Zero Vibration Position Control of a Spherical Pendulum for Control Systems Demonstration

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Abstract—This paper presents the control design and interface implementation of a spherical pendulum system for control education demonstration. The Zero Vibration input shaper is implemented in the robot controller to minimize command induced cable swing. To maximize user perception, the interface is developed on a LabVIEW environment within a standard PC which is interface to the robot controller by a T1 connection. The system demonstration enables a broad range of audience to appreciate control systems and its effects in improving system performance.

I. INTRODUCTION

Control systems are a multidisciplinary subject encompassing almost all fields of engineering applications. However, the traditional presentation of this subject tends to be highly theoretical and mathematical with heavy emphasis on equation derivation and algorithmic development. Such appearance may at times be discouraging to students/audiences with different learning styles [1-4], especially when about 70% of learners are not analytic learners [5], [6]. Experiential learning theory advocates a holistic approach to combine experience, perception, cognition and behavior [7-9]. Therefore, control demonstration should be adapted accordingly to address a broad range of audience. In the past decade, the availability of many commercial hardware demonstration units such as magnetic bearing and inverted pendulum has increased the interests of control systems among prospective students. It can be argued that many such technologies have significant interest to our students. However, the subject of robots has long fascinated many young minds and can be a very effective demonstration platform to develop interest in control systems and mechatronics technology. For example, the FIRST competition [10] has made major progress in introducing robotics into high schools. A number of schools also utilized the LEGO system for robotic design [11-13]. Over the years, high speed Internet also serves as an enabling factor for remote programming [14, 15] and laboratory exploration [16-22]. To maximize motivation and interest, a competitive, game set up tends to have the best impact [10, 23-26]. Various approaches using robots as a learning tool have also been reported [19, 20, 24, 27-33].

The spherical pendulum encapsulates the properties of many different types of engineering applications such as a building crane, mid-air refueling, etc. [34-37]. It is also a phenomenon that a general audience can easily relate to from past experience such as playing a yo-yo, swing line, etc. In these cases, fast and precise position manipulation is essential: a task well suited for computer control. In general, the spherical pendulum system consists of three parts: actuating pivot, string/rod, and a bob (load). For a non stretchable/compressive string/rod, the bob may have up to three degrees of freedom in the angular directions while moving in a constrained 3d space with respect to the string or rod. Due to the low damping factor, command induced oscillation may present a serious degradation to the performance of the system [38]. The paper reports on the design, implementation, and demonstration of the Zero Vibration (ZV) [39] control of the bob position using a modified ZV shaper which is derived based on the linearized dynamical model of the spherical pendulum. An Adept Technology Cobra 600 SCARA robot [15] serves as the actuation platform. Both simulation and experiment data confirm that the ZV method achieves an overshoot-free response with a settling time of one half of the vibration period. The organized of this paper is organized as follows: In section II The experimental setup is described. Section III outlines the model and control design for the pendulum. Control algorithm and simulation results are given in Section IV. In Section V, experimental data are presented. This is followed by a description of the demonstration and conclusions in Sections VI and VII respectively.

II. EXPERIMENTAL SETUP

The experimental set up is shown in Figure 1. An Adept Technology 4-axis SCARA Cobra 600 robot is used to move the pivot of the pendulum. The Cobra 600 has a reach of 600 mm in the horizontal (x-y) plane and a reach of 210 mm in the vertical (z) direction. The robot’s maximum payload is 5.5 kg. The ranges of motion for the joints are as follows: joint 1: ±105°, joint 2: ±150°, joint 3: ±210 mm, joint 4: ±360°. Joint 1 is the joint connected directly to the base of the robot; its motion is in the x-y plane. Joint 2 is the next link out attached to the end of joint 1. Its motion is also in the x-y plane. Joint 3 is located at the end of joint 2 and oriented perpendicular to joint 2. Joint 3’s motion is purely up and down along the z-axis. Joint 4 is the rotation of the pivot.
The robot is powered and operated through the Adept MV-1060 controller. This controller consists of two separate towers, a PA-4 power amplifier chassis and a MV-10 Adept Windows controller chassis. The MV-10 controller has internal memory with V+ programming environment as well as free space for storing program code. The robot controller is connected to a PC, via a T1 Ethernet connection (10Mbit/s). The PC runs LabVIEW which animates and interfaces the robot movement.

As shown in Figure 1, the top end of the pendulum string is attached to the end effector of the Cobra robot. With this setup the robot has the ability to move the pivot of the pendulum in all three dimensions. Network File System (NFS), which enables the controller to access files over a network as if they were on its local disks, is implemented using a connectionless protocol (UDP) in order to make it stateless as shown in Figure 2.

Data communication setup in LabVIEW utilizes four Virtual Instruments (VI’s) to establish a connection. In Figure 3, the first VI (blocks from left to right) opens, creates or replaces a text file, in this case the file is called ABC.dat, so that the second VI can determine where to place the data inside the text file, in our example the data gets placed at the beginning (start) of the file. The third VI writes the given binary code into the text file, and the last VI closes the connection. The MV10/PC communication rate was measured to be about 14 Hz, too slow for controller implementation on the PC side. Therefore, time sensitive real time control is exclusively embedded on the MV10.

### III. MODELING AND CONTROL OF THE SPHERICAL PENDULUM

In this experiment, the pendulum cable length is given as 1.06 meter. Nomenclature and axis definitions are shown in Figure 4.

Differential equations of the spherical pendulum are readily derived using Lagrange Equation (1)-(2):

\[
K = T - V = E_{kin} - E_{pot} \quad (1)
\]

\[
\frac{d}{dt} \left[ \frac{\partial}{\partial q_k} [L] \right] = \frac{\partial}{\partial q_k} [L] \quad (2)
\]

Kinetic energy (3) of the Spherical Pendulum is defined as:

\[
E_{kin} = \frac{1}{2} mv^2 = \frac{1}{2} ml^2 \dot{\Theta}^2 + \frac{1}{2} ml^2 \sin^2 \theta \dot{\Theta}^2 \quad (3)
\]

While potential energy (4) is defined as:

\[
E_{pot} = mgh = lmg(1 - \cos \theta) \quad (4)
\]

resulting in the following Lagrangian (5) equation:

\[
L = \frac{1}{2} ml^2 \dot{\Theta}^2 + \frac{1}{2} ml^2 \sin^2 \theta \dot{\Phi}^2 - lmg(1 - \cos \theta) \quad (5)
\]

Without considering cable torsion, the two angular variables are described by (6) and (7) below:
\begin{align*}
\dot{\Theta} &= \frac{1}{2} \Phi^2 \sin 2\theta - \frac{\mathcal{G}}{l} \sin \theta \\
\dot{\Phi} &= -\frac{\sin 2\theta \cdot \dot{\Theta}}{\sin \theta} \Phi 
\end{align*}

\begin{align*}
\text{A linearized state space model of the spherical pendulum with } 
\Theta = \frac{\pi}{8}, \quad \omega = 0, \quad \Phi = 0, \quad \dot{\Phi} = 0 
\text{can be readily derived as:} 
\begin{bmatrix}
0 & 1 & 0 & 0 \\
-8.54 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix} 
\begin{bmatrix}
A \\
B \\
C
\end{bmatrix} 
\begin{bmatrix}
\dot{\Theta} \\
\dot{\Phi}
\end{bmatrix} 
\begin{bmatrix}
D \\
E
\end{bmatrix} 
\begin{align*}
V(\omega, \zeta, t_n) &= |y(t_n)| 
&= e^{-\omega_d t_n} \left[ \sum_{i=1}^{m} B_i e^{\omega_d t_i} \cos(\omega_d t_i) \right]^2 
&= \frac{\pi}{\omega_t \sqrt{1 - \zeta^2}} 
\end{align*}
\end{align*}

IV. CONTROL ALGORITHM AND SIMULATION RESULT

The primary goal for the controller is to control the position of the bob in a way where the response is vibration free and has a fast and smooth transient characteristic. The Zero Vibration control [39] is a feed forward control that shapes the control input of a system. An overview of this method based on a second order system is given below:

\begin{align*}
G_s(s) &= \frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2} 
\end{align*}

This system has the following impulse response, where \(u(t)\) is the unit step:

\begin{align*}
h(t) &= \frac{\omega_n}{\sqrt{1 - \zeta^2}} e^{-\zeta \omega_n t} \sin \left( \omega_n \sqrt{1 - \zeta^2} t \right) u(t) 
\end{align*}

If the system is excited with an impulse train \(A_i \delta(t-t_i)\), where \(i\) represents the impulse index, the output is given by (11).

\begin{align*}
y_{(t_i)} &= \sum_{i=1}^{m} B_i e^{\omega_d t_i} \cos(\omega_d (t_n - t_i)) 
\end{align*}

Where \(B_i = \frac{A_i \omega_n}{\sqrt{1 - \zeta^2}}\) and \(\omega_d = \omega_n \sqrt{1 - \zeta^2}\).

Equation (11) can be rewritten as:

\begin{align*}
y(t_n) &= e^{-\zeta \omega_n t_n} \left[ \sum_{i=1}^{m} B_i e^{\omega_d t_i} \cos(\omega_d t_i) \right] \sin(\omega_d t_n) 
\end{align*}

By eliminating the sinusoidal terms that depend on \(\omega_d t_n\), the residual Vibrations can be shown as:

![Fig. 5 step response of the ZV shaper.](image)

![Fig. 6 Pendulum movement in terms of the theta axis.](image)
\[ X_{\text{displacement}} = l \sin(\Theta) \]
\[ Y_{\text{displacement}} = l \cos(\Theta) \]  

The residual vibration error (13) with respect to amplitude (14) and timing (15) is shown in Figure 7 where it is observed that the ZV shaper is more sensitive against a timing error than an amplitude error. It therefore again supports the argument that the time sensitive control be implemented on the MV10 controller.

The second automatic task is that the bob follows a V-shape trajectory that includes 2 movements (both in X-axis and Y-axis simultaneously. These movements are shown in Figure 10 where the robot tip traverses movement (D & E).

In this case, the coordinates for the bob trajectories are, 
\[ P1=(415,-415), P2=(200,0), P3=(415,415) \]

The ZV shaper had the same properties for A1, A2 & \( \Delta T \) as in the U-shape task. It should be noted that it is also possible to design a shaper if the step response of the system is measurable such as that of Figure 10. There are two properties which are important to measure, the period of the vibration and the overshoot of the step response. With this
two measured values it is possible to define $A_1, A_2 & ΔT$.

$$K_{OS} = \frac{I}{\Pi}, \quad A_1 = \frac{1}{1 + K_{OS}}, \quad A_2 = \frac{K_{OS}}{1 + K_{OS}}, \quad ΔT = \frac{T_d}{2} \quad (17)$$

![Diagram of pendulum system](image)

Fig. 11 Experimental determination of shaper parameters.

VI. DEMONSTRATION

The demonstration is intended for a broad audience who may or may not have a Science, Technology, Engineering, and Mathematics (STEM) background. Group size varies from 10-20 per demonstration which begins by asking a volunteer to move a second but identical pendulum by hand with the requirement of minimum pendulum swing. The typical human operator either cannot satisfy this requirement or has to move very slowly. This is followed by the robot manipulation of the pendulum using the ZV shaper. The pendulum system has been demonstrated to a number of groups at different occasions:

1. High school summer experience
2. University open house
3. Career day

For high school experience, the students are typically 11th graders with some STEM background. For University open house, the audience consists of both high school students and parents with the latter typically being the non-STEM type. Career day is aimed at electrical & computer engineering students who are deciding a major such as communication, control, solid states, etc. Therefore this last group has the strongest STEM background and readily understands the ZV shaper construction. Although such a cross spectrum of audience poses a strong challenge to control systems demonstration where the goal is to impress, interest, and impale control engineering into the audiences’ mind. However, the pendulum demonstration is highly successful in each case as the audience can readily gauge the difficulty of such manipulation by hand. The GUI LabVIEW front panel also improves visualization by its customized design and compact layout. Feedback from the audience after the demonstration confirms the effective of the robot/spherical pendulum as an effective demonstration instrument.

VII. CONCLUSIONS

This paper presents the control design and interface implementation for a spherical pendulum demonstration system. A zero vibration shaper minimizes command induced pendulum cable swing. However, due to the relatively low communication rate between the robot controller and the user PC as well as the high temporal resolution of the shaper, time sensitive codes are embedded in the robot controller. Consequently, it is demonstrated experimentally that the ZV shaper results in a precise and vibration free pendulum movement in a three dimensional space. Demonstration to various student/parent groups has been very successful in that the audience can relate to the nature of the experiment and the level of difficulty in manipulating the pendulum manually. It was also observed that the general audience readily comprehends operating principles of the ZV shaper.

VIII. REFERENCES