Adaptive robust dynamic surface control of DC torque motors with true parameter estimates

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Abstract—This paper presents an adaptive robust dynamic surface control (ARDSC) algorithm for the position control of DC torque motors which are modeled as third-order nonlinear systems with parametric and nonlinear uncertainties. Based on the dynamic surface control (DSC) approach, the “explosion of terms” problem in the normal adaptive robust control (ARC) is avoided, which enhances the practicability of the controller. Furthermore, besides theoretically proving that the tracking error is uniformly ultimate bounded or asymptotically converge to zero in presence of parametric uncertainties only, the true estimates of the unknown parameters are achieved by designing a novel adaptation law, which improves the system performance and can be used in system monitor and diagnosis on-line as byproducts. Finally, the comparative simulation results illustrate that the proposed algorithm is very effective.

I. INTRODUCTION

DC torque motors are widely used in industrial systems, such as robotic manipulators, flight tables, because their controls are relatively simple and they can be directly attached to the load. The direct attachment will eliminate some unnecessary nonlinearity (e.g. backlash or dead-zone) produced by mechanical transmissions. Meanwhile it will bring about some other problems. For example the effect of model uncertainties, such as parameter variations (e.g. unknown payload), unknown nonlinearities (e.g. friction torque) and external disturbance, will become more serious. Thus in order to exploit the high performance of the direct-drive DC motors, the controller has to be elaborately designed. In recent years, a great deal of attention has been paid to solving these problems in controlling DC motors [1~5]. As we know, a DC motor servo system is usually modeled as a second-order or third-order nonlinear system subjected to parametric uncertainties and unknown dynamic nonlinearities [1~3]. And for this class of nonlinear system, in recent literatures, there are many control methods to achieve the required performance [4~6]. An adaptive robust control approach [6] which combines the advantages of the adaptive and robust control is able to deal with parametric and nonlinear uncertainties simultaneously, It is popularized in community of servomechanisms control. For instance, it has been widely used in linear motor control [7], hard-disk servomechanism [8] and hydraulic systems [9]. However, it still has some drawbacks: first, since it integrates the backstepping approach the “explosion of terms” problem would occur, which eventually leads to a complicated control law and makes the controller difficult to implement in practice; second, its update law is driven by tracking error signals only, which will yield low convergent speed and bad accuracy of parameter estimation. Consequently, an improved ARC using DSC technique[10] to replace backstepping was proposed [3][11]. It solves the “explosion of terms” problem perfectly. However, it can only guarantee a uniformly ultimate bounded result of the system tracking error even if there only parametric uncertainties exist. Furthermore, it did not concern about the convergence performance of unknown parameters. On the other hand, to overcome the second drawback, Bin Yao et al. introduced a least square method in lieu of the gradient type update law to identify the unknown parameters in [12] and [13] according to the modularized design philosophy. Namely, the unknown parameters were identified dependently on the controller design and the system stability was guaranteed by a strong robust controller. However, the “explosion of terms” problem has not been taken into account in [12] and [13].

In this paper, the main purpose is to achieve faster and more accurate position tracking and parameter convergence simultaneously with a simple and effective controller. Thus, the main contributions of this paper can be concluded as follows.

- An adaptive robust dynamic surface control with a composite adaptation law is firstly proposed for the position tracking control of DC torque motors.
- An improved DSC approach is proposed to solve the “explosion of term” problem and the asymptotical position tracking error is proved theoretically in presence of the parametric uncertainties only.
- A composite gradient type of adaptive update law which integrates with parameter estimation error signals is constructed to guarantee faster and more accurate parameter estimates. Furthermore, when the PE condition is satisfied and there only parametric uncertainties exist, true parameter estimates are achieved.

The remainder of this paper is organized as follows. In section 2, the model of a DC torque motor is constructed. Then the controller design procedure is provided in section 3. The composite adaptation law and stability and performance of the system are given in section 4. In section 5 the advantages of the proposed method are illustrated by some comparative simulations. Section 6 concludes this paper.
II. MODEL OF THE DC TORQUE MOTOR AND PROBLEM FORMULATION

A DC torque motor servomechanism (seen Fig.1) is usually modeled as an uncertain nonlinear system by considering the friction torque and load disturbance and ignoring the effect of armature reaction. Its mathematical model is given by the following three functions[14].
\[
\begin{align*}
J\ddot{q} + T_f + T_{\text{dis}} + T_l &= T_m, \\
K_p\dot{q} + L_a \frac{di_a}{dt} + R_a i_a &= u, \\
T_m &= K_I I_a, \\
\end{align*}
\]
where \(q\) is the angular position and \(\dot{q}\) is the angular velocity. \(J, T_f, T_l, T_{\text{dis}}, T_m\) are the mechanical parameters: motor inertia, friction torque, load disturbance torque and generated torque respectively. \(u, I_a, R_a, I_a\) are the electrical parameters: input voltage, armature current, armature resistance and armature inductance respectively.

A simple friction model considering only Coulomb and viscous friction in this paper is given by[12]
\[
T_f(\dot{q}) = T_c sgn(\dot{q}) + B\dot{q},
\]
where \(T_c\) is the minimum level of Coulomb friction torque, \(B\) is the viscous coefficient, and \(sgn(*)\) is an standard sign function that has the following form
\[
sgn(*) = \begin{cases} 
1, & \text{if } * > 0, \\
-1, & \text{if } * < 0, \\
[-1, 1], & \text{if } * = 0.
\end{cases}
\]
As we known, in practical application, the DC motor cannot generate discontinuous torque to compensate the friction. Thus, we use a differentiable function \(S_f(*)\) to approximate (3). Then (2) can be rewritten as
\[
T_f(\dot{q}) = T_c S_f(\dot{q}) + B\dot{q} + \tilde{T}_f,
\]
where \(\tilde{T}_f\) is the friction approximation error.

Defining \([x_1, x_2, x_3]^T = [q, \dot{q}, i]^T\) and from (1), the state space model of the DC servomechanism is described as
\[
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= p_1 x_1 + p_2 S_f(x_2) + p_3 x_2 + p_4 + \Delta(x_1, x_2, t), \\
\dot{x}_3 &= p_5 u + p_6 x_2 + p_7 x_3, \\
y &= x_1.
\end{align*}
\]
where \(p_1 = K_f, p_2 = -\frac{T_f}{J}, p_3 = -\frac{B}{J}, p_4 = -\frac{T_l}{J}, p_5 = \frac{1}{T_c}, p_6 = -K_p, p_7 = -R_a\) are all unknown parameters; \(\Delta(x_1, x_2, t) = -\frac{T_{\text{dis}}}{T_c + T_{\text{dis}}}\) is the lumped disturbance. From (5), we known that the DC torque servo system is subjected to parametric uncertainties \(p_i\) and nonlinear uncertainty \(\Delta\).

In addition, all the sates in (5) are measurable.

Throughout the paper, the following notations will be used. In general, the operation \(\leq\) for two vectors is performed in terms of the corresponding elements of the vectors. Let \(\hat{x}\) denote the estimate of \(*\) (e.g., \(\hat{\theta}_i\) for \(\theta_i\)) and \(\hat{x}\) is defined as \(\hat{x} = x - \hat{x}\). \(|x|\) denotes the Euclidean norm of \(x\).

**Lemma 1**[15]: Consider the function \(\phi: R_+ \rightarrow R\). If \(\phi, \dot{\phi} \in L_{\infty}\), and \(\phi \in L_p\) for some \(p \in [1, \infty)\), then
\[
\lim_{t \to \infty} \phi(t) = 0.
\]

**Assumption 1**: The lower and upper bounds of the unknown parameters are all known, i.e.
\[
p_{i(\text{min})} \leq p_i \leq p_{i(\text{max})}, i = 1, 2, \cdots, 7,
\]
where \(p_{i(\text{min})}, p_{i(\text{max})}, i = 1, 2, \cdots, 7\) are known constants, moreover, \(p_{i(\text{min})} > 0, p_{i(\text{min})} > 0\).

**Assumption 2**: The nonlinear uncertainty \(\Delta\) is assumed to be bounded by
\[
|\Delta(x_1, x_2, t)| \leq \delta,
\]
where \(\delta\) is a known positive constant.

**Assumption 3**: The desired trajectory is continuous and available, and \([x_r, \dot{x}_r, \ddot{x}_r]^T \in \Omega_r\) with a known compact set \(\Omega_r = \{[x_r, \dot{x}_r, \ddot{x}_r]^T : x_r^2 + \dot{x}_r^2 + \ddot{x}_r^2 \leq B_0\} \subset R^3\), whose size \(B_0\) is a known positive constant.

The objective of this paper is to design a bounded control law for the input \(\dot{u}\) such that the output position \(y\) tracks the desired position trajectory \(x_r\) as closely as possible in spite of the aforementioned uncertainties.

III. ADAPTIVE ROBUST CONTROL USING DYNAMIC SURFACE APPROACH

In this section, we introduce an improved DSC approach instead of the integral backstepping method for the ARC design to avoid the so-called “explosion of terms” problem. The concrete design procedure is given as follows.

**Step 1**: Noting that the first equation of (5) does not have any uncertainties, a dynamic surface can be omitted via defining a switching-function-like quantity as
\[
z_2 = \dot{z}_1 + k_1 z_1 = x_2 - x_{2r}, \quad x_{2r} = \dot{x}_{2r} - k_1 z_1,
\]
where \(z_1 = x_1 - x_{1r}(t), k_1\) is any positive feedback gain.
If \(z_2\) is small or converges to zero exponentially, then the output tracking error \(z_1\) will be small or converge to zero.
exponentially since $G_s(s) = z_1(s)/z_2(s) = 1/(s + k_1)$ is a stable transfer function. So the rest of the design is to make $z_2$ as small as possible with a guaranteed transient performance. Differentiating (9) and noting (5), we have

$$\ddot{z}_2 = \ddot{x}_2 - \dot{x}_2 = p_1 x_3 + p_2 S_f(x_2) + p_3 x_2 + p_4 + \Delta - \dot{x}_2$$

where $\dot{x}_2 = \hat{a} \dot{x}_1 - k_1 \dot{x}_1$. In (10), if we treat $x_3$ as the input, a virtual control law $\alpha_2$ for $x_3$ can be synthesized such that $z_2$ is as small as possible. Herein we utilize an ARDCP approach to accomplish the objective since both parametric uncertainties and nonlinear uncertainty exist in (10).

The control function $\alpha_2$ consists of two parts given by

$$\alpha_2(x_1, x_2, \hat{\theta}, t) = \alpha_{2a} + \alpha_{2s},$$

$$\alpha_{2a} = \frac{1}{p_1} \left( -\hat{p}_2 S_f(x_2) - \hat{p}_3 x_2 - \hat{p}_4 + \dot{x}_2 \right),$$

(11)

where $\hat{\theta} = [\hat{p}_1, \hat{p}_2, \hat{p}_3, \hat{p}_4]^T$, $\alpha_{2a}$ represents the adaptive control law to overcome the parametric uncertainties through online parameter adaptation, and $\alpha_{2s}$ is a robust control law to be synthesized later.

Unlike the norm ARC approach design procedure, in this paper, we introduce an improved DSC approach to simplify the structure of controller.

$$\tau_2 \ddot{z}_2 + f_2 \dot{z}_2 = \alpha_2 - k_v \tau_2 sgn(z_2 - \alpha_2), \quad z_2(0) = \alpha_2(0).$$

(12)

Let $z_3 = x_3 - x_2$ as the second error surface and $y_2 = x_2 - \alpha_2$ as the input-output error of the filter. Substituting (11) into (10) leads to

$$\ddot{z}_2 = p_1 (z_3 + y_2 + \alpha_{2s}) + \hat{\theta}_2^T \phi_2 + \Delta - \dot{x}_2,$$

where $\theta_2 = [p_1, p_2, p_3, p_4]^T$, $\hat{\theta}_2 = [\hat{p}_1, \hat{p}_2, \hat{p}_3, \hat{p}_4]^T$, and $\phi_2 = [\alpha_{2a}, S_f(x_2), 1]^T$.

(13)

In (13), if $z_3, y_2$ are small or converge to zero, then we can elaborately design the robust control law $\alpha_{2s}$ to make sure that $z_2$ is small. It is easily to deduce the dynamic equation of $y_2$ that

$$\dot{y}_2 = \dot{x}_2 - \dot{\alpha}_2 = -\frac{y_2}{p_1} - \dot{\alpha}_2 - k_v sgn(y_2),$$

(14)

which indicates that if $\dot{\alpha}_2$ is bounded, the filter error $y_2$ will exponentially converge to zero in finite time by choosing proper gain $k_v$.

The robust control law $\alpha_{2s}$ consists of two terms given by

$$\alpha_{2s} = \alpha_{2s1} + \alpha_{2s2}, \quad \alpha_{2s1} = -\frac{1}{p_1} k_{2s1} z_2,$$

(15)

where $\alpha_{2s2}$ is a robust control function designed in the following and $k_{2s1}$ is a positive feedback gain that will be determined later to stabilize the normal system. Substituting (15) into (13), we have

$$\ddot{z}_2 = p_1 (z_3 + y_2) - \frac{p_1}{p_1} k_{2s1} z_2 + p_1 \alpha_{2s2} + \hat{\theta}_2^T \phi_2 + \Delta.$$

(16)

Define a positive semi-definite (psd.) function $V_2$ as $V_2 = (1/2) z_2^T z_2$. From (16), its time derivative is

$$\dot{V}_2 = p_1 z_2 (z_3 + y_2) - \frac{p_1}{p_1} k_{2s1} z_2^2 + z_2 (p_1 \alpha_{2s2} + \hat{\theta}_2^T \phi_2 + \Delta).$$

(17)

The robust control function $\alpha_{2s2}$ is now chosen to satisfy the following conditions:

- condition (i) $z_2 \{ p_1 \alpha_{2s2} + \hat{\theta}_2^T \phi_2 + \Delta \} \leq \varepsilon_2$,
- condition (ii) $z_2 \alpha_{2s2} \leq 0$, (18)

One of smooth example of $\alpha_{2s2}$ satisfying (18) can be chosen as

$$\alpha_{2s2} = -\frac{h_2}{2p_{1\min}} z_2,$$

(19)

with $h_2 \geq \|\theta_{2\max} - \theta_{2\min}\|_2^2 \|\phi_2\|_2^2 + \Delta^2$.

**Step 2**: The next step is to force $x_3 \rightarrow x_{2f}$. Consider the second dynamic surface $z_3 = x_3 - x_{2f}$, whose time derivative is

$$\dot{z}_3 = p_5 u + p_6 x_2 + p_7 x_3 - \dot{x}_2,$$

(20)

Similar to step 1, the system control law $u$ consists of two parts

$$u(x_1, x_2, x_3, \hat{\theta}, t) = u_a + u_s,$$

$$u_a = \frac{1}{p_5} (-p_6 x_2 - p_7 x_3 - \dot{x}_2),$$

(21)

where $\hat{\theta}_3 = [\hat{p}_5, \hat{p}_6, \hat{p}_7]^T$, $u_a$ represents the adaptive control law to overcome the parametric uncertainties and $u_s$ represents the robust control law to be synthesized later.

Substituting (21) to (20) leads to

$$\dot{z}_3 = p_5 u_a + p_5 u_s + p_5 u_s + p_6 x_2 + p_7 x_3 - \dot{x}_2,$$

(22)

where $\hat{\theta}_3 = [\hat{p}_5, \hat{p}_6, \hat{p}_7]^T$, and $\phi_3 = [u_a, x_2, x_3]^T$. Our purpose is to stabilize the error $z_3$. From (22), the robust control law $u_s$ can be designed as

$$u_s = u_{s1} + u_{s2}, \quad u_{s1} = -\frac{1}{p_5} k_{us1} z_3,$$

(23)

where $k_{us1}$ is a positive feedback gain that will be determined later and $u_{s2}$ is a robust control law to be chosen below. Choose a psd. function $V_3 = (1/2) z_3^2$. From (22), (23), its time derivative is

$$\dot{V}_3 = z_3 \{ \hat{\theta}_3^T \phi_3 + p_5 u_{s2} \} - \frac{p_5}{p_5} k_{us1} z_3^2.$$

(24)

Hence, if $u_{s2}$ satisfies the following two conditions

- condition (i) $z_3 \{ p_5 u_{s2} + \hat{\theta}_3^T \phi_3 \} \leq \varepsilon_3$,
- condition (ii) $z_3 u_{s2} \leq 0$, (25)

then $z_3$ will exponentially converge to a small value corresponding to the arbitrary positive constant $\varepsilon_3$. Similar to the step 1, the $u_{s2}$ is chosen as

$$u_{s2} = -\frac{h_3}{2p_{5\min}} z_3,$$

(26)

with $h_3 \geq \|\theta_{3\max} - \theta_{3\min}\|_2^2 \|\phi_3\|_2^2$.

**Remark 1**: The derivative of virtual control $\hat{\alpha}_2$ is calculated from a first-order filter via using the DSC approach [3,11]. It indicates that the operation of differentiation in norm ARC design procedure can be replaced by simpler algebraic operation. As a result, a simpler controller is obtained even for a higher order system.

**Remark 2**: Viewing above the control law design procedure,
we know that the second term of robust control law $\alpha_{2s, u_{2s}}$ can stabilize the whole uncertainties including parametric and nonlinear uncertainties. That is to say even though the adaptive control law is closed, the tracking system is also stable. The adaptive term herein is to enhance the tracking accuracy, which will be analyzed in next section in detail.

IV. ADAPTATION LAW DESIGN AND STABILITY ANALYSIS

In this section, we will design new adaptation laws for the unknown parameters to guarantee the parameters estimates converge to their true values under some conditions. Then, the stability and performance will be analyzed in detail concluded as a theorem.

A. Adaptation law design

The discontinuous projection operator will be used in adaptation design, which has the following expression and properties:

$$\text{Proj}_{\Theta}() = \begin{cases} 0 & \text{if } \Theta_i = \Theta_{i(\max)} \text{ and } \bullet > 0 \\ 0 & \text{if } \Theta_i = \Theta_{i(\min)} \text{ and } \bullet < 0 \\ \bullet & \text{otherwise} \end{cases} \quad (27)$$

where $\Theta, \hat{\Theta}$ represent the unknown parameter vector to be online updated and its estimate respectively. The footnotes $i, \text{min, max}$ denote the $i$th element, minimum and maximum values of the $i$th element of $\Theta$, respectively. $\bullet$ represents any reasonable adaptation function. Choose the parametric adaptation given by

$$\dot{\hat{\Theta}} = \text{Proj}_{\Theta}(\Gamma\pi) \quad (28)$$

where $\Gamma > 0$ is a diagonal matrix, $\pi$ is an adaptation function. It can be shown that for any adaptation function $\pi$, the projection mapping used in (27) guarantees

Property 1: $\hat{\Theta} \in \Omega_\Theta \triangleq \{ \hat{\Theta} : \Theta_{\text{min}} \leq \hat{\Theta} \leq \Theta_{\text{max}} \}; (29a)$

Property 2: $\hat{\Theta}^T(\pi - \Gamma^{-1}\text{Proj}_{\Theta}(\Gamma\pi)) \leq 0, \forall \pi. \quad (29b)$

In order to achieve the accurate parameters estimate, unlike in[7][13], a new adaptation function is elaborately designed. Let $x = [x_1, x_2, x_3]^T$ and $\theta = [\theta_1^T, \theta_2^T]^T$. The DC-motor model (5) is rewritten as the following compact form

$$\dot{x} = F(x, u, t)\theta + f(x, u, t) + d \quad (30)$$

where $F(x, u, t) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ x_3 & S_f(x_2) & x_2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & u & x_2 \end{bmatrix}$, $f(x, u, t) = \begin{bmatrix} x_2 \\ 0 \\ 0 \end{bmatrix}$, $d = \begin{bmatrix} 0 \\ \Delta \end{bmatrix}$. Construct the following two filters:

$$\dot{z} = -k(t)z + F(x, u, t); \quad (31a)$$

$$\dot{z}_0 = -k(t)(x + z_0) - f(x, u, t), \quad (31b)$$

where $k(t) = k_0 + k_1(t)$, $k_0$ is a positive constant and $k_1(t)$ is any positive function. Let $\gamma = x + z_0$. From (30)(31), its time derivative is

$$\dot{\gamma} = F(x, u, t)\theta - k(t)(x + z_0) + d. \quad (32)$$

Define $\eta = \gamma - \Theta$, then its derivative is

$$\dot{\eta} = -k(t)\eta + d, \quad (33)$$

which indicates that $\eta \to 0$, when $t \to \infty$ on the condition of $d = 0$. Defining $P(t) = \int_0^t \Xi^T(s)\Xi(s)ds$, $Q(t) = \int_0^t \Xi^T(s)(\Gamma(s) - \eta(s))ds$, we have $P(t)\theta = Q(t)$. Let $z = [z_2, z_3]^T$ and $\sigma_0 = [z_2\phi_2^T, z_3\phi_3^T]^T$. A novel adaptation law is chosen as

$$\dot{\hat{\theta}} = \text{Proj}(\Gamma(\sigma_0 - \gamma(P\hat{\theta} - Q))), \quad (34)$$

where $\gamma > 0$ is a learning factor.

B. Stability analysis

**Theorem 1:** Consider the third order DC servomechanism (5). If all the assumptions (1-3) are established, the parameter adaptation law is chosen as (34), and given any positive number $\mu$, for all initial conditions satisfying $z_2^2 + z_3^2 + y_2^2 \leq 2\mu$, then by choosing proper $k_1, \rho_0, k_{u_{2s}}$, $\Gamma$, and $\gamma$, the control law designed as (11), (21) guarantees that:

A) All the signals such as $z_1, z_2, z_3, y_2, \alpha_2, u$ are uniformly bounded.

B) If there only parametric uncertainties exist, i.e. $\Delta = 0$, then, in addition to results in A), asymptotic output tracking is achieved, i.e. $z_1 \to 0$. Moreover, the unknown parameters will converge to their true values asymptotically, i.e. $\hat{\theta} \to 0$, when the persistent exciting condition is satisfied, i.e. $P(t) = \int_0^t \Xi^T(s)\Xi(s)ds \geq \lambda I$, $t_e \in [0, \infty]$.

**Proof:** Part A). Choose a Lyapunov function

$$V = V_2 + V_3 + \frac{1}{2}y_2^2. \quad (35)$$

Differentiating (35) and noting (14) (17) (24), we obtain that

$$\dot{V} = \eta^T(\Gamma - \Gamma^{-1}\text{Proj}_{\Theta}(\Gamma\pi)) \leq 0, \forall \pi. \quad (36)$$

Using the facts

$$z_2z_3 \leq z_2^2 + \frac{1}{2}z_3^2; \quad (37)$$

$$z_2y_2 \leq z_2^2 + \frac{1}{2}y_2^2; \quad (38)$$

and noting (18) (25), the following inequality can be deduced

$$\dot{V} \leq -k_2z_2^2 - k_{u_{3s}}z_3^2 + \rho_0 z_2z_3 + \frac{1}{2}z_2^2 + \frac{1}{2}z_3^2 + \frac{1}{2}y_2^2$$

$$+ \varepsilon_2 + \varepsilon_3 - \frac{\gamma^2}{\Gamma} - k_vy_2sgn(y_2) - y_2\dot{y}_2. \quad (39)$$

From (11), the explicit term of $\dot{\alpha}_2$ is described as

$$\dot{\alpha}_2 = -\frac{\rho_0}{\rho_1}\{\dot{\rho}_2S_f(x_2) - \dot{\rho}_3x_2 - \dot{\rho}_4 + \dot{x}_2\}$$

$$+ \frac{1}{\rho_1}\{\dot{\rho}_2S_f(x_2) - \dot{\rho}_3x_2 - \dot{\rho}_4 + \dot{x}_2\}. \quad (40)$$

Obviously, $\dot{\alpha}_2$ is bounded by a continuous function represented by

$$|\dot{\alpha}_2| \leq \left|C(z_1, z_2, y_2, \dot{\theta}_2, x_{1r}, \ddot{x}_{1r})\right|. \quad (41)$$

Since assumption 3 is established, and initial condition satisfies that $V(0) \leq \mu$, we have

$$\left|C(z_1, z_2, y_2, \dot{\theta}_2, x_{1r}, \ddot{x}_{1r})\right| \leq M_0 < \infty. \quad (41)$$
Then choosing $k_{2s1}, k_{us1}, \tau, k_v$ such that
\[
\begin{align*}
k_{2s1} &\geq 2p_1 \max + \rho_0 \\
k_{us1} &\geq \frac{1}{2} p_1 \max + \rho_0 \\
\frac{1}{2} &\geq \frac{1}{2} p_1 \max + 1 + \rho_0 \\
k_v &\geq M_0
\end{align*}
\]  \(42\)

The inequality (38) is rewritten as
\[
\dot{V} \leq -2\rho_0 V + c.
\] \(43\)

where $c = \varepsilon_2 + \varepsilon_3$ is bounded. If we choose $\rho_0 \geq \frac{c}{2\rho}$, then $\dot{V} \leq 0$. Thus, $V(t) \leq \mu$ is an invariant set on $t \in [0, \infty)$, i.e., (44) holds for all $t \in [0, \infty)$. Therefore,
\[
0 \leq V(t) \leq \left(\frac{c}{2\rho_0} + (V(0) - \frac{c}{2\rho_0})e^{-2\rho_0 t}\right).
\] \(44\)

This indicates that $z_2, z_3, y_2$ are uniformly ultimate bounded. From (9) (11) (21), $z_1, \alpha_2, u$ are also uniformly ultimate bounded. Furthermore, the tracking error can converge to a small neighborhood of zero by choosing sufficient large $\rho_0$. Part A) is proved. Part B). Define a new Lyapunov function $V_\theta$ as
\[
V_\theta = V + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta}.
\] \(45\)

From (34) and (36), the derivative of $V_\theta$ is
\[
\begin{align*}
\dot{V}_\theta &= -\frac{p_1}{p_1 \min} k_{2s1} z_2^2 - \frac{p_1}{p_1 \min} k_{us1} z_3^2 + p_1 z_2(z_3 + y_2) \\
&\quad + z_1(\alpha_1 \alpha_2 + \Delta) + z_3(p_{us2}) + y_2\left(-\frac{\tilde{\alpha}_2}{\tilde{\alpha}_2} - \tilde{\alpha}_2 - k_v \text{sgn}(y_2)\right) + \tilde{\theta}^T \Gamma \tilde{\theta} - \tilde{\theta}^T \hat{\theta} \tilde{\theta} - \tilde{\theta}^T \Gamma \tilde{\theta} \\
&\quad - \text{Pro} \hat{\theta}(\Gamma \tilde{\theta} - \tilde{\theta}^T (P \tilde{\theta} - Q)) + \tilde{\theta}^T (P \tilde{\theta} - Q)
\end{align*}
\] \(46\)

Noting (29b) and (38), if $\Delta = 0$, the following inequality is established
\[
\begin{align*}
\dot{V}_\theta &\leq -\rho_0(z_2^2 + z_3^2 + y_2^2) - \gamma \tilde{\theta}^T P \tilde{\theta} - y_2 \tilde{\alpha}_2 - k_v y_2 \text{sgn}(y_2). \\
&\leq -\rho_0(z_2^2 + z_3^2 + y_2^2) - \gamma \tilde{\theta}^T P \tilde{\theta}
\end{align*}
\] \(47\)

Define $V_0 = \rho_0(z_2^2 + z_3^2 + y_2^2) + \gamma \tilde{\theta}^T P \tilde{\theta}$. Integrating both sides of the inequality (47) on $t \in [0, \infty)$, we have
\[
\int_0^\infty V_0(\varsigma) d\varsigma \leq V_0(0) - V_0(\infty).
\] \(48\)

From (45), $V_0(0) - V_0(\infty)$ is bounded since $z_2, z_3, y_2$ is bounded which is proved in A). Thus, if $P(t) = \int_0^t \tilde{\Gamma}^T \tilde{\Gamma} \xi(t) d\xi(t) \geq \lambda I$ is satisfied and from (48), we know that $z, y_2, \tilde{\theta}$ are all square integrable, i.e. $z, y_2, \tilde{\theta} \in L_2(0, \infty)$. Furthermore, from the conclusion of Part A) and noting (16),(22),(34), $z, y_2, \tilde{\theta} \in L_\infty(0, \infty)$ and $\dot{z}, \dot{y}_2, \dot{\tilde{\theta}} \in L_\infty(0, \infty)$ are established. Thus, according to Lemma 1, we conclude that when $t \to \infty$, the error signals $z, y_2, \tilde{\theta}$ will converge to zero asymptotically.

**Remark 3:** In this paper, we propose an improved DSC approach to simplify the controller design. The filter error $y_2$ will converge to zero in finite time due to using sliding mode leakage term $k_v \text{sgn}(y_2)$. As a result, we can prove that the tracking error converge to zero asymptotically when there parametric uncertainties exist, which differs from the approaches proposed in [3] and [11].

**Remark 4:** A novel adaptation law which is derived from the Lyapunov stability analysis is elaborately designed in this paper. It guarantees the true parameter estimates and improves the tracking accuracy. This is a considerable improvement compared with [3] and [11].

**V. APPLICATION EXAMPLE**

In order to verify the above proposed control approach, simulation results are obtained from a DC torque motor which is applied in the azimuth axis of the two-axis turntable. This axis is easily interfered by parametric uncertainties and external disturbance because of the rolling of the pitch axis.

Nominal values of the DC motor are shown in TABLE I. The friction force used for simulation is given by (2), where $T_c = 0.2 \text{Nm}$, $B = 0.05 \text{N/m/s}$. The approximation function is chosen as $S_f(x_2) = \frac{x}{\pi} \tan(90x_2)$. The external disturbance caused by the pitch axis is supposed to be $T_{dis} = 0.01 \sin(\pi x_1) \text{Nm}$ and the payload $T_l = 1 \text{Nm}$. Hence, the mean load of $T_n = 1 \text{Nm}$. From above nominal parameter values, we can obtain that $\theta = \left[p_1, p_2, p_3, p_4, p_5, p_6, p_7\right]^T = \left[386.2, -69.0, -17.2, -344.8, 161.3, -180.2, -1161.3\right]^T$.

Parameter variations are supposed as following:

- $R_a = 5 \sim 9 \Omega, L_a = 4.5 \sim 7.6 \text{mH}$, $J = 2.6 \sim 3.2 \times 10^{-3} \text{kgm}^2$,
- $K_E = K_p = \text{1.03} \sim \text{1.25V/(rad/s)}$,
- $T_c = 0.1 \sim 0.3 \text{Nm}$, $B = 0.02 \sim 0.08 \text{N/m/s}$

and $T_f = -0.04 \sim 0.04 \text{Nm}$. As such, the bounds of the unknown parameters are given by $\theta_{\min} = \left[312.5, -115.4, -30.8, -384.6, 131.6, -277.8, -2000.0\right]^T$.

Therefore, $\theta_{\max} = \left[473.1, -31.3, -6.3, -312.5, 222.2, -135.5, -657.9\right]^T$  is the initial parameter estimates are

- $\theta(0) = \left[320, -38, -30, -320, 200, -150, -1000\right]^T$,
- $\theta_{\min} = \left[386.2, -69.0, -17.2, -344.8, 161.3, -180.2, -1161.3\right]^T$

and $\theta_{\max} = \left[473.1, -31.3, -6.3, -312.5, 222.2, -135.5, -657.9\right]^T$ to test the effect of parametric uncertainties.

The controller parameters are chosen as: $k_1 = 5, k_{2s1} = 1200, k_{us1} = 1500, k_{2s1} = 5 \times 10^6, \varepsilon_3 = 2 \times 10^6$, and $\tau = 0.002$. The parameter learning factors are selected as $\Gamma = \text{diag}(305, 125, 200, 75.5, 28, 36)$ and $\beta = 100$.

The purpose of the simulation is to show the tracking ability and parameter convergence. Thus, to illustrate the advantages of the proposed algorithm, the desired input is given by $x_1(t) = \sin t + 1.5 \sin 0.5t + 2 \sin 0.1t$ and three cases are considered.

Case I: ARDSC for DC torque motors with parametric and nonlinear uncertainties using novel adaptation laws.

Case II: ARDSC for DC torque motors with only parametric uncertainties.
 uncertainties using novel adaptation laws.

Case III: ARDSC for DC torque motors with only parametric uncertainties using adaptation laws as in [3,7], i.e., $\gamma = 0$. The simulation results are shown in Fig.2 and Fig.3. As we see from the Fig.2, the position tracking error converges to a small prescribed bound in case I which is coherent to the part A) of theorem 1 and the tracking error in case II converges to zero gradually because the parametric uncertainties can be compensated completely by the novel parameter learning process whose effect can be seen in the case II of Fig.3. Comparing case II with case III in Fig.2 and Fig.3, we conclude that the new adaptation law is very effective for improving the tracking performance.

VI. Conclusion

In this paper, an adaptive robust dynamic surface controller was proposed for the position tracking of a DC torque motor. By using the DSC technique, the implementation of the controller is much simpler than the norm ARC designed by using backstepping approach. Theoretical proof and simulation results show that though it has simpler form and design procedure than the norm ARC, it does not lose its original advantages. Furthermore, in this paper, a novel parameter adaptation law was proposed to guarantee the true estimates of the unknown parameters, which is good for improving the tracking performance.

REFERENCES


