Model Matching Control for MIMO Systems with Multiple Time Delays and Its Applications in Adaptive Scheme

Haixia Su, Yingmin Jia, Junping Du and Fashan Yu

Abstract—This paper considers the model matching problem of multiple-input multiple-output (MIMO) systems with multiple time delays. The reference model is chosen to be the diagonal delay transfer function matrix. A controller is designed for nominal systems. Furthermore, an adaptive control scheme is proposed for uncertain systems with parameter variation. The resulting scheme can guarantee the global stability of the closed-loop systems and the convergence of the tracking errors. A simulation example is included to illustrate the proposed scheme.

I. INTRODUCTION

Controller design of time delay systems is more challenging than delay-free systems. Researchers have paid attention to this problem in the last four decades [1]-[6]. Controller design and adaptive control problems were extensively studied for single-input single-output (SISO) delay systems. Smith predictor proposed in [1] could eliminate the time delay from characteristic equation of the closed-loop systems. But it depended on accurate model and the control plant needed to be stable. Finite spectrum assignment method in [3], [4] could be applied to delay systems which were not necessarily stable. The controller involved an integral of past input over the delay interval. The model matching technique proposed in [9] is more ideal for controllers than the methods mentioned above. This technique can make the closed-loop transfer function coincide exactly with the reference model transfer function. This paper investigates the controller design and adaptive control for general MIMO delay systems based on this technique.

Compared with finite spectrum assignment method used in time domain, the model matching method used in frequency domain needs less computational work and it can be easily extended to adaptive control. In addition, this method avoids the use of a trial and error design procedure compared to results in [10], [11] where the time delay was considered as a perturbation of the nominal system and then the problem reduced to robust control problem. This method has been applied to delay systems for these apparent advantages. For example, a model matching controller was designed in [5] for SISO delay systems. Consequently, this result was extended to adaptive control in [6]-[8] where adaptive schemes were designed for systems with parameter uncertainty.

There also have been some results about the model matching control for MIMO delay systems. For example, a model matching controller was designed for a two-input two-output system in [12]. Further, a model matching controller was developed in [14] for multiple-output-delay (MOD) systems whose each output involved a fixed time delay. It is known that delays may occur in inputs, outputs or every control loop of the systems. This paper considers the model matching problem for general MIMO delay systems with different delays in every control loop. Controller structure is designed for nominal systems first, and then an adaptive control scheme is designed for systems with parameter variation.

This paper is organized as follows. Section 2 is problem statement. In section 3, a model matching controller is designed for nominal systems. An adaptive control scheme is designed for uncertain systems with parameter variation in section 4. A simulation example is included in section 5. The last section is the conclusion.

II. PROBLEM STATEMENT

Consider MIMO systems

\[ y(s) = T(s)u(s) \]  \hspace{1cm} (1)

where \( y(s) = [y_1(s), \ldots, y_n(s)]^T \) and \( u(s) = [u_1(s), \ldots, u_n(s)]^T \) are the output and the input of the systems, respectively. The transfer function matrix \( T(s) \) is in the form of

\[
T(s) = \begin{bmatrix}
  g_{11}r_{11}(s)e^{-L_{11}s} & \cdots & g_{1n}r_{1n}(s)e^{-L_{1n}s} \\
  \vdots & \ddots & \vdots \\
  g_{n1}r_{n1}(s)e^{-L_{n1}s} & \cdots & g_{nn}r_{nn}(s)e^{-L_{nn}s} \\
  p_1(s) & \cdots & p_n(s)
\end{bmatrix}
\]

where \( r_{ij}(s) \) and \( p_i(s) \) are monic polynomials, the degrees of them are \( m_{ij} \) and \( n_i \), denote \( \partial[r_{ij}(s)] = m_{ij} \), \( \partial[p_i(s)] = n_i \), \( g_{ij} \) is gain, \( L_{ij} > 0 \) are known different time delays. In particular, we take \( g_{ij} = 0 \) and \( r_{ij}(s) = 1 \) if the \( ij \)th element of \( T(s) \) is zero.

Controlling each output independently is ideal for multi-variable systems. Therefore, the reference model is chosen to be the diagonal MIMO delay systems

\[ y_r(s) = T_r(s)u(s) \]  \hspace{1cm} (3)

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the transfer function matrix is
\[ T_r(s) = \text{diag}\left( \frac{y_1r_{11}(s)}{p_{11}(s)}, \ldots, \frac{y_nr_{n1}(s)}{p_{n1}(s)} \right) e^{-Lr_1s}, \ldots, \frac{y_nr_{nn}(s)}{p_{nn}(s)} e^{-Lr_n s} \] (4)
where \( y_r(s) = [y_1(s), \ldots, y_r(s)]^T \) and \( v(s) = [v_1(s), \ldots, v_n(s)]^T \) are output and input, respectively. Without loss of generality, all \( p_{ij}(s)(i = 1, 2, \ldots, n) \) may be selected to be stable polynomials, and \( n_r = \partial[p_{ij}(s)], m_r = \partial[r_{ij}(s)], \) \( L_r = \text{min} \{L_{ij} | j = 1, 2, \ldots, n \} \). The objective is to design a model matching controller for nominal systems first, and then extend it to adaptive control of uncertain systems with parameter variation. The following assumptions are needed.

Assumption 1: Denote
\[ R(s) = \begin{bmatrix} g_{11}r_{11}(s) & \cdots & g_{1n}r_{1n}(s) \\ \vdots & & \vdots \\ g_{n1}r_{n1}(s) & \cdots & g_{nn}r_{nn}(s) \end{bmatrix} \] (5)
\( R(s) \) is asymptotically stable, i.e., minimum phase.

Assumption 2: \( n_r - m_r \geq n_i - m_i (i = 1, 2, \ldots, n) \), where \( m_i = \max \{m_{ij} | m_{ij} = \partial[r_{ij}(s)], j = 1, 2, \ldots, n \} \).

Assumption 3: The constant matrix
\[ B^* = \{b_{ij}\} = \lim_{s \to \infty} \text{diag}[s^{d_i+1}] \cdot T(s) \] (6)
is nonsingular, where the integer \( d_i \) is chosen such that \( \lim_{s \to \infty} s^{d_i+1} T(s) \) is a nonzero finite vector, where \( T(s) \) is the \( i^{th} \) row of \( T(s) \).

Remark 1: Clearly, we should choose a reference model which involves the delays that are not shorter than the smallest time delay in every row, i.e., \( L_{ri} \geq L_{ij} \). Otherwise, the model matching controller does not exist.

III. MODEL MATCHING CONTROLLER DESIGN

Introduce any monic stable polynomials \( r^*_i(s), p^*_i(s) (i = 1, 2, \ldots, n) \), \( t^*_i(s) = m_i, \partial[p^*_i(s)] = n_i \). Then \( T_r(s) \) in (4) can be rewritten as
\[ T_r(s) = T^*(s) \text{diag}\left( \frac{r_{11}(s)p^*_1(s)}{r^*_1(s)p_{11}(s)}, \ldots, \frac{r_{n1}(s)p^*_n(s)}{r^*_n(s)p_{n1}(s)} \right) \] (7)
where
\[ T^*(s) = \text{diag}\left( \frac{g_{11}r_{11}(s)}{p^*_1(s)} e^{-Lr_1s}, \ldots, \frac{g_{nr}(s)}{p^*_n(s)} e^{-Lr_n s} \right) \] (8)
From Assumption 2, each \( r_{ij}(s)p^*_i(s)/r^*_i(p_{ij}(s)) \) is proper and stable. Denote
\[ \bar{v}_i(s) = \frac{r_{ij}(s)p^*_i(s)}{r^*_i(s)p_{ij}(s)}v_i(s), \quad \bar{v}(s) = [\bar{v}_1(s), \ldots, \bar{v}_n(s)]^T \] (9)
Then \( T^*(s) \) becomes transfer function of the new reference model \( y_r(s) = T^*(s)\bar{v}(s) \). As a result of this transformation, the objective can be viewed as to design a controller so that the transfer function of the closed-loop from \( \bar{v}(s) \) to \( y(s) \) coincides with \( T^*(s) \). In the following controller design procedure, we only consider one case where \( p_i(s) \) and \( r^*_i(s) \) have single or distinct roots. The controller for another case where they have roots with their multiplicities greater than one can be designed similarly.

By employing virtual precompensators
\[ r^*_i(s)p_i(s) - g_{ij}r_{ij}(s)p^*_i(s) \]
\[ = \sum_{k=1}^{m_i+n_i} \frac{\beta^k_{ij} e^{L_{ij}-L_i z^k_i}}{s - z^k_i} + 1 - b_{ij}, \quad j = 1, 2, \ldots, n \] (10)
where \( z^k_i \) are roots of \( r^*_i(s) \) for \( k = 1, \ldots, m_i \) and roots of \( p_i(s) \) for \( k = m_i + 1, \ldots, m_i + n_i \), respectively. Define the polynomials \( \phi_{ij}(s) \) satisfying the equations
\[ r^*_i(s)p_i(s) - \phi_{ij}(s) \]
\[ = \sum_{k=1}^{m_i+n_i} \frac{\beta^k_{ij} e^{L_{ij}-L_i z^k_i}}{s - z^k_i} + 1 - b_{ij}, \quad j = 1, 2, \ldots, n \] (11)
Additionally, choose arbitrarily monic polynomial \( \bar{r}_{ij}(s) \) satisfying \( \partial[\bar{r}_{ij}(s)] = m_i \), write
\[ -g_{ij}r_{ij}(s) = \sum_{k=1}^{m_i+n_i} \frac{\beta^k_{ij} e^{L_{ij}-L_i z^k_i}}{s - z^k_i} - b_{ij}, \quad j \neq l \] (12)
and define polynomials \( \bar{\phi}_{ij}(s) \) by
\[ \frac{-\bar{\phi}_{ij}(s)}{r^*_i(s)} = \sum_{k=1}^{m_i+n_i} \frac{\beta^k_{ij} e^{L_{ij}-L_i z^k_i}}{s - z^k_i} - b_{ij}, \quad j \neq l \] (13)
From (13), it is known that \( \partial[\bar{\phi}_{ij}(s)] \leq m_i \). Choose any monic stable polynomial \( \tau_{ij}(s) \) satisfying \( \partial[\tau_{ij}(s)] = n_i - m_i - 1 \), define polynomial equations by
\[ k_{ij}p_i(s) + g_{ij}h_{ij}(s)r_{ij}(s) = -\tau_{ij}(s)\phi_{ij}(s), \quad i, j = 1, 2, \ldots, n \] (14)
where \( k_{ij} \) and \( h_{ij} \) are unknown polynomials. Their solutions are given in the following theorem.

Theorem 1: The polynomial equations in (14) have solutions \( k_{ij}(s) \) and \( h_{ij}(s), \partial[h_{ij}(s)] \leq n_i - 2, \partial[k_{ij}(s)] \leq n_i - 1 \). Furthermore, for a fixed \( i, \) all \( h_{ij}(s) \) are the same and they can be written as \( h_i(s) \).

Proof: Define \( \mu_k(s) = \prod_{i=1, k \neq k}(s - z^k_i) \), then (10) and (11) can be rewritten as
\[ r^*_i(s)p_i(s) - g_{ij}r_{ij}(s)p^*_i(s) \]
\[ = \sum_{k=1}^{m_i+n_i} \beta^k_{ij}(k) e^{L_{ij}-L_i z^k_i} + (1 - b_{ij})r^*_i(s)p_i(s) \] (15)
and
\[ r^*_i(s)p_i(s) - \phi_{ij}(s) \]
\[ = \sum_{k=1}^{m_i+n_i} \beta^k_{ij}(k) e^{L_{ij}-L_i z^k_i} + (1 - b_{ij})r^*_i(s)p_i(s) \] (16)
respectively. Because \( z^k_i (k = m_i + 1, \ldots, m_i + n_i) \) are roots of \( p_i(s) \), (15) and (16) imply that
\[ -g_{ij}r_{ij}(z^k_i)p^*_i(z^k_i) = \beta^k_{ij}(k) e^{L_{ij}-L_i z^k_i} \] (17)
and
\[ -\phi_{ij}(z^k_i) = \beta^k_{ij}(k) e^{L_{ij}-L_i z^k_i} \] (18)
Therefore, we can get
\[ \sum_{k=1}^{m_i+n_i} [g_{ij} r_{ij}(z^k_i), \ldots, g_{in} r_{in}(z^k_i)] e^{-L_i z^k_i} \]
\[ = [\phi_{ij}(z^k_i), \ldots, \phi_{in}(z^k_i)] e^{-L_i z^k_i} \] (19)
From (14), a necessary condition that the solution \( h_{ij}(s) \) can be written as \( h_i(s) \) is that the following equation holds at \( z_k^i \):

\[
\begin{align*}
  h_i(z_k^i)[g_1r_1(z_k^i), \ldots, g_nr_n(z_k^i)] \\
  = -\tau_i(z_k^i)[\phi_1(z_k^i), \ldots, \phi_n(z_k^i)]
\end{align*}
\]  

(20)

In view of (19) and (20), it is known that for \( k = m_i + 1, \ldots, m_i + n_i \), we have

\[
  h_i(z_k^i) = -\tau_i(z_k^i)p_i^r(z_k^i)e^{L_{ii}z_k^i}
\]

(21)

The polynomial \( h_i(s) \) can be written as

\[
  h_i(s) = h_{i,1}^{n_i-1}s^{n_i-1} + h_{i,2}^{n_i-2}s^{n_i-2} + \cdots + h_{i,0}
\]

(22)

where the coefficients \( h_{i,1}^0, h_{i,2}^0, \ldots, h_{i,0} \) are unknown constants. Because \( z_k^i \) are distinct roots, the coefficient matrix of the Vandermonde matrix. Variables \( h_{i,1}^0, h_{i,2}^0, \ldots, h_{i,0} \) can be solved from (21), i.e., \( h_i(s) \) is solved. According to (20), when \( s = z_k^i \), the right hand of the above equation is equal to zero. Then \( k_{ij}(s) \) can be solved. Thus, the proof is completed.

In the following theorem, we present the model matching controller design procedure.

**Theorem 2**: There is a controller for system (1) such that the transfer function of the closed-loop system coincides exactly with \( T(s) \).

**Proof** The controller design mainly based on the virtual precompensators and solutions of the polynomial equations. In view of (10) and (11), we have

\[
\begin{align*}
  \int_{-(L_{ij}-L_{ii})}^{-(L_{ij}-L_{ii})} \sum_{k=1}^{m_i+n_i} \beta_{ij}^k e^{-\sigma z_k^i} u_j(s) e^{\sigma \sigma} d\sigma \\
  = \sum_{k=1}^{m_i+n_i} \beta_{ij}^k e^{-(L_{ij}-L_{ii})z_k^i} \\
  - \sum_{k=1}^{m_i+n_i} \beta_{ij}^k e^{-z_k^i} u_j(s) e^{-(L_{ij}-L_{ii})s} \\
  = \sum_{k=1}^{m_i+n_i} \beta_{ij}^k e^{-(L_{ij}-L_{ii})z_k^i} \\
  - \sum_{k=1}^{m_i+n_i} \beta_{ij}^k e^{-z_k^i} u_j(s) e^{-(L_{ij}-L_{ii})s}
\end{align*}
\]

(23)

Moreover, using (12) and (13), we have another integral which is calculated by

\[
\begin{align*}
  \int_{-(L_{ij}-L_{ii})}^{0} \sum_{k=1}^{m_i} \beta_{ij}^k e^{-\sigma z_k^i} u_j(s) e^{\sigma \sigma} d\sigma \\
  = \sum_{k=1}^{m_i} \beta_{ij}^k e^{-(L_{ij}-L_{ii})z_k^i} \\
  - \sum_{k=1}^{m_i} \beta_{ij}^k e^{-z_k^i} u_j(s) e^{-(L_{ij}-L_{ii})s} \\
  = -\tau_i \sum_{k=1}^{m_i} \beta_{ij}^k e^{-(L_{ij}-L_{ii})z_k^i} \\
  + b_{ij} u_j(s) - b_{ij} u_j(s)e^{-(L_{ij}-L_{ii})s}, \quad j \neq l
\end{align*}
\]

(24)

For \( j = 1, 2, \ldots, n \), combining (23) with (24) yields

\[
\begin{align*}
  \sum_{j=1}^{n} \int_{-(L_{ij}-L_{ii})}^{0} \sum_{k=1}^{m_i+n_i} \beta_{ij}^k e^{-\sigma z_k^i} u_j(s) e^{\sigma \sigma} d\sigma \\
  = \sum_{j=1}^{n} \int_{-(L_{ij}-L_{ii})}^{0} \sum_{k=1}^{m_i} \beta_{ij}^k e^{-\sigma z_k^i} u_j(s) e^{\sigma \sigma} d\sigma \\
  - \sum_{j=1}^{n} \int_{-(L_{ij}-L_{ii})}^{0} \sum_{k=1}^{m_i} \beta_{ij}^k e^{-\sigma z_k^i} u_j(s) e^{\sigma \sigma} d\sigma
\end{align*}
\]

(25)

Substituting (26) into (25), we obtain

\[
\begin{align*}
  \sum_{j=1}^{n} \int_{-(L_{ij}-L_{ii})}^{0} \sum_{k=1}^{m_i+n_i} \beta_{ij}^k e^{-\sigma z_k^i} u_j(s) e^{\sigma \sigma} d\sigma \\
  + \sum_{j=1}^{n} \int_{-(L_{ij}-L_{ii})}^{0} \sum_{k=1}^{m_i} \beta_{ij}^k e^{-\sigma z_k^i} u_j(s) e^{\sigma \sigma} d\sigma
\end{align*}
\]

(27)

It is noticed that in the above equation

\[
\begin{align*}
  \sum_{j=1}^{n} \int_{-(L_{ij}-L_{ii})}^{0} \sum_{k=1}^{m_i+n_i} \beta_{ij}^k e^{-\sigma z_k^i} u_j(s) e^{\sigma \sigma} d\sigma \\
  - \sum_{j=1}^{n} \int_{-(L_{ij}-L_{ii})}^{0} \sum_{k=1}^{m_i} \beta_{ij}^k e^{-\sigma z_k^i} u_j(s) e^{\sigma \sigma} d\sigma
\end{align*}
\]

(28)

Therefore, we can choose the controller \( u \) satisfying

\[
[b_{i1}, b_{i2}, \ldots, b_{in}] [u_1, u_2, \ldots, u_n]^T = c_i
\]

(29)

where

\[
\begin{align*}
  c_i = \sum_{j=1}^{n} \frac{k_{ij}(s)}{\tau_i r_i^s(s)} u_j(s) e^{-(L_{ij}-L_{ii})s} + \frac{h_i(s)}{\tau_i r_i^s(s)} u_i(s)
\end{align*}
\]

(30)
Because polynomials $r_i^*(s)$ and $p_i^*(s)$ are chosen to be monic stable polynomials, substituting (29) into (27) yields
\[
\sum_{j=1}^{n} \frac{g_{ij} r_i^*(s)}{p_i^*(s)} u_j(s) e^{-L_{ij}s} = \frac{r_i^*(s)}{p_i^*(s)} g_{ri} \bar{v}_i(t) e^{-L_{ri}s} \quad (31)
\]
that is
\[
y_i(s) = y_{ri}(s), \quad i = 1, 2, \ldots, n \tag{32}
\]
The system output is identical with the reference model output, which means that the transfer function of the closed-loop system coincides exactly with $T_i(s)$. Therefore, the controller in (29) is the desired model matching controller. From assumption 3, it is known that $B^*$ is nonsingular. In view of (29), it follows that the controller is in the form of
\[
[u_1, u_2, \ldots, u_n]^T = (B^*)^{-1} [c_1, c_2, \ldots, c_n]^T \tag{33}
\]
This completes the proof.

IV. ADAPTIVE CONTROL SCHEME DESIGN

In this section, an adaptive control scheme is designed for systems with parameter variation. $B^*$ is nonsingular, we can always find a constant matrix that the systems with it post-multipled satisfies $b_{ii} = g_{ii} \neq 0$. Consider the case where $b_{ii} = g_{ii} \neq 0$. From (29) and (30), $u_i$ can be rewritten as
\[
u_i = \frac{1}{b_{ii}} \left\{ \sum_{j=1}^{n} \frac{k_{ij}(p)}{r_i^*(p)} u_j(t-L_{ij}) + \frac{h_{ij}(p)}{r_i^*(p)} y_i(t)
+ \sum_{j \neq i} \int_{-L_{ij}}^{0} \sum_{k=1}^{m_{ij}+1} \beta_{ij}^k e^{-\sigma} u_j(t) d\sigma
+ \sum_{j \neq i} \int_{-(L_{ij}-L_{il})}^{0} \sum_{k=1}^{m_{il}+1} \beta_{il}^k e^{-\sigma} u_j(t) d\sigma + \sum_{j \neq i} \sum_{k=1}^{m_{il}+1} \beta_{il}^k e^{-\sigma} u_j(t-L_{ij})
- \sum_{j \neq i} b_{ij} u_j(t) + \sum_{j \neq i} b_{ji} u_j(t-L_{ij})
+ g_{ri} \bar{v}_i(t-L_{ri}) \right\}, \quad i = 1, 2, \ldots, n \tag{34}
\]
Because $\partial[\phi_{ij}(s)] \leq m_i, \partial[k_{ij}(s)] \leq n_i - 2$. Thus, $\bar{\phi}_{ij}(s)$ and $k_{ij}(s)$ can be written as
\[
\omega_{ij}(s) = q_{ij}^m s^m + q_{ij}^{m-1}s^{m-1} + \cdots + q_{ij}^1, \quad j \neq l \tag{35}
\]
respectively, where $q_{ij}^m, \ldots, q_{ij}^1$ are unknown constants. Define parameter vectors by
\[
\theta_i = \frac{1}{g_{ii}} \left[ k_{il}^{m_{li}} - 2, k_{il}^{m_{li}-1} - 2, \ldots, k_{il}^1 - 2, k_{in}^{m_{in}} - 2, k_{in}^{m_{in}-1} - 2, \ldots, k_{in}^1 - 2, k_{il}^{m_{il}} - 2, k_{il}^{m_{il}-1} - 2, \ldots, k_{il}^1 - 2, i, j = 1, 2, \ldots, n \tag{36}
\]
\[
\lambda_{ij} = \sum_{k=1}^{m_{ij}+1} \beta_{ij}^k e^{-\sigma} z_i^k, \quad i, j = 1, 2, \ldots, n
\]
\[
\rho_{ij} = \sum_{k=1}^{m_{il}+1} \beta_{il}^k e^{-\sigma} z_i^k, \quad i, j = 1, 2, \ldots, n
\]
and the signal vector by
\[
\omega_i(t) = \left[ \frac{p_{ni}^{-2}}{r_i(p)r_i^*(p)} u_1(t-L_{i1}), \ldots, \frac{p_{ni}^{-2}}{r_i(p)r_i^*(p)} u_n(t-L_{in}), \ldots, \frac{1}{r_i(p)r_i^*(p)} u_1(t), \ldots, \frac{1}{r_i(p)r_i^*(p)} y_i(t), \ldots, \frac{1}{r_i(p)r_i^*(p)} y_1(t), \ldots, \frac{1}{r_i(p)r_i^*(p)} y_n(t), \ldots \right]
\]
\[
\bar{v}_i(t) = \left[ \frac{p_{ni}}{r_i^*(p)} u_n(t-L_{il}), \ldots, \frac{p_{ni}}{r_i^*(p)} u_1(t-L_{il}), \ldots, \frac{1}{r_i^*(p)} u_n(t), \ldots, \frac{1}{r_i^*(p)} u_1(t), \ldots, \frac{1}{r_i^*(p)} u_n(t), \ldots \right] \tag{37}
\]
Then the controller (34) can be represented as
\[
u_i(t) = \hat{\theta}_i T(\omega_i(t)) + \sum_{j=1}^{n} \int_{-L_{ij}}^{0} \lambda_{ij}(t, \sigma) u_j(t) d\sigma + \sum_{j \neq i} \int_{-(L_{ij}-L_{il})}^{0} \rho_{ij}(t, \sigma) u_j(t) d\sigma \tag{38}
\]
\[
\hat{\theta}_i(t), \hat{\lambda}_{ij}(t, \sigma) \quad \text{and} \quad \hat{\rho}_{ij}(t, \sigma) \quad \text{is the estimates of the parameters} \quad \theta_i, \lambda_{ij} \quad \text{and} \quad \rho_{ij}, \quad \text{respectively. Define tracking errors by} \quad e_i(t) = y_i(t) - y_{ri}(t), \quad i = 1, 2, \ldots, n. \quad \text{Its parametric representation is given in the following theorem.}
\]

Theorem 3: Tracking errors can be represented by
\[
e_i(t) = b_{ii} \frac{r_i^*(p)}{p_i^*(p)} q_{-L_{ii}} \left[ \hat{\theta}_i T(\omega_i(t)) + \sum_{j=1}^{n} \int_{-L_{ij}}^{0} \hat{\lambda}_{ij}(t, \sigma) u_j(t) d\sigma + \sum_{j \neq i} \int_{-(L_{ij}-L_{il})}^{0} \hat{\rho}_{ij}(t, \sigma) u_j(t) d\sigma \right], \tag{39}
\]
where $\hat{\theta}_i(t) = \hat{\theta}_i(t) - \theta_i, \quad \hat{\lambda}_{ij}(t, \sigma) = \hat{\lambda}_{ij}(t, \sigma) - \lambda_{ij}$ and $\hat{\rho}_{ij}(t, \sigma) = \hat{\rho}_{ij}(t, \sigma) - \rho_{ij}$. $q_{-L_{ii}}$ denotes a time delay operator, i.e., $q_{-L_{ii}} u_i(t) = u_i(t-L_{ii})$. 

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Proof From (27), noting the controller (34) and its parametric representation (38), we can easily get the representation of the parametric representation of the tracking error (39).

In view of the error equation (39), the operator \( \frac{r_i^*(p)}{p_j^*(p)} q^{-L_{ii}} \) is not passive because its Laplace transformation is not stable. Therefore, an augmented error has to be generated. Define a signal by

\[
\eta_i(t) = \begin{cases} \\
\tilde{\eta}_i(t) \omega_i(t) \\
+ \sum_{j=1}^{n} \int_{-L_{ij}}^{-(L_{ij}-L_{il})} \lambda_{ij}(t, \sigma) \bar{u}_j(t + \sigma) d\sigma \\
+ \sum_{j \neq l} \int_{-(L_{ij}-L_{il})}^{0} \tilde{\lambda}_{ij}(t, \sigma) \bar{u}_j(t + \sigma) d\sigma \\
- \frac{r_i^*(p)}{p_j^*(p)} q^{-L_{ii}} \left\{ \frac{\tilde{\eta}_i(t) \omega_i(t)}{\omega_i(t)} \right\} \\
\sum_{j=1}^{n} \int_{-L_{ij}}^{0} \lambda_{ij}(t, \sigma) \bar{u}_j(t + \sigma) d\sigma \\
+ \sum_{j \neq l} \int_{-L_{ij}}^{0} \tilde{\lambda}_{ij}(t, \sigma) \bar{u}_j(t + \sigma) d\sigma \\
\end{cases}
\]

(40)

where

\[
\omega_i(t) = \frac{r_i^*(p)}{p_j^*(p)} q^{-L_{ii}} \omega_i(t), \bar{u}_j(t) = \frac{r_i^*(p)}{p_j^*(p)} q^{-L_{ii}} u_j(t)
\]

(41)

Define the augmented error by

\[
\varepsilon_i(t) = e_i(t) + \hat{g}_{ii}(t) \eta_i(t)
\]

Define the signal vector by

\[
\Omega_i(t) = \begin{bmatrix} \\
\omega_i(t), & -L_{ii} \leq \sigma \leq -(L_{ii}-L_{il}) \\
\bar{u}_1(t + \sigma), & -L_{i1} \leq \sigma \leq 0 \\
\bar{u}_i(t + \sigma), & -L_{ii} \leq \sigma \leq -(L_{ii}-L_{il}) \\
\bar{u}_{i-1}(t + \sigma), & -L_{i1} \leq \sigma \leq -(L_{i1}-L_{il}) \\
\bar{u}_{i+1}(t + \sigma), & -L_{i1} \leq \sigma \leq -(L_{i1}+1-L_{il}) \\
\bar{u}_n(t + \sigma), & -L_{ii} \leq \sigma \leq -(L_{ii}-L_{il}) \\
\bar{u}_{i-1}(t + \sigma), & -(L_{i1}-L_{il}) \leq \sigma \leq 0 \\
\bar{u}_{i+1}(t + \sigma), & -(L_{i1}+1-L_{il}) \leq \sigma \leq 0 \\
\bar{u}_n(t + \sigma), & -(L_{ii}-L_{il}) \leq \sigma \leq 0 \\
\end{bmatrix}^T
\]

(42)

Choose adaptive law

\[
\hat{g}_{ii}(t) = -\tau_{g_{ii}} \eta_i(t) \frac{\bar{u}_i(t + \sigma)}{1 + \left\| \Omega_i(t) \right\|^2}, \quad i = 1, 2, \ldots, n
\]

\[
\hat{\theta}_{ij}(t) = -\tau_{\theta_{ij}} \eta_i(t) \frac{\bar{u}_i(t + \sigma)}{1 + \left\| \Omega_i(t) \right\|^2}, \quad i = 1, 2, \ldots, n
\]

\[
\hat{\lambda}_{ij}(t, \sigma) = -\tau_{\lambda_{ij}} \frac{\left\| \Omega_i(t) \right\|^2}{1 + \left\| \Omega_i(t) \right\|^2} \varepsilon_i(t),
\]

\[
\hat{\rho}_{ij}(t, \sigma) = -\tau_{\rho_{ij}} \frac{\left\| \Omega_i(t) \right\|^2}{1 + \left\| \Omega_i(t) \right\|^2} \varepsilon_i(t),
\]

where \( \tau_{g_{ii}}, \tau_{\theta_{ij}}, \tau_{\lambda_{ij}}, \) and \( \tau_{\rho_{ij}} \) are some positive constants to be chosen.

**Theorem 4:** Consider system (1), controller (38), and adaptive law (44), all signals in the closed-loop system are bounded and \( \lim_{t \to \infty} e_i(t) = 0 \) for all \( i = 1, 2, \ldots, n \).

**Proof** Without loss of generality, \( g_{ii}(i = 1, 2, \ldots, n) \) are assumed to be positive. Consider a candidate Lyapunov function

\[
V_i(t) = \frac{1}{2} \hat{g}_{ii}^2(t) + \frac{1}{2} \theta_{ij}^2(t) \tilde{g}_{ii}(t)
\]

\[
+ \frac{n}{2} \int_{-(L_{ii}-L_{il})}^{-(L_{ii}-L_{il})} \lambda_{ij}(t, \sigma) \bar{u}_j(t + \sigma) d\sigma
\]

(45)

Because \( \hat{g}_{ii}(t) = \hat{g}_{ii}(t) \tilde{g}_{ii}(t) \hat{\lambda}_{ij}(t, \sigma) = \hat{\lambda}_{ij}(t, \sigma) \), and \( \hat{\rho}_{ij}(t, \sigma) = \hat{\rho}_{ij}(t, \sigma)(j \neq l) \), the deviation of \( V_i \) along the trajectories of (44) is given by

\[
\dot{V}_i(t) = -\frac{1}{1 + \left\| \Omega_i(t) \right\|^2} \hat{g}_{ii}(t) \eta_i(t) + g_{ii} \left\| \tilde{\eta}_i(t) \right\| \omega_i(t)
\]

\[
+ \sum_{j=1}^{n} \frac{g_{ij}}{2} \int_{-(L_{ii}-L_{il})}^{-(L_{ii}-L_{il})} \lambda_{ij}(t, \sigma) \bar{u}_j(t + \sigma) d\sigma
\]

(46)

Because \( \hat{\lambda}_{ij}(t, \sigma) = \hat{\lambda}_{ij}(t, \sigma) \), \( \hat{\rho}_{ij}(t, \sigma) = \hat{\rho}_{ij}(t, \sigma)(j \neq l) \), the deviation of \( V_i \) along the trajectories of (44) is given by

\[
\dot{V}_i(t) = -\frac{1}{1 + \left\| \Omega_i(t) \right\|^2} \hat{g}_{ii}(t) \eta_i(t) + g_{ii} \left\| \tilde{\eta}_i(t) \right\| \omega_i(t)
\]

\[
+ \sum_{j=1}^{n} \frac{g_{ij}}{2} \int_{-(L_{ii}-L_{il})}^{-(L_{ii}-L_{il})} \lambda_{ij}(t, \sigma) \bar{u}_j(t + \sigma) d\sigma
\]

\[
+ \sum_{j \neq l} \int_{-(L_{ii}-L_{il})}^{0} \lambda_{ij}(t, \sigma) \bar{u}_j(t + \sigma) d\sigma
\]

\[
+ \sum_{j \neq l} \int_{-(L_{ii}-L_{il})}^{0} \tilde{\lambda}_{ij}(t, \sigma) \bar{u}_j(t + \sigma) d\sigma
\]

(47)

Therefore, \( \hat{g}_{ii}(t), \hat{\theta}_{ij}(t), \hat{\lambda}_{ij}(t, \sigma) \), and \( \hat{\rho}_{ij}(t, \sigma) \) are bounded as long as their initial values are bounded, and

\[
\frac{1}{1 + \left\| \Omega_i(t) \right\|^2} \varepsilon_i(t) \in L_2
\]

In view of (44), we have

\[
\dot{\varepsilon}_i(t) = -\tau_{\varepsilon_{ii}} \frac{\eta_i(t)}{1 + \left\| \Omega_i(t) \right\|^2} \frac{e_i(t)}{1 + \left\| \Omega_i(t) \right\|^2}
\]

(48)

It follows that \( \dot{\varepsilon}_i(t) \in L_2 \), similarly, \( \dot{\bar{\varepsilon}}(t), \dot{\tilde{\varepsilon}}(t), \dot{\tilde{\varepsilon}}(t), \dot{\tilde{\varepsilon}}(t), \dot{\tilde{\varepsilon}}(t), \dot{\tilde{\varepsilon}}(t), \dot{\tilde{\varepsilon}}(t), \dot{\tilde{\varepsilon}}(t) \in L_2 \). Using the similar proof procedure in [14] and [15], we can prove that all signals in the closed-loop system are bounded and \( \lim_{t \to \infty} e_i(t) = 0 \), similarly, we can prove \( \lim_{t \to \infty} \eta_i(t) = 0 \). Therefore, we obtain \( \lim_{t \to \infty} e_i(t) = 0 \). Thus, the proof is completed.
V. SIMULATION
Consider a MIMO system with the transfer function
\[
\begin{bmatrix}
g_{11}(s + a_{12})e^{-7s} \\
(s + a_{11})(s + a_{12})e^{-8s} \\
(s + a_{21})(s + a_{22})
ge_{12}(s + 5s) \\
(s + a_{11})(s + a_{12})e^{-10s} \\
(s + a_{21})(s + a_{22})
\end{bmatrix}
\]

(49)

where \(g_{ij}, a_{ij}(i, j = 1, 2)\) are parameters of the system. The transfer function \(T_r(s)\) of the reference model is
\[
T_r(s) = \text{diag} \left( \frac{3e^{-6s}}{s^2 + 5s + 6}, \frac{5e^{-8s}}{s^2 + 6s + 1} \right)
\]

(50)
The parameters of the nominal system and two systems with parameter variation are listed in Table 1.

<table>
<thead>
<tr>
<th>parameter</th>
<th>(g_{11})</th>
<th>(g_{12})</th>
<th>(g_{21})</th>
<th>(g_{22})</th>
<th>(a_{11})</th>
<th>(a_{12})</th>
<th>(a_{21})</th>
<th>(a_{22})</th>
</tr>
</thead>
<tbody>
<tr>
<td>nominal system</td>
<td>13</td>
<td>1</td>
<td>0.5</td>
<td>8</td>
<td>0.5</td>
<td>6</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>system 1</td>
<td>12</td>
<td>0.5</td>
<td>2</td>
<td>6</td>
<td>0.8</td>
<td>7</td>
<td>2.5</td>
<td>1.5</td>
</tr>
<tr>
<td>system 2</td>
<td>10</td>
<td>0.3</td>
<td>1</td>
<td>7</td>
<td>0.6</td>
<td>5</td>
<td>1.5</td>
<td>2.5</td>
</tr>
</tbody>
</table>

When reference input \(v_i(i = 1, 2)\) are unit step signal, choose stable polynomials
\[
\begin{align*}
r_1^*(s) &= s + 5, \quad r_2^*(s) = s + 4, \quad \tau_1(s) = \tau_2(s) = 1 \\
p_1^*(s) &= s^2 + 10s + 1, \quad p_2^*(s) = s^2 + 6s + 1
\end{align*}
\]

(51)

Applying the designed controller in this paper and choosing parameters \(\tau_{g_{11}} = \tau_0 = 0.01, \tau_{g_{12}} = \tau_{g_{21}} = 0.01, \tau_{g_{22}} = 0.05\), simulation results are shown in Fig. 1, Fig. 2 and Fig. 3. From the simulation results, we can see that errors of the nominal systems, systems 1 and system 2 can all converge to zero. This shows that the designed adaptive control scheme meets the objective.

VI. CONCLUSION
This paper solves the model matching problem for MIMO systems with multiple time delays. Time delays occur in every control loop, i.e., the considered systems are general multivariable delay systems. A controller structure is designed for nominal systems and an adaptive control scheme is proposed for systems with parameter variation. Robust adaptive control for MIMO delay systems with other kinds of uncertainties needs further research.

REFERENCES