Control-based p-persistent Adaptive Communication Protocol

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Abstract—An enhancement to the CSMA p-persistent protocol family is proposed, based on a fully decentralized control that adjusts the message transmission rate of each node to the estimated density of surrounding transmitting nodes. The system does not require enumeration of nodes nor control messages, the only input to the control coming from the physical medium occupation. Stability conditions are given in both cases of fixed and switching topologies. We show that good channel exploitation levels can be assured as well by suitably tuning the control parameters.

Index Terms—Distributed control, Transmission control, Consensus, Switching, Lyapunov Functions

I. INTRODUCTION

The CSMA p-persistent protocol family represents an interesting solution for coordinator-free ad–hoc–networks. In highly mobile applications, reachability among nodes may quite often endure for a too short time, if compared to the time that is typically necessary for network discovery, coordination and connection setup. This problem becomes crucial if these operation are centralized and performed by a coordinator. Moreover, in currently available technologies (e.g., Zigbee, Bluetooth) it is likely to happen that the coordinator exploits RF circuits more than the other nodes, making this approach quite unfair since coordinator is like to burn down sooner than the other nodes. Finally, in highly dynamic mobile scenarios (e.g., vehicle–to–vehicle communication [7], swarm robot systems, affinity matching [14][11]) the communication protocol should be able to adapt to sudden changes in network topology.

In this paper we aim at improving p-persistent protocols by including an innovative distributed control which adapts the protocol behavior to changes in nodes density. We adopt a continuous–time model which is amenable for a theoretical investigation. Then we propose the control law and we analyze its performances in terms of stability, robustness in the presence of network topology abrupt changes, channel exploitation and fairness among nodes. Furthermore we show how to design a specific digital communication protocol. Such a protocol does not require any bandwidth exploitation for control purposes being purely based on the overall channel utilization measured at the physical level. Therefore, neither complete digital signal reception nor correct message decoding are required. This makes it extremely simple so allowing implementation on low-cost and low-computation-power devices. The theoretical results have been validated by simulation. As expected, the protocol guarantees fairness among nodes and robustness. A side-effect of the proposed protocol is the ability of estimating density of nodes without counting them (actually, even without exchanging messages, but only on behalf of RF channel occupation).

II. PROTOCOL DESCRIPTION

The operational method followed by the protocol we are proposing resembles that of the CSMA p-persistent decentralized protocols family [21][13][8][4][20][7].

The protocol we are proposing requires that every node, asynchronously and independently, subdivides the time dimension into time slots with duration $T_s$. At the beginning of each slot, the node has to choose whether to transmit or to switch to receiving mode. The choice is made by tossing a fictional biased coin.

With a probability $p$ the node starts transmitting a packet of length $T_{pk}$. At the end of transmission, the node has preemption over the channel, thus if it chooses to perform another transmission (again with probability $p$) it is allowed to do it immediately. This preemption technique guarantees a better throughput in case of bulk transmissions. If a node chooses to switch to receiving mode (probability $1-p$), it remains in that mode until either it reaches the beginning of the next slot or the channel becomes busy. In the first case, the process starts all over again by tossing another fictional coin. In the second case, the node waits until the channel returns idle and then performs a random back-off.

![Fig. 1. Conceptual scheme of the protocol](image-url)

This is a typical decentralized and distributed control paradigm [16] in which several agents make a local decisions
in the absence of a supervisor. The adopted policy must be very simple for both computation complexity and energy consumption limitations. Besides, in the presence of very complex systems it is apparent that simple control algorithm offer better robustness properties than more sophisticated (possibly optimal) ones which suffer of fragility phenomena in the presence of network variations (see e.g. [3]).

The adopted approach is related to the consensus-oriented control [15]: this subject is currently receiving attention especially in communication control problems [5][6][9][1][12][17]. Generally speaking, in a transmission context consensus among agents would require that all individuals transmit at the same rate (i.e., transmission fairness). We assume that all agents agree on the transmission protocol, so selfish behavior is ignored. This goal can be achieved under the strong assumption of full connection, or in the presence of a known network topology. Both requirements are highly unrealistic. Therefore the fundamental issue is how to deal with networks topologies that can change in time.

What we show is that

- Under ideal conditions of full connections among all nodes, we prove stability and consensus (i.e. fairness), namely all the nodes transmit exactly at the same rate. Almost full channel exploitation, namely up to an arbitrarily small tolerance, can be assured as well.
- Under realistic conditions we prove stability. Exact consensus is not possible unless we unrealistically assume that each node knows the number of neighboring nodes. Good channel exploitation and "local fairness", namely essentially uniform transmission rate inside subsets of close nodes, can be assured by means of a tuning parameter.
- The theoretical investigation provides efficient tuning rules for the control parameters, which have been tested by simulation.

III. MODEL AND CONTROL: THEORETICAL INVESTIGATION

In this section we propose the control algorithm and a rigorous stability analysis. The following variables will be adopted.

- \( x_i(t) \) message rate of the \( i \)th node;
- \( z_i(t) \) aggregate message rate of the complementary nodes \( z_i(t) = \sum_{j \neq i} x_j(t) \)
- \( u(t) \) transmission rate variation—the control input;
- \( \bar{y}_{opt} \) optimal channel occupation.
- \( y(t) \) total message rate \( y(t) = \sum_{i=1}^{\infty} x_i(t) \)

These are "ideal" variables, suitable for theoretical investigation, which are strongly related to the "real" ones. For instance \( x_i \) is essentially proportional to the probability \( p_i \) already mentioned. Similarly, \( z_i \) will be actually statistically estimated, by each node, from the external transmission intensity. Note that

\[ y(t) = z_i(t) + x_i(t) \]

for all \( i \). Without restrictions, assume that the variables are normalized so that

\[ \bar{y}_{opt} = 1. \]

The relation between \( x_i \) and its variation \( u_i \) turns out to be

\[ \dot{x}_i(t) = u_i(t) \tag{1} \]

so that the corresponding control scheme is

\[ \text{control} \quad u_i \quad \mathcal{S} \quad X_i \]

Fig. 2. the control scheme

In the ideal case we assume the following.

Assumption 1: Each node has an exact information about the complementary message rate \( z_i(t) \) and, of course, about its own \( x_i(t) \). The node interacts with all the remaining nodes but it has no information about the total number of nodes. We select a fully decentralized control algorithm of the form

\[ u_i(t) = -\alpha (1+\mu) x_i(t) - \alpha z_i(t) + \alpha \tag{2} \]

with \( \alpha \) and \( \mu \) positive parameters. The control goal is to achieve the ideal steady–state condition

\[ \lim_{t \to \infty} x_i(t) = \frac{1}{n} \tag{3} \]

which implies that \( \lim_{t \to \infty} y(t) = 1 \). The next theorem states that condition (3) can be assured up to an arbitrarily small tolerance.

**Theorem 3.1:** Under Assumption 1, system (1) with the control algorithm (2) is asymptotically stable. Moreover, given \( e \) we have, asymptotically,

\[ \lim_{t \to \infty} x_i(t) = \frac{1}{n+\mu} \]

a value which is arbitrarily close to the target one \( 1/n \) provided that \( \mu \) is small enough.

**Proof:** Denoting by \( x(t) = [x_1(t) \ x_2(t) \ldots x_n(t)]^T \), the system can be written as

\[ \dot{x}(t) = Ax(t) + Bu(t), \]

with constant input \( u(t) \equiv \alpha \), where \( \bar{1} \) is the one–vector \( \bar{1} = [1 \ 1 \ldots 1]^T \) and

\[ A = \alpha \left[ \begin{array}{cccc}
-1 & 1+\mu & -1 & \ldots & -1 \\
-1 & -1 & 1+\mu & \ldots & -1 \\
& & \ddots & \ddots & \ddots \\
-1 & -1 & -1 & \ldots & 1+\mu
\end{array} \right] \]

The matrix can be written as

\[ A = -\alpha \tilde{O} \tilde{M} \]

where \( \tilde{O} \) is the "ones matrix", i.e. \( O_{ij} = 1 \). The matrix \( \tilde{O} \) has rank 1 and therefore it has eigenvalues 0 of multiplicity \( n-1 \). The last eigenvalue is \( n \) as it can be immediately seen. Indeed the one–vector \( \bar{1} \) is an eigenvector and that \( \bar{1} \bar{1} = n \bar{1} \).
It is known that the eigenvalue of $A = -\alpha [O + \mu I]$ are those of $O$ translated by $-\mu$, and multiplied by $\alpha$. Therefore $A$ has strictly negative eigenvalues and thus the overall system is stable.

The following aspect is worth pointing out. The eigenvalues of the system are:

- $\lambda_i = -\alpha \mu$, $i = 1, 2, \ldots, n-1$ which are associated with slow modes $e^{-\alpha \mu t}$
- $\lambda_n = -\alpha (n + \mu)$ which is associated with the fast mode $e^{-\alpha (n+\mu) t}$

Since the parameter $\mu$ must be small to meet the ideal requirement, convergence is compromised. However, the global message rate $y(t)$ is not affected by the slow modes. Indeed, denoting by $\tilde{1}^T = [1 \ 1 \ldots \ 1]$ we have

$$y(t) = \tilde{1}^T x(t)$$

The slow eigenvalues $-\alpha \mu$ is not observable and not reachable since, considering that $-\alpha \mu I - A = \alpha O$, using Popov criterion we have

$$\text{rank} \left[ \begin{array}{c} \alpha O \\ \tilde{1}^T \end{array} \right] = 1$$

As a result, there is a single reachable and observable mode associated with $\lambda_n$. This implies that

$$y(t) = \left[ y(0) - \frac{n}{n + \mu} \right] e^{-\alpha (n+\mu) t} + \frac{n}{n + \mu}$$

and the message rate $y(t)$ converges quickly to the desired value, being function of the fast mode only. Clearly, there is also a “slower” redistribution between the nodes which assures fairness asymptotically.

This phenomenon is evidenced in the next figure in which we report the transient with 5 initial nodes and 20 entering nodes. The initial condition is $x_i(0) = 1/(5 + \mu)$ for $i = 1, \ldots, 5$ and $x_i(0) = 0$ for $i = 6, \ldots, 20$. We assume $\mu = 1$ and $\alpha = 0.5$. The slow eigenvalues are $-0.5$ and the fast one is $-10.5$. According to our expectation, the transient of $y$ is, roughly, 20 times faster in terms of time constant.

**A. Partial and time–varying connection**

In practice some of the nodes may be no connected to others and the connection topology may switch in time. This requires the definition of new variables.

**Definition 3.1:** Define $C_i$, $i = 1, \ldots, n$ as the set of nodes whose messages reach node $i$, let $c_i$ the connectivity degree, namely the number of nodes in $C_i$, and let

$$w_i(t) = \sum_{j \in C_i} x_j(t)$$

the transmission rate of such nodes.

Since no signal is received other than that originating from nodes in $C_i$, the control law is replaced by

$$u_i(t) = -\alpha (1 + \mu_i) x_i(t) - \alpha w_i(t) + \alpha p$$

where $\rho$ is a suitable coefficient, which has no effects on stability and will be used later to adjust the bandwidth exploitation. Define also the local output

$$y_i(t) = x_i(t) + w_i(t)$$

representing the local transmission rate namely the transmission rate in the “region of node $i$”.

The next theorem proves that global stability is preserved as long as parameter $\mu_i$ is large enough compared to the degree of connectivity of node $i$.

**Theorem 3.2:** System (1) with the control algorithm (5) is asymptotically stable under arbitrary occlusion namely under arbitrary choice of the $C_i$ provided that $\mu_i > c_i$.

**Proof:** The system can be still represented in the form

$$\dot{x}(t) = Ax(t) + \tilde{1}u(t),$$

where $u(t) \equiv \alpha p$, but the new matrix $A$ is

$$A = \alpha \begin{bmatrix}
-\delta_{11} & -\delta_{12} & -\delta_{13} & \ldots & -\delta_{1n} \\
-\delta_{21} & -\delta_{22} & -\delta_{23} & \ldots & -\delta_{2n} \\
-\delta_{31} & -\delta_{32} & -\delta_{33} & \ldots & -\delta_{3n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
-\delta_{n1} & -\delta_{n2} & -\delta_{n3} & \ldots & -\delta_{nn}
\end{bmatrix}$$

where the $\delta_{ij}$ are 1 if and only if the nodes $j$ reaches node $i$, formally,

$$\delta_{ij} = \begin{cases} 1 & \text{if node } j \in C_i \\ 0 & \text{else} \end{cases}$$

This matrix has the property of being diagonally sub–dominant, namely for any $i$

$$A_{ii} = -(1 + \mu_i) < -c_i \leq -\sum_{j \neq i} |\delta_{ij}|$$

According to the theory developed in [19], [18], the unit ball of the norm $V(x) = \max_i |x_i|$ is $\mu$–contractive, (see also [2] page 128 for details) and therefore $V(x)$ is a Lyapunov function. This implies that the system is asymptotically stable.

The next corollary which states that the condition assures stability even under arbitrary switching.
**Corollary 3.1:** Denote by \( G_i(t) \) the set of nodes whose transmission reaches node \( i \) at time \( t \) and \( c_i(t) \) the number of such nodes. The condition \( \mu_i > c_i(t) \) assures stability even in the case in which the sets \( G_i(t) \) change arbitrarily in time. The proof of Corollary 3.1 is immediate, since as long as we involve a Lyapunov function stability is assured even under variations of the coefficient \( \delta_j \).

**Remark 3.1:** Stability is assured even if there is no symmetry, namely nodes \( i \) can reach \( j \) but no vice-versa.

To apply the result a bound of the degree of connectivity of each node must be known. For brevity let us consider the case in which the algorithm is common to all nodes

\[ \mu_i = \mu, \quad \text{for all } i \]

which is reasonable in application. In view of Theorem 3.2, we must take \( \mu > c_i \) to assure stability. The number \( c_i \) is typically unknown and changes with time. We must therefore assume an upper bound for the local degree of connection, namely a value \( \bar{c} \geq c_i \). Such a bound \( \bar{c} \) is reasonably estimated in most cases.

Stability is an essential requirement; however, channel exploitation is also very important. In the ideal case of full transmission, i.e.

\[ \bar{c} = n - 1, \]

which requires \( \mu > n - 1 \).

Expression (4) suitably adapted in view of the coefficient \( \rho \) evaluated at \( t = \infty \) yields the following steady-state value

\[
y(\infty) = \rho - \frac{n}{n + \mu} < \frac{1}{2} \rho
\]

thus only half of the transmission target is achieved as long as we let \( \rho = 1 \) as in the ideal case. Clearly we can compensate this low bandwidth exploitation by \( \rho \) to improve transmission. In view of (6) it is reasonable to take \( \rho = 2 \).

In this way we almost restore the original situation getting closer to the target \( \bar{y}_{opt} = 1 \).

In practice the situation is much more complex than the ideal one since full connection is not a realistic scenario in most cases. Let us explore the situation at steady state, in the ideal case in which the sets \( G_i \) are generic. First of all, let us notice that the true target is full occupancy of the local channel capacity, namely, \( y_i(\infty) \approx x_i(\infty) + w_i(\infty) = 1 \) a condition which cannot be assured in general. Since at the equilibrium we must have \( \dot{x}_i = u_i = 0 \), we get the following condition

\[
(1 + \mu) x_i(\infty) + w_i(\infty) = \rho
\]

This property assures some kind of *local fairness*, roughly, the fact that nodes in similar conditions transmit at the same rate at steady-state. It is understood that assuming local fairness, is possible only among nodes \( i-j \) which are “equivalent” from the connection point of view in the sense that \( c_i \approx c_j \). If the network is highly unbalanced, a strong mismatch between the capacity exploitation among different regions and nodes is unavoidable.

As far as the bandwidth exploitation is concerned, as already mentioned, we can tune it by means of parameter \( \rho \). Take for instance \( \rho = 2 \), as suggested above, and write

\[
\mu x_i(\infty) + (w_i(\infty) + x_i(\infty)) = 2
\]

If we assume, for brevity, that node \( i \) transmits approximately at the local average of its neighborhood (“local fairness”) we have

\[
x_i(\infty) \approx \frac{w_i(\infty) + x_i(\infty)}{c_i + 1}
\]

then

\[
\left( \frac{\mu}{c_i + 1} + 1 \right) \left( w_i(\infty) + x_i(\infty) \right) \approx 2
\]

We know that we must take \( \mu > \bar{c} \), thus we may select

\[ \mu = \bar{c} + 1 \]

that is to say

\[
w_i(\infty) + x_i(\infty) \approx \frac{2}{1 + \frac{\bar{c} + 1}{\mu}}
\]

This basically means that under good estimates of the connection level \( c_i \approx \bar{c} \), the right hand term of the previous expression is close to 1 and a good local channel exploitation is then possible.

It is worth pointing out that the proposed value of \( \mu \) is taken by considering the worst case scenario, namely the case in which the node topology can arbitrarily switch among all possible configurations. Certainly this is not realistic in practice, and as we will be able to see later, good performance and stability can be achieved by reasonably small values of \( \mu \), allowing for better performances.

Finally note that to digitally implement the scheme we replace the continuous–time equation

\[
\dot{x}_i(t) = -\alpha(1 + \mu)x_i(t) - \alpha w_i(t) + \alpha \rho
\]

by the corresponding discrete–time equation. To this aim we fix a sampling time \( T \) and we consider a standard 0-order hold control

\[
u_i(t) = u_i(kT) = \alpha[-(1 + \mu)x_i(kT) - w_i(kT) + \rho]
\]

\( t \in [kT,(k+1)T) \) The resulting discrete–time equation is

\[
x_i((k+1)T) = x_i(kT) + T \alpha[-(1 + \mu)x_i(kT) - w_i(kT) + \rho]
\]

It can be proved that, if \( \mu \) satisfies the proper conditions, then the discrete–time system remains stable stable provided that \( T < \frac{\mu}{\alpha(1 + \mu)} \). The details are omitted for brevity.

**IV. Simulation results**

To validate the proposed approach we performed several extensive simulations in which stability as well as good performances have always been verified. For the simple implementation, we adopted the approximation of replacing the transmission rate \( x_i \) by the probability \( p_i \) according to the “average” relation

\[
x_i = \frac{p_i}{T}
\]
Then equation (1) becomes $\dot{p} = T_s u$ and the resulting system
(7)
$$\dot{p}_i(t) = -\alpha T_s (1 + \mu_i) p_i(t) - \alpha w(t_i) + \alpha \eta$$
with $w_i = z_i T_s$ and $\eta \doteq T_s p$.

The proposed simulations have been designed to test the algorithm in a realistic dynamic situation of 80 nodes representing vehicles, whose cluster configuration changes in time. As expected the theoretical limit $\mu > 81$ to have

worst–case stability under arbitrary occlusions leads to unsatisfactory performances in terms of channel utilization.

Figure 4 shows the simulation results using $\alpha = 0.01$ and $\eta = 1$. Several values have been tested for $\mu$, starting from 1 up to 81, being the latter a value taken to assure stability under arbitrary topology. According to the results of Figure 4, although $\mu = 81$ assures theoretical stability in every possible configuration, it provides very low values for $p$ and therefore poor performance in terms of channel utilization.

As a case study consider a scenario in which two groups of nodes are initially present. Then a subset of one of them abandons the initial group to reach the other. This evolution and its timing are reported in Figure 5. One time unit corresponds to one packet transmission time while space is measured in meters. At the beginning, a group of 75 nodes (B) is at position 0 and a group of 5 nodes (A) is 100 meters away. At time $t = 8000$, 15 nodes (C) leave the bigger group (D) and stop halfway. At $t = 20000$ the 5 nodes group reaches the 5 nodes group (E). In order to test the response of the algorithm to sudden changes, both the transitions are assumed to be instantaneous. Each node is within the reachability range of all its neighbors, whereas groups are isolated. Initial values are randomly assigned.

Figure 6 and 7 show the values of the variable $p$ for each node with the parameter settings $\alpha = 0.01$, $\eta = 1$ $\mu = 3$ , and $\alpha = 0.01$, $\eta = 1$ $\mu = 81$, respectively. As expected, the smaller value $\mu = 3$ leads to a quite better channel occupation, with respect to the value $\mu = 81$ without stability problems, which confirms that, generally speaking, the sufficient condition $\mu \leq n + 1$ is quite conservative.

Figure 8 represents a scenario of vehicles distributed along a road. Each bar represents the value $1/p$ evaluated for a vehicle, along a four-lanes, 60-vehicle-length long simulated road (without full connection). The transmission probability $p$ is higher in low–density regions. In fact the quantity $1/p$ is a rough but simple approximation of the density of vehicles (number of vehicles within a few-lengths range) and could be immediately used for traffic congestion control or safety applications.

V. CONCLUSION AND OPEN PROBLEMS

In this paper we have proposed a decentralized control for distributed transmission control. The main property of the control protocol is that it is extremely simple, allowing for low cost implementation. Stability conditions have been provided supported by rigorous mathematical considerations. The effectiveness of the proposed approach has been confirmed by means of simulations of non–trivial systems.

Our results obtained via simulation are promising, especially if compared with the performances shown by other efficient $p$-persistent protocols, like those analyzed in [7] and [20]. In particular the simulations evidence very low values...
of idle time, a crucial aspect in $p$-persistent protocols, and relatively few collisions among the transmissions of competing nodes. Moreover, our results are attracting because they assure some fairness among nodes. Again, this issue has been both theoretically developed and experimentally confirmed by simulations.

Among the open problems we mentioned that finding better conditions for parameter $\mu$ assuring stability would be important. We believe that including a–priori information on the graph topology and reasonable restriction on its variations might essentially reduce the values of $\mu$ and, consequently, improve bandwidth/channel exploitation and global fairness.

REFERENCES


