Attitude Reference Generation for Leader-Follower Formation With Nadir Pointing Leader

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Abstract—In this paper we present a solution for real-time reference generation for a leader-follower spacecraft formation. The solution is assuming a nadir pointing leader spacecraft and a follower spacecraft which is complimenting the measurement of the leader by tracking the point located at the intersection on the Earth surface between Earth center and leader spacecraft. Desired relative attitude, angular velocity and angular acceleration are generated in real-time based on the relative position, velocity and acceleration between the leader and follower spacecraft. A passivity based PD+ controller is derived and the equilibrium points of the closed-loop system are shown uniformly asymptotically stable, and simulation results are presented to show the performance of both the reference generation method and the control law during a formation reconfiguration.

I. INTRODUCTION

In recent years, formation flying has become an increasingly popular subject of study. This is a new method of performing space operations, by replacing large and complex spacecraft with an array of simpler micro-spacecraft bringing out new possibilities and opportunities of cost reduction, redundancy and improved resolution aspects of onboard payload. One of the main challenges is the requirement of synchronization between spacecraft; robust and reliable control of relative position and attitude are necessary to make the spacecraft cooperate to gain the possible advantages made feasible by spacecraft formations. For fully autonomous spacecraft formations both path- and attitude-planning must be performed on-line which introduces challenges like collision avoidance and restrictions on pointing instruments towards required targets, with the lowest possible fuel expenditure. The system model is a key element to achieve an reliable and robust controller. Basically there are two different approaches for modeling spacecraft formations: Cartesian coordinates and orbital elements, which both have their pros and cons. The orbital element method is often used to design formations concerning low fuel expenditure because of the relationship towards natural orbits, while Cartesian models often are used where an orbit with fixed dimensions are studied, which is the case in this paper.

The simplest Cartesian model of relative motion between two spacecraft is linear and known as the Hill [1] or Clohessy-Wiltshire [2] equations; a linear model based on assumptions of circular orbits, no orbital perturbations and small relative distance between spacecraft compared with the distance from the formation to the center of the Earth. As the visions for tighter spacecraft formations in highly elliptic orbits appeared, the need for more detailed models arose, especially regarding orbital perturbations. This resulted in nonlinear models as presented in e.g. [3], [4], and later in [5] and [6], derived for arbitrary orbital eccentricity and with added terms for orbital perturbations. Most previous work on reference generation are concerned with translational trajectory generation for fuel optimal reconfiguration and formation keeping such as in [7] where a formation located at the Sun-Earth $L_2$ Langrange Point is considered, while [8] proposed two approaches to design perturbed satellite formation relative motion orbits using least-square techniques. Trajectory optimization for satellite reconfiguration maneuvers coupled with attitude constraints have been investigated in [9] where a path planner based on rapidly-exploring random tree is used in addition to a smoother function. Coupling between the position and attitude is introduced by the pointing constrains, and thus the trajectory design must be solved as a single 6N Degrees of Freedom (DOF) problem instead of $N$ separate 6 DOF problems. Ground target tracking for spacecraft has been addressed by several other researchers, such as [10], [11], [12] and [13] where only one spacecraft spacecraft has been addressed by several other researchers, such as [10], [11], [12] and [13] where only one spacecraft is considered. The usual way to generate target tracking reference is to find a vector pointing from the spacecraft towards a point on the planet surface where the instrument is supposed to be pointing, and then the desired quaternions and angular velocities are generated to ensure high accuracy tracking of the specified target point. The control problem for spacecraft formation was issued in [3] using nonlinear control to upkeep the shape of a ring formation. In [4] the control of relative position was addressed and later extended to include relative rotation [14]. The PD+ controller was first presented for robot manipulators [15], later for single spacecraft [16] and spacecraft formations [6].

In this paper we present a solution for real-time generation of attitude references for a leader-follower spacecraft formation with nadir pointing leader and followers complementing the measurement by pointing their instruments at the common target on Earth surface. The solution is based on a 6DOF model where each follower generates the attitude references in real-time, based on relative position and translational motion between the leader and its followers, which also ensures that the spacecraft are pointing at the target during formation reconfiguration. We are also utilizing a passivity-based PD+ controller for relative position and attitude tracking between spacecraft, and proving uniform asymptotic stability according to standard Lyapunov theorems. Simulation results are
presented to show how the attitude references are generated during a formation reconfiguration using the derived control law.

II. MATHEMATICAL BACKGROUND

In the following, we denote by $\dot{x}$ the time derivative of a vector $x$, i.e. $\dot{x} = d\mathbf{x}/dt$, and moreover, $\ddot{x} = d^2\mathbf{x}/dt^2$. The cross product operator $a \times b$ is denoted $\mathbf{S}(a)b$, where $a, b \in \mathbb{R}^3$ is the angular velocity of frame $a$ relative to frame $b$, expressed in frame $c$, $\mathbf{R}_{\mathbf{b}}^c$ is the rotation matrix from frame $a$ to frame $b$, and $\|\|_2$ denotes the L2-norm. Coordinate reference frames are denoted by $\mathcal{F}$. The set $\mathbb{S}^3$ is the special orthogonal group of order three, where $\mathbb{S}^3$ is defined as $\mathbb{S}^3 = \{ R \in \mathbb{R}^{3 \times 3} : R^T R = I, \det R = 1 \}$, where $\mathbb{R}$ is the scalar part and $\varepsilon = [\varepsilon_x, \varepsilon_y, \varepsilon_z]^T$ is the vector part. The rotation matrix may be described by (17)

$$
\mathbf{R} = \mathbf{I} + 2\mathbf{S}(\varepsilon) + 2\mathbf{S}^2(\varepsilon),
$$

where the matrix $\mathbf{S}(\cdot)$ is the cross product operator described as

$$
\mathbf{S}(\varepsilon) = \varepsilon \times = \begin{bmatrix}
0 & -\varepsilon_z & \varepsilon_y \\
\varepsilon_z & 0 & -\varepsilon_x \\
-\varepsilon_y & \varepsilon_x & 0
\end{bmatrix}.
$$

The inverse rotation can be performed by using the inverse conjugated of $\mathbf{q}$ as $\bar{\mathbf{q}} = [\eta, -\varepsilon]^T$. The set $\mathbb{S}^3$ forms a group with quaternion multiplication, which is distributive and associative, but not commutative, and the quaternion product of two arbitrary quaternions $\mathbf{q}_1$ and $\mathbf{q}_2$ is defined as

$$
\mathbf{q}_1 \otimes \mathbf{q}_2 := \begin{bmatrix}
\eta_1 \eta_2 - \varepsilon_1 \varepsilon_2 \\
\eta_1 \varepsilon_2 + \eta_2 \varepsilon_1 + \mathbf{S}(\varepsilon_1)\varepsilon_2
\end{bmatrix}.
$$

It should be noted that the quaternion representation is an inherent redundant representation, giving two different equilibriums $\mathbf{q} = [\pm 1, 0]^T$ which in fact represents the exact same physical orientation, however one is rotated $2\pi$ relative to the other about an arbitrary axis. The time derivative of equation (4) can be written as (17)

$$
\dot{\mathbf{R}}_{\mathbf{b}}^a = \mathbf{S} \left( \mathbf{\omega}_{\mathbf{a},b}^b \right) \mathbf{R}_{\mathbf{b}}^a = \mathbf{R}_{\mathbf{b}}^a \mathbf{S} \left( \mathbf{\omega}_{\mathbf{a},b}^b \right). \tag{7}
$$

The kinematic differential equations may be expressed as (17)

$$
\dot{\mathbf{q}} = \mathbf{T}(\mathbf{q})\mathbf{\omega}, \tag{8}
$$

where

$$
\mathbf{T}(\mathbf{q}) = \frac{1}{2} \left[ -\varepsilon^T \eta \mathbf{I} + \mathbf{S}(\varepsilon) \right] \in \mathbb{R}^{4 \times 3}. \tag{9}
$$

C. Relative Translation

The fundamental differential equation of the two-body problem can be expressed as (cf. (18))

$$
\dot{\mathbf{r}}_s = -\frac{\mu}{r_s^3} \mathbf{r}_s + \frac{f_{ds}}{m_s} + \frac{f_{xs}}{m_s}, \tag{10}
$$

Fig. 1. Reference frames
where $f_{ds} \in \mathbb{R}^3$ is the perturbation term due to external effects and $f_{as} \in \mathbb{R}^3$ is the actuator force. Spacecraft masses are assumed to be small relative to the mass of the Earth $m_e$, so $\mu \approx Gr_e$. Using the true anomaly $\nu$ of the leader spacecraft, and denoting relative velocity as $v = \dot{p}$, the nonlinear position dynamics can be represented in the $F^l$ frame on the form (cf. [5], [6])

$$m_f \ddot{v} + C_t v + D_l p + n_t = f_a + F_d,$$  

(11)

where

$$C_t(\dot{v}) = 2m_f \dot{\nu} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \in SS(3)$$  

(12)

is a skew-symmetric Coriolis-like matrix,

$$D_l(\dot{\nu}, \dot{r}_f) = m_f \begin{bmatrix} \dot{r}_f & -\nu^2 & -\dot{\nu} \\ \nu & \dot{r}_f & -\nu^2 \\ 0 & 0 & \frac{\dot{r}_f}{r_f} \end{bmatrix}$$  

(13)

may be viewed as a time-varying potential force matrix, and

$$n_t(r_l, r_f) = m_f \mu \begin{bmatrix} r_l & 0 & 0 \end{bmatrix}^\top \begin{bmatrix} r_l & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} + 0.$$  

(14)

The composite perturbation force $F_d$ and the relative control force $F_a$ are given by

$$F_d = f_d - \frac{m_f}{m_l} f_{dl}, \quad F_a = f_a f - \frac{m_f}{m_l} f_{a}. $$  

(15)

### D. Relative Rotation

With the assumptions of rigid body movement, the dynamical model of a spacecraft can be found from Euler’s momentum equations as [19]

$$J_f \dot{\omega} + J_f S(R_{lb} \dot{\omega}_{lb}) - J_f R_{lb} \hat{J}_f \hat{S}(\dot{\omega}_{lb}) J_f \omega_{lb} + \tau_{as}^{sb} = \omega_{s,b} - \omega_{s,b} - R_{lb} \omega_{t,s}; $$  

(16)

$$J_f \dot{\omega} + J_f S(R_{lb} \dot{\omega}_{lb}) - J_f R_{lb} \hat{J}_f \hat{S}(\dot{\omega}_{lb}) J_f \omega_{lb} + \tau_{as}^{sb} = \omega_{s,b} - \omega_{s,b} - R_{lb} \omega_{t,s}; $$  

(17)

where $J_f = \text{diag}(J_{xx}, J_{yy}, J_{zz}) \in \mathbb{R}^{3 \times 3}$ is the spacecraft moment of inertia, $\tau_{as}^{sb} \in \mathbb{R}^3$ is the total disturbance torque, $\omega_{s,b} \in \mathbb{R}^3$ is the total actuator torque and $\omega_{t,s} = \dot{r} \times \nu / r$ $r$ is the orbit angular velocity. The relative attitude dynamics may be expressed as [6]

$$J_f \dot{\omega} + J_f S(R_{lb} \dot{\omega}_{lb}) - J_f R_{lb} \hat{J}_f \hat{S}(\dot{\omega}_{lb}) J_f \omega_{lb} + \tau_{as}^{sb} = \omega_{s,b} - \omega_{s,b} - R_{lb} \omega_{t,s}; $$  

(18)

where

$$-\dot{\omega} = \omega_{f}^{lb} - R_{lb} \omega_{t,lb}$$  

(19)

is the relative angular velocity between the follower body reference frame and the leader body reference frame expressed in the follower body reference frame,

$$\tilde{Y}_d = \tau_{as}^{lb} J_f - J_f R_{lb} \hat{J}_f \hat{S}(\dot{\omega}_{lb}) J_f \omega_{lb} + \tau_{as}^{sb} \hat{S}(\dot{\omega}_{lb}) J_f \omega_{lb} = \tilde{Y}_a \quad (20)$$

are the relative perturbation torques and actuator torques respectively. For simplicity (18) may be rewritten as

$$J_f \dot{\omega} + C_r(\omega) \omega + n_r(\omega) = \tilde{Y}_d + \tilde{Y}_a,$$  

(21)

where

$$C_r(\omega) = J_f S(R_{lb} \dot{\omega}_{lb}) - J_f R_{lb} \hat{J}_f \hat{S}(\dot{\omega}_{lb}) J_f \omega_{lb} + \tau_{as}^{sb} \hat{S}(\dot{\omega}_{lb}) J_f \omega_{lb}$$  

(22)

is a skew-symmetric matrix, and

$$n_r(\omega) = S(R_{lb} \dot{\omega}_{lb}) J_f R_{lb} \dot{\omega}_{lb} - J_f R_{lb} \hat{J}_f \hat{S}(\dot{\omega}_{lb}) J_f \omega_{lb}$$  

(23)

is a nonlinear term.

#### E. 6DOF Model

For a 6DOF model including relative translation and rotation the state vectors are defined as $x_1 = [p^\top, q^\top]^\top$ and $x_2 = [v^\top, \omega^\top]^\top$, and based on the models (11) and (21) it may be written as [6]

$$\dot{x}_1 = \Lambda(x_1) x_2, \quad \Lambda(x_1) = \begin{bmatrix} I & 0 \\ 0 & \frac{1}{2} \begin{bmatrix} 0 & -\epsilon^\top \\ \epsilon & -I \end{bmatrix} \end{bmatrix}$$  

(24)

$$M_f x_2 = U + W - C(\nu, \omega) x_2 - D(\dot{\nu}, \dot{\nu}, r_f)x - n(\nu, r_l, r_f),$$  

(25)

where $M_f = \text{diag}(m_f I, J_f)$ is a symmetric positive definite matrix, $U = [F_f^a, Y_a]^\top$ and $W = [F_f^a, T_f]^\top$ are the relative input forces and orbital perturbations, $C(\nu, \omega) = \text{diag}(C_v(\dot{\nu}), C_r(\omega))$, is a skew-symmetric Coriolis-like matrix, $D(\dot{\nu}, \dot{\nu}, r_f) = \text{diag}(D(\dot{\nu}, \dot{\nu}, r_f), 0)$ is the time-varying potential force, and $n(\nu, r_l, r_f) = [n_t(r_l, r_f), n_r(\omega)]$ is a nonlinear term.

#### III. REFERENCE GENERATION

For minimizing the workload on the ground station operator it is important for a spacecraft formation to be as autonomous as possible. The topic of reference generation is to have the spacecraft themselves generate the needed references to fulfill the mission requirements in real-time. In this Section we present rotational references which are generated for the follower spacecraft to ensure tracking of the target located at the footprint of the leader spacecraft, based on relative translation between the leader and its followers.

#### A. Attitude Reference

The target reference frame $F^l$, as depicted in Figure 1, is located at the earth surface specified by the vector $r_e(\lambda, \phi)$ relative to $F$, where $r_e$ is the Earth radii. It is assumed a perfect spherical Earth; alternatively a function of the Earth radii may be used $r_e(\lambda, \phi)$ with longitude and latitude as arguments. Unit vectors of the target reference frame align with the basis vectors of the leader orbit reference frame $F^l$. A vector pointing from the follower spacecraft towards the pointing target is first derived in target and follower orbit frames as

$$\ell^t = -\gamma e_r - R_e^p, \quad \ell^f = -\gamma R_f^e e^t - R_f^t p,$$  

(26)

where $\gamma = \|r_l\| - r_e$ is the orbit height. A desired reference frame $F^d$, as depicted in Figure 1, is located at the center of mass of the follower spacecraft, and defined as $z_d = U_f^t/\|U_f^t\|$, $y_d = z_d \times p/\|z_d \times p\|$ and $x_d = y_d \times z_d$. It
should be noted that the vectors \( p \) and \( l' \) are not necessarily perpendicular, and in some instances, when \( p = [c, 0, 0] \top \), the vectors are parallel, which will give a zero vector in the defined reference frame. When this occurs it means that the two spacecraft are lined along the \( r_l \) vector and one of the spacecraft is blocking the field of view for the other. As a solution for this instance the generated rotational reference is set to nadir pointing.

**B. Angular Velocity Reference**

The angular velocity of the desired frame relative the target frame is expressed in the desired reference frame \( \omega_{d,t} = [\omega_{dx}, \omega_{dy}, \omega_{dz}] \top \). The derivative of the target pointing vector in the follower orbit frame can be expressed in the desired frame as [12]

\[
l'^{d} = (l'^{f} \cdot x_d)x_d + (l'^{f} \cdot y_d)y_d + (l'^{f} \cdot z_d)z_d. \tag{27}
\]

Alternatively the vector may be expressed as

\[
l'^{f} = \frac{dl^{d}}{dt} + \omega_{f,d} \times l, \tag{28}
\]

where \( \frac{dl^{d}}{dt} \) is the derivative taken in the desired reference frame, and \( l = \|l'^{f}\|z_d \). By combining (27) and (28) we get the expression

\[
\omega_{f,d} = \left[ \frac{-l'^{f} \cdot x_d}{\|l'^{f}\|} \frac{l'^{f} \cdot y_d}{\|l'^{f}\|} 0 \right] \top. \tag{29}
\]

It should be noted that the desired angular velocity about the \( z_d \)-axis is set to zero to ensure that the instrument is not rotating while performing measurements. The derivative of (26) is

\[
l'^{f} = -\gamma(S(\omega_{f,d}^{I})R_{r_t}^{f}l - R_{r_t}^{f}l^{f}S(\omega_{f,d}^{I})R_{r_t}^{f}p - R_{r_t}^{f}p), \tag{30}
\]

where \( p \) and \( \dot{p} = v \) are state variables. Since \( f^{f} \) and \( f^{I} \) are parallel, the relative velocity \( \omega_{r,f} = 0 \) and \( R_{r_t}^{f} = I \). The same is also true for \( f^{f} \) and \( f^{I} \) which means that \( R_{r_t}^{f} = I \) and \( \omega_{r,f} = \omega_{r,f}^{I} + \omega_{r,f}^{I} + \omega_{r,f}^{I} - \omega_{r,f}^{I} + \omega_{r,f}^{I} = 0 \), since \( \omega_{r,f}^{I} = \omega_{r,f}^{I} \), as both reference frames are located along the same vector \( r_l \) rotating about \( f^{f} \) with equal angular velocity. Finally, \( \dot{e}_r = 0 \) since a change in the radial vector will have no impact on the angular velocity between the target and follower orbit frame. Equation (30) is then rewritten as

\[
l'^{f} = -\dot{\gamma}e_{r} - \dot{p}. \tag{31}
\]

**C. Angular Acceleration Reference**

The angular acceleration reference \( \omega_{f,d}^{I} = [\omega_{dx}, \omega_{dy}, \omega_{dz}] \top \) is obtained by differentiating the angular velocity reference (29) leading to

\[
\omega_{f,d}^{I} = \left[ \frac{-l'^{f} \cdot x_d + 2x_dx_{d,t}l'^{f}l'^{I}}{\|l'^{f}\|^2} + \omega_{dy}\omega_{dz} \right]; \tag{32}
\]

Since the angular velocity about the \( z \)-axis is set to zero in (29), the last terms of \( \omega_{dx} \) and \( \omega_{dy} \) are zero. The derivative of (31) is

\[
l'^{f} = -\ddot{\gamma}e_{r} - \ddot{p} - \ddot{\dot{e}}_{r} - \ddot{\dot{p}}. \tag{33}
\]

**IV. CONTROLLER DESIGN**

For control of the relative translation and rotation, we incorporate a passivity based PD+ controller, similar to the one derived in [15]. For the control law it is assumed that the follower spacecraft has available information of relative position \( p \), relative velocity \( v \), relative attitude \( q \), relative angular velocity \( \omega_{f,b}^{I} \), leader true anomaly rate \( \dot{\nu} \) and leader true anomaly rate of change \( \dot{\dot{\nu}} \).

The control problem is to design a controller that makes the state \( x(t) \) converge towards the generated reference specified as \( x_{d1} = [p_{d1}^{I}, q_{d1}^{I}] \top \) and \( x_{d2} = [v_{d2}^{I}, \omega_{d2}^{I}] \top \). The leader spacecraft is controlled to be nadir pointing, \( q_{d1} = [1, 0] \top \), but as it is not perfectly controlled we take the quaternion product \( q_{d1} = q_{d1} \otimes q_{d1} \), since the error quaternion is defined as \( \bar{q} = q \otimes q_{d1} \) and \( q = q_{d1} \otimes \bar{q} \), we get \( \bar{q} = (q_{d1} \otimes q_{d1}) \otimes (q_{d1} \otimes \bar{q}) \), which again may be written as \( \bar{q} = q_{d1} \otimes q_{d1} \), because all quaternions are unit quaternions. For 6DOF the error vector is written as \( e_1 = [\bar{p}^{I}, 1 - \bar{\nu}, \epsilon^{I}] \top \), where the relative position error is defined as \( p = p - p_d \). It should be noted that this is the error vector for the positive equilibrium point \( q_\pm = [1, 0] \top \) on the rotational sphere and that there exists a duality in the negative equilibrium point \( q_\mp = [-1, 0] \top \) such that \( e_1 \pm = [\bar{p}^{I}, 1 + \bar{\nu}, \epsilon^{I}] \top \), and (24) is rewritten as

\[
e_1 = \Lambda_{e}(e_1 \pm) e_2, \quad \Lambda_{e}(e_1 \pm) = \begin{bmatrix} I & 0 \\ 0 & \frac{1}{2} \begin{bmatrix} 1 & \bar{\epsilon}^	op \\ \bar{\epsilon} & S(\bar{\epsilon}) \end{bmatrix} \end{bmatrix}, \tag{34}
\]

where the relative velocity error is defined as \( \bar{v} = v - v_d \), the relative angular velocity error as \( \bar{\omega} = \omega_{f,b}^{I} - \omega_{f,b}^{I} \), and \( e_2 = [\bar{v}^{I}, \epsilon^{I}] \top \). To express (29) as the required angular velocity we use

\[
\omega_{f,b}^{I} = \omega_{f,b}^{I} - S(\omega_{f,b}^{I} \omega_{f,b}^{I})R_{f,b}^{I} \omega_{f,b}^{I} + R_{f,b}^{I} \omega_{f,b}^{I}, \tag{35}
\]

and the desired angular acceleration (32) may be rewritten by differentiation of (35)

\[
\omega_{f,b}^{I} = \omega_{f,b}^{I} - S(\omega_{f,b}^{I} \omega_{f,b}^{I})R_{f,b}^{I} \omega_{f,b}^{I} + R_{f,b}^{I} \omega_{f,b}^{I} + R_{f,b}^{I} \omega_{f,b}^{I}. \tag{36}
\]

The control law for the passivity-based PD+ controller is expressed as [6]

\[
u = M_f x_d + C(\dot{\nu}, \omega)x_d + D(\dot{\nu}, \dot{\nu}, r_f)x_1 \tag{37}
\]

\[
+ n(\omega, r_1, r_f) - w_f - K_p \Lambda_{e}^	op e_1 - K_d e_2,
\]

where \( \omega = \omega_{f,b}^{I} \), and \( K_p = \text{diag}(k_p I, k_q I) = K_p^{1} > 0 \) and \( K_d = \text{diag}(k_c I, k_c I) = K_d^{1} > 0 \) are feedback gain matrices. The control law (37) is composed of feed-forward of the inertial forces, compensation for nonlinear terms, and proportional and derivative error feedback. By combining the 6DOF model (25) and the control law (37) we get a closed-
loop system
\[ M_f \dot{e}_2 + (C + K_d)e_2 + K_p \Lambda^\top e_1 = 0. \] (38)
The proof consists of two parts; first, a suitable radially unbounded, positive definite Lyapunov function candidate is chosen as
\[ V(e_1, e_2) = \frac{1}{2} e_1^\top K_p e_1 + \frac{1}{2} e_2^\top M_f e_2, \] (39)
and by differentiation and insertion of (34) and (38), we obtain
\[ \dot{V} = -e_2^\top K_d e_2. \] (40)
Accordingly, by Lyapunov arguments (cf. [20]), it follows that the equilibrium point \((\Lambda^\top e_1, e_2) = (0, 0)\) for the closed-loop system in (38) is uniformly stable (US). For the second part, an auxiliary function is defined as
\[ W(\Lambda^\top e_1, e_2) = e_1^\top \Lambda_p K_p M_f e_2, \] (41)
which depends on time through the reference function \(x_{d2}\), and is continuous in both arguments. By differentiation of (41) and inserting (34) and (38) yields
\[ \dot{W} = -e_1^\top \Lambda_p K_p K_p \Lambda^\top e_1, \] (42)
in the set \( E : \{ \dot{V} = 0 \} = \{ e_2 = 0 \} \), which is negative definite in the position error, continuous in both arguments, and depends on time through the reference function \(x_{d1}\), which proves that the equilibrium point is uniformly asymptotically stable (UAS). It should be noted that we do not get global results because of the duality of the equilibrium point. The proof for the negative equilibrium point is performed in a similar way.

V. SIMULATION

In this section we present illustrative simulation results of the theory presented in this paper. The simulation was performed in Simulink using a variable sample-time Runge-Kutta ODE45 solver, with tolerance of \(1 \cdot 10^{-12}\). The spacecraft masses were set to 100 kg, and moments of inertia given as \( J = \text{diag}\{4.350, 4.3370, 3.6650\} \) kgm\(^2\) each. The leader spacecraft was chosen to orbit Earth circularly at altitude of 600 km, inclination at 79°, and the argument of perigee and the right ascension of the ascending node at 0°. The maximum available force and torque was limited to 0.2 N and 0.05 Nm respectively, while the controller gains were chosen as \( K_p = \text{diag}\{0.003I, I\} \) and \( K_d = \text{diag}\{2I, 2I\} \). The initial relative position and attitude were chosen as standstill at \( p_{init} = [200, 100, -300]^\top \) m and \( \mathbf{q}_{init} = [1, 0]^\top \), while the desired relative position was chosen as standstill at \( p_d = [-200, 1000, -300]^\top \) m and desired attitude and rotational motion according to what was presented in Section III. The simulation was performed without perturbations to better illustrate its purpose.

Figure 2 shows the generated references for relative attitude, angular velocity and angular acceleration which are evolving in the first 3000 s because of the translational motion of the follower spacecraft. In Figure 3 the relative position and velocity are shown showing the translational motion during the reconfiguration. The bottommost plot depicts the actuator force during two consecutive orbits and as can be seen, is quite active during all parts of the simulation. This is because the follower spacecraft is commanded to keep a fixed relative position to the leader spacecraft thus challenging the natural orbital motion. The relative attitude and angular velocity is depicted in Figure 4 which shows that the controller is able to track the generated reference. Lastly, Figure 5 shows both latitude and longitude of where the
follower and leader spacecraft are pointing their instruments. The follower spacecraft is not pointing at the Earth surface during the first eight seconds, and after this it can be seen that both measuring points coincide.

VI. CONCLUSIONS AND FUTURE WORKS

In this paper we have presented a solution for real-time reference generation of relative rotation for a spacecraft formation, both during normal operation and reconfiguration, where a leader spacecraft is assumed performing nadir measurements while the follower spacecraft is required to complement the measurement by performing target tracking. The method was derived based on the changes in the orbit altitude and relative position and motion between the spacecraft by calculating references for desired attitude, angular velocity and angular acceleration. The spacecraft was controlled using a passivity based PD+ controller and proven UAS under the assumptions of known orbital perturbations. A simulation results was presented where the follower spacecraft showed to satisfactorily track the desired target during a formation reconfiguration. For future work the authors would like to extend the result to target tracking for a spacecraft formation where all the members are able to track a given point on the Earth surface.

REFERENCES