLLER

An ILC scheme to lift the arm using FES is now developed. To produce a discrete-time system representation of the lifted system with sampling period \( T_{s} \), let the linear activation dynamics be represented by the state-space system in standard form \( \Phi_{m}, \Gamma_{m}, H_{m} \). The relationship between the signals \( w_{1} \) and \( w_{2} \) appearing in Fig. 4 is given by
\[
x_{m}(t + T_{s}) = \Phi_{m}x_{m}(t) + \Gamma_{m}w_{1}(t) \\
w_{2}(t) = H_{m}x_{m}(t) \quad t \in [T_{s}, T_{s} + T_{s}]
\]
where the initial condition \( x_{m0} = x_{m}(T_{s}) \) is inherited from the planar arm system. Similarly, the integrators in Fig. 4 can be represented by the state-space system \( \Phi_{b}, \Gamma_{b}, H_{b} \) so that the relationship between \( w_{1} \), \( \delta \) and \( \dot{\delta} \) is given by
\[
x_{b}(t + T_{s}) = \Phi_{b}x_{b}(t) + \Gamma_{b}w_{1}(t) \\
\begin{bmatrix} \delta(t) \\ \dot{\delta}(t) \end{bmatrix} = \begin{bmatrix} H_{b1} \\ H_{b2} \end{bmatrix} x_{b}(t) + H_{b}x_{b}(t) \quad t \in [T_{s}, T_{s} + T_{s}]
\]
with \( x_{b0} = x_{b}(T_{s}) = 0 \). The full lifted arm system is therefore given on trial \( k \) by

Fig. 4. Continuous-time model of stimulated lifted human arm.
\[ x_{i,k}(t + T_s) = \Phi_l x_{i,k}(t) + \Gamma_l [d(x_{i,k}(t)) - f_l(x_{i,k}(t), u_{k}(t))] \]
\[
\delta_k(t) = \bar{H}_{b_l} x_{i,k}(t) = h_l(x_{i,k}(t)) \quad x_{i,k}(T_1) = x_{i,0}
\]

where \( x_{i}(t) = [x_{p}(t) x_{m}(t)]^T \), \( x_{i,0} = [x_{p0} x_{m0}]^T \), \( \Phi_l = \text{diag}(\Phi_{b_l}, \Phi_{m}) \), \( \Gamma_l = \text{diag}(\Gamma_{b_l}, \Gamma_{m}) \), \( H_m = [0 H_m] \), \( H_{b_l} = [H_{b_l}^1 0] \) and \( H_{b_2} = [H_{b_2}^1 0] \), and

\[
d(x_{i,k}) = \left\{ \begin{array}{ll}
\max \{ \varepsilon (x_{i,k}), 0 \} & \text{if } H_{b_1} x_{i,k} = 0 \\
\varepsilon (x_{i,k}) & \text{otherwise}
\end{array} \right.
\]

with

\[
\varepsilon (x_{i,k}) = H_m x_{i,k} f_m (H_{b_1} x_{i,k}) - K_{M_2}
\]

\[
+ \left( c_{i,k} (H_{b_1} x_{i,k}) H_{b_2} x_{i,k} - K_{B_2} \right) H_{b_2} x_{i,k} - F_\delta (H_{b_1} x_{i,k}, H_{b_2} x_{i,k}) - \left( m_{1/2} + m_{1/2} \right) \cos (H_{b_1} x_{i,k}) g
\]

in which the explicit time dependence of \( x_{i,k} \) has been omitted. To replace (20) with a set of algebraic equations in \( \mathbb{R}^{N_2-N_1} \), where \( N_1 = T_i/T_s \), \( i = 1, 2 \), define the shifted input and output vectors as

\[
u_{k} = [u_{k}(N_1), u_{k}(N_1 + 1), \ldots, u_{k}(N_2 - 1)]^T
\]

\[
\delta_k = [\delta_k(N_1 + 1), \delta_k(N_1 + 2), \ldots, \delta_k(N_2)]^T
\]

and following the procedure of [8], the relationship between the input and output time-series can be expressed by the following algebraic functions

\[
\delta_k(N_1 + 1) = h_l(x_{i,k}(N_1 + 1)) = h_l(f_l(x_{i,k}(N_1), u_{k}(N_1)))
\]

\[
= g_{l,1}(x_{i,k}(N_1), u_{k}(N_1))
\]

\[
\delta_k(N_1 + 2) = h_l(x_{i,k}(N_1 + 2)) = h_l(f_l(x_{i,k}(N_1 + 1), u_{k}(N_1 + 1)))
\]

\[
= g_{l,2}(x_{i,k}(N_1), u_{k}(N_1), u_{k}(N_1 + 1))
\]

\[
\vdots
\]

\[
\delta_k(N_2) = h_l(x_{i,k}(N_2)) = h_l(f_l(x_{i,k}(N_2 - 1), u_{k}(N_2 - 1)))
\]

\[
= g_{l,N_2-N_1+1}(x_{i,k}(N_1), u_{k}(N_1), u_{k}(N_1 + 1), \ldots, u_{k}(N_2 - 1))
\]

so that the system (20) can be represented as

\[
\delta = g_l(u_k), \quad g_l(\cdot) = [g_{l,1}(\cdot), g_{l,2}(\cdot), \ldots, g_{l,N_2-N_1}(\cdot)]^T
\]

(21)

The ILC task of finding the input which drives the dynamic system (20) to track the desired output, becomes finding the solution that satisfies the non-linear function (21) with \( \delta_{k} \) substituted by \( \delta^* = [\delta^*(N_1 + 1), \delta^*(N_1 + 2), \ldots, \delta^*(N_2)]^T \).

The Newton method is selected to solve this non-linear equation, and is given in ILC notation as

\[
u_{k+1} = u_k + \alpha_{k+1} g_l^{-1}(u_k) e_k
\]

(22)

where \( e_k = \delta^* - \delta_k \), and the scalar \( \alpha_{k+1} \geq 0 \) is a relaxation parameter. The derivative \( g_l(u_k) \) is equivalent to the linearisation of (20), on the \( k^{th} \) trial at \( (u_k, x_k) \) which can be represented by the time-varying system

\[
\delta = g_l(u_k) \bar{u}
\]

(23)

which is given by

\[
x_{i}(t + T_s) = A_{I}(t)x_{i}(t) + B_{I}(t)\bar{u}(t)
\]

\[
\delta(t) = C_{I}(t)x_{i}(t) \quad t \in [T_1, T_2]
\]

with

\[
A_{I}(t) = \left( \frac{\partial f_l}{\partial x_l} \right)_{u_l(t), x_l(t)} = \Phi_l + \Gamma_l \begin{bmatrix} p(t) \\ 0 \end{bmatrix}
\]

\[
B_{I}(t) = \left( \frac{\partial f_l}{\partial u_l} \right)_{u_l(t), x_l(t)} = \Gamma_l \begin{bmatrix} 0 \\ h_{l,IRC}(u_{k}(t)) \end{bmatrix}
\]

\[
C_{I}(t) = \left( \frac{\partial h_{l}}{\partial x_{i}} \right)_{u_l(t), x_l(t)} = \bar{H}_{b_l}
\]

where \( \bar{x}_{i} = x_{i,1,k+1} - x_{i,k+1} \), \( \bar{u} = u_{k+1} - u_{k} \), \( \bar{\delta} = \delta_{k+1} - \delta_{k} \), \( x_{i}(t) = x_{i,1,k+1}(1) - x_{i}(1) = 0 \), and

\[
p(t) = \begin{cases}
0 & \text{if } \varepsilon (x_{i,k}) < 0 \text{ and } \bar{H}_{b_1} x_{i,k} = 0 \\
\frac{\beta_{i} h_{l,IRC}(u_l) e_{x_i}}{(h_{l,IRC}(u_l) - \bar{K}_{M_2})^2} + \varepsilon'(x_{i,k}) & \text{otherwise}
\end{cases}
\]

If the system (23) can be made to track \( e_k \), the corresponding input is \( \bar{u} = g_l^{-1}(u_k) e_k \) which is then used in the update (22).

As in [7] the Norm Optimal ILC (NOILC) method has been applied to this (see [9]), thereby obviating the problem of calculating the inverse of \( g_l^{-1}(u_k) \) directly. Using NOILC, the input to (23) on the \((n+1)^{th}\) trial is chosen to minimise

\[
J_{m+1} = \sum_{i=1}^{N_2-N_1+1} \| e_{k+1}(i) - \delta_{m+1}(i) \|^2 + \sum_{i=1}^{N_2-N_1} \| \bar{u}_{m+1}(i) - \bar{u}_{m}(i) \|^2
\]

(24)

where \( Q \) and \( R \) are scalar weights. The parameter \( \alpha_{k+1} \) has been chosen to minimise

\[
J_{k+1}(\alpha_{k+1}) = \| e_{k+1} \|^2 = \| \delta^* - g_l^{-1}(u_k + \alpha_{k+1} \bar{e}_k) \|^2
\]

(25)

VII. EXPERIMENTAL RESULTS

Having developed robotic assistance and FES control schemes for (7) over \( t \in [0, T_1] \), and (11) over \( t \in [T_1, T_2] \), it is necessary to ensure that switching duly occurs at \( T_1 \). The criteria adopted to initiate the latter controllers on trial \( k \) is

\[
\delta_{f,k}(N_1) < s_1 \quad \text{and} \quad \sum_{i=1}^{N_1} (\delta_f(t) - \delta_{f,k}(t))^2 < s_2
\]

(25)

where \( s_1 \) and \( s_2 \) are small positive scalars. These criteria approximately fulfill the switching demand \( \delta_f \approx 0 \) at \( T_1 \), and additionally reduce the initial condition variation for the lifted system. It is desirable that these be met within a small number of trials to reduce the total number necessary.

A secondary consideration is that torque generated by the planar system at \( t = T_1 \) is sufficiently close to violating (10) so that forearm lift will subsequently occur without excessive increased stimulation. This can be addressed through selection of \( K_{M_2} \) and \( K_{B_2} \). Selection of \( K_{M_2} \) and \( K_{B_2} \) must also ensure that changes in the direction of \( \delta \) at the point of switching do not transfer into large changes in robotic assistance torque. In practice, the accurate tracking of \( \delta_{f} \) observed has meant that such large changes do not occur.

Transient response of the ILC scheme can be tuned through selection of the NOILC parameters \( Q \) and \( R \), and
convergence speed can be traded for robustness by replacing automatic selection of $\alpha_k$ via (24) by a smaller fixed value.

Parameters appearing in the planar arm system (7) have been identified using tests described in [7], and this has been extended in [4] for the lifted arm system (11). These procedures are far simpler than identifying the full human arm system (1). In particular, muscles parameters were found using isometric tests [5], and stimulation sequences and kinematic trajectories were applied to yield the remaining model parameters through LMS optimisation.

Experiments have been performed on a 60 year old subject to verify the approach. Elliptical trajectories requiring full arm extension have been selected on a clinical basis and then decomposed into $\Psi_1$, $\Psi_2$, $\Upsilon$, and $\Lambda$ components. The constant velocity movement considered here is $7.5s$ long, with a $5s$ region appended to the beginning and a $2.5s$ region appended to the end. Figure 5 shows the tracking of the planar arm system over 6 trials. By trial 6, $\theta_f$ tracks the reference well (a) with stimulation which is not excessive (b), resulting in a small error (d). At this point the switching criteria (25) are satisfied and the lifted control scheme is initiated. Figure 6 shows the resulting tracking over 8 further trials. The overall movement, comprising both the planar and lifted components, is shown in Fig. 7. The planar system gains are $R = 1$, $Q = 50$, and the lifted system gains are $R = 1$, $Q = 100$. Further experimental results appear in [4].

VIII. CONCLUSIONS AND FUTURE WORK

A Newton method based nonlinear ILC scheme has been developed for use in stroke rehabilitation, and will subsequently be used in clinical trials with patients. Future research will extend the number of muscles stimulated to maximise the effectiveness of treatment.

IX. ACKNOWLEDGMENTS

This work is dedicated to the memory of Iain L. Davies.

REFERENCES


Fig. 5. Planar system with 35° trajectory: a) tracking of $\theta_f$, b) applied stimulation, $u_k$, c) variation in $\alpha_k$, and d) error norm, $\|e_k\|$.

Fig. 6. Lifted system with 35° trajectory: a) tracking of $\delta$, b) applied stimulation, $u_k$, c) variation in $\alpha_k$, and d) error norm, $\|e_k\|$.

Fig. 7. Reference trajectory, position of subject, and final trial tracking.