Towards Robust Control with Constraints for a Class of Dynamical Systems: Theory and Application

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Abstract—In this paper we are mainly interested in a generalization of backstepping nonlinear control technique, applied to a wider class of dynamical nonlinear systems than the so-called strict-feedback form, namely a special type of pure feedback systems. This extension of backstepping is interesting from our point of view, both theoretically and practically, since a large number of industrial applications may benefit from it. We will briefly present theoretical aspects concerning design procedure (stability and optimality) and then pass to an application example. Robustness and performance issues with respect to other linear and nonlinear control techniques will be addressed by running a series of simulations.

Index Terms—nonlinear control, backstepping, Lyapunov theory, pure-feedback nonlinear systems, optimization.

I. INTRODUCTION

The closest subject found in known literature, related to our present study is backstepping control design, an efficient and popular tool to address global asymptotic stabilization (GAS) and tracking problems, for nonlinear systems with unknown or uncertain parameters (see [1]). Backstepping is present in nowadays applications, some of which include flight control [17], Voltage Source Converter High Voltage Direct Current (VSC-HVDC) [18], robot joint manipulators [6]. These prove the increased interest towards nonlinear control techniques, in the same time with microcontroller technology advancements. Although nonlinear control solutions might not be considered as cost-appealing (mainly due to increased complexity, which translates itself into increased prototype-production costs), they address efficiently applications where uncertainty and accuracy are non negligible. Classical backstepping is a type of recursive design control, that can be applied for linear systems with lower triangular structure, and nonlinear systems in strict-feedback form (see [1], [2], [4], [6]). Systems which have the upper triangular form, are handled by forwarding technique (see [21], [22]). The class of strict-feedback and the class of strict feedforward nonlinear systems have been extended by the so called interlaced systems (see [20]). In this context, our present study where we propose and develop an efficient control solution to address an extension of strict-feedback form class, extends interlaced systems, for which GAS can be systematically approached. We intend to develop theoretical fundamentals, so that proposed design controller may be ready to be implemented numerically.

In this paper we are concerned with a subclass of pure feedback nonlinear systems, affine in control (see [1], [3]), which includes strict-feedback form (see [19]), thus extending it:

\[
\begin{aligned}
\dot{x}_1 &= f_1(x_1) + g_1(x_1)x_2^n  \\
\dot{x}_2 &= f_2(x_2, x_1) + g_2(x_2, x_1)x_3^{n-1}  \\
&\vdots  \\
\dot{x}_{n+1} &= f_{n+1}(x_{n+1}, \ldots, x_n) + g_{n+1}(x_{n+1}, \ldots, x_n)x_1^{\alpha_{n+1}}  \\
\dot{x}_n &= f_n(x_n, \ldots, x_1) + g_n(x_n, \ldots, x_1)u
\end{aligned}
\]

with \( f_i(\cdot) \) and \( g_i(\cdot) \) known nonlinear functions \( (i = 1, \ldots, n) \). Single-Input Single-Output system (1) contains \( n \in \mathbb{N} \) state-space equations, with input-output pair \((x, u)\) and has relative degree \( n \). Let us define the vector of state variables \( x = (x_1, x_2, \ldots, x_n)^T \). The naming convention of pure-feedback system will be used throughout this paper, to address system (1). The difference between strict-feedback and pure-feedback systems (1), resides in the exponents \( \alpha_i \in \mathbb{R}_+ \) of quantities \( x_i^{\alpha_i} \) \((j = 2, \ldots, n)\) which are constant values in the case of strict-feedback form: \( \alpha_{n+1} = 1 \). Numerous applications might benefit from extension (1), as for instance the widely used solenoid electrical actuators class found in aeronautical industry, which includes solenoid valves [10]: magnetic bearings and magnetic levitation systems (see [14]-[16]). An application of proposed control scheme, referring to an example which can represents a levitation system, will be used in this paper, in order to offer a concrete example of modeling and nonlinear control, within this framework. The classical approach of backstepping gives a continuous analytical form of control law. Depending on the type of system to be controlled, a discontinuous command might be appealing for better robustness. A part of the current study is dedicated to the illustration of both continuous and discontinuous analytical control law designs, for any nonlinear system belonging to class (1). Other extensions of backstepping like integrator backstepping have been illustrated, for instance in [6], [7]. Throughout the paper, we assume that all state variables are available for measurement, and consequently used for full state-feedback. The paper is...
organized as follows: Section II deals with theoretical issues concerning the design of proposed nonlinear command for any system belonging to class (1); in Section III we apply this nonlinear control to an example which may represent a levitation system; Section IV provides simulation results and the conclusions are drawn in Section V.

II. NONLINEAR CONTROL

The classical approach of backstepping consists of n steps constructive and systematical procedure, applied to strict feedback nonlinear systems, in order to obtain ultimately the stabilizing control law, in the form of analytical command \( u \). As to our specific class of pure-feedback systems (1), we address stability by applying the same design procedure, although with some changes, and once feedback stability is achieved, we tune control law \( u \) using an optimization based technique.

A. Stability Analysis

Throughout this subsection, we will systematically build the state-feedback control \( u \), capable of ensuring tracking, regardless of real constraints.
The scheme hereafter may be applied to any nonlinear system belonging to class (1) and is intended to be briefly presented, keeping the formalism as general as possible.

Let us start the design procedure, by rewriting the subsystem consisting of first equation from (1):

\[
\dot{x}_1 = f_1(x_1) + g_1(x_1)x_2^m
\]  
(2)

We would like to know what should be the value of \( x_2^m \), so that one may guarantee asymptotic stability of (2) and asymptotic convergence of time-varying error signal \( e_1 = x_1 - x_{1\text{ref}} \) towards 0, with \( x_{1\text{ref}}(t) \) being the desired reference trajectory for \( x_1(t) \). In other words, we are dealing with a tracking problem. Using the terminology of backstepping, we will introduce the virtual control \( \phi_1^m \), which states for the desired analytical value of \( x_2^m \). This virtual control replaces \( x_2^m \) in (2), and at this first step of design procedure, we will use the following control Lyapunov function (CLF) \[20], or as commonly used in literature, the so-called candidate Lyapunov function [21]:

\[
V_1(x_1) = \frac{1}{2} e_1^2
\]  
(3)

By imposing \( \dot{V}_1 < 0 \), we will be able to obtain explicitly the virtual control \( \phi_1^m \). For example, one may choose

\[
V_1(x_1) = -k_1 e_1^2 < 0 \quad \text{with} \quad k_1 \in R^*_+ \quad \text{an adjustment parameter, used to control the slope of} \quad V_1(x_1) \quad \text{and thus impose the rate of convergence of} \quad e_1 \rightarrow 0.
\]

For the second step of our design procedure, let’s take the first two equations of (1):

\[
\begin{align*}
\dot{x}_i &= f_i(x_i) + g_i(x_1)x_2^m \\
\dot{x}_2 &= f_2(x_i, x_2) + g_2(x_1, x_2)x_3^m
\end{align*}
\]  
(4)

We address subsystem (4) similarly: we would like to know what should be the value of \( x_3^m \), so that asymptotic stability of (4) and asymptotic convergence of the error signal \( e_2 = x_2 - \phi_2^m \) to zero are guaranteed. In other words, we want to calculate virtual control \( \phi_2^m \), which states for the desired analytical value of \( x_3^m \); consider this virtual control instead \( x_3^m \) in (4). The analytical form of \( \phi_2^m \) will be calculated by imposing \( \dot{V}_2(x_1, x_2) < 0 \), where the CLF is defined by:

\[
V_2(x_1, x_2) = \frac{1}{2} e_2^2 + \frac{1}{2} e_1^2
\]  
(5)

It might be possible, for example, to impose

\[
\dot{V}_2(x_1, x_2) = -k_2 e_2^2 - k_1 e_1^2 < 0 \quad \text{with} \quad k_1, k_2 \in R^*_+ \quad \text{being a second adjustment parameter. Notice that at this step, we have been able to ensure asymptotic stability of the partial subsystem (4). The two adjustment parameters} \quad k_1, k_2 \quad \text{are intended to impose the shape of} \quad V_1(k_1, x_1, x_2) \quad \text{and thus control rapidity of asymptotic convergence.}
\]

This design procedure will continue step-by-step as described before. We will now focus on the \( n^{\text{th}} \) step (last one): let us consider whole system (1); we define \( \epsilon_n = x_{n+1} - \phi_{n+1} \) and the final CLF ensuring GAS [1]-[6]:

\[
V_n(x_1, \ldots, x_n) = \sum_{i=1}^{n} \frac{1}{2} e_i^2
\]  
(6)

In order to give an insight of the meaning of overall Lyapunov function (6), from an intuitive point of view, we might notice that since the goal is to control the evolution of state variables \( x_i \), we have constructed a CLF capable of reducing tracking errors between state variables and their desired values. These last ones have been calculated step-by-step according to the systematic procedure presented above.

Again, we impose \( V_n(x_1, \ldots, x_n) < 0 \) and get the final control law \( u \). At this point we have (at least) two choices for \( \dot{V}_n \):

1) **Continuous Control**: If we are interested in getting continuous analytical form of \( u = u_1 \) one may seek:

\[
\dot{V}_n(x_1, \ldots, x_n) = - \sum_{i=1}^{n} k_i e_i^2
\]  
(7)

2) **Discontinuous Control**: In the case of discontinuous analytical form of \( u = u_2 \), our idea is to make the choice:
\[ \dot{V}_f(x_i, \ldots, x_n) = \sum_{i=1}^{n_k} k_i \varepsilon_i - k_i \varepsilon_i^2 \]  

(8)

Special care should be taken to this discontinuous command due to possible chattering effect (see [2], [4]–[6]). Chattering is known to be undesired not only from practical point of view (i.e., fast-switching the command increases wear and damages actuators, other than electrical ones), but also from simulation point of view, results becoming very sensitive to simulation sample time for instance. Thus it might be confusing to conclude upon validity of results. One way to address chattering is to use \( \text{sat}(\cdot) \) function instead of \( \text{sign} \) function: \( R \rightarrow [\pm 1] \) according to:

\[
\text{sat}(\xi) = \begin{cases} 
0 & \text{if } |\xi| < \kappa \\
\text{sgn}(\xi) & \text{elsewhere}
\end{cases}
\]

(9)

with \( \kappa > 0 \). The subscript value of 2 in (9) was preferred, in order to avoid confusion with other independent saturation function definitions, found in literature [2], [4]. We conclude this subsection by saying that, at the end of \( n \)th step, this design procedure simultaneously yields the construction of a globally stabilizing feedback and of a CLF.

B. Constraints handling: an optimization-based approach

We recall that at the last step (\( n \)th) of afore-introduced state-feedback control design procedure (subsection II-A), we got \( n \) adjustment parameters: \( k_i \in R_0^+ (i = 1 \ldots n) \) as means to control the rapidity of asymptotic convergence, acting on \( \dot{V}_f(x_i, \ldots, x_n) \), that appear on final command \( u \) and ensuring GAS. As it might be difficult to choose the “right” adjustment parameters, we make use of optimization theory. The second part of our proposed control design procedure deals with structural constraints and performance issues. Since both might be difficult to address analytically, due to nonlinear structure of system (1), we propose a numerical approach, seeking an appropriate solution, by translating actual context into an optimization problem. Multi-objective, multi-variable optimization problems solved numerically become more and more popular in nowadays modern, complex applications (see for example [23]).

Optimal solution for the final command \( u = u(k) \), \( k \in R_0^+ \) (\( i = 1 \ldots n \)) might be difficult to infer, so we propose to use numerical calculation search algorithms (see [12]), capable of solving numerically minimization problems (for example, by using MATLAB/Optimization ToolboxTM).

Let us come back to system (1) and define all nonlinear constraints as:

\[ g_i(x) - b_i \leq 0: (i = 1 \ldots p) \]

(10)

with \( b_i \in R \). Using an external penalty function [12], one can reinforce the constraints satisfaction by adding them within objective function, so that we are now faced to a minimization problem without constraints. Keeping in mind that \( u = u(k_1, \ldots, k_n) \), we get the following multi-variable, non-convex minimization problem:

\[
\begin{align*}
\min_{u, t, H} \int_0^H & \left( \varepsilon_i^2 + \rho_1 \max \left| u(t) - u_{\max} \right| + \rho_2 \max \left| g_i(x(t)) - b_i \right| \right) dt \quad (11)
\end{align*}
\]

\( \rho_1, \rho_2 > 0 \) and \( \rho_2 > 0 \) barrier parameters [12], used to define the importance of each term appearing inside the objective function.

Notice that the objective function contains an integral over non-receding time-horizon \( H \in [t_s, t_f] \), with \( t_s > t_f \) being the initial simulation time, and \( t_f \) simulation end time. In other words, optimization problem (11) is intended to run offline, and only afterwards, one may test the solution on actual plant. The choice for naming convention coincidence horizon \( H \) is according to Predictive Functional Control (PFC) theory (see [13]). The significance of \( H \) is the same, the context is different only. Still, the same ideas regarding \( H \) in PFC can be adapted to our present nonlinear control design procedure: inadequate choice of \( t_s \) and \( t_f \) might lead to unstable closed-loop system response, raising feasibility issues. When implementing the above problem into numerical routines, one might be surprised to notice the efficiency of some search algorithms like Nelder-Mead method (see [12]). They prove to be fast and accurate enough so that we will be able to get optimal adjustment parameters \( k_{\text{opt}} \), and doing so, the optimal command \( u_{\text{opt}} \). Most likely the optimum found will be local. Due to the nature of system (1) and minimization problem (11), it might be very difficult to find the global solution (the choice of initial conditions being a sensitive parameter for feasibility). At the best, we might be able to guarantee that local solution \( k_{\text{opt}} \) is the minimum minimum for a certain closed ball in \( R^n \), centered at \( k_{\text{opt}} \).

We recall that stability is guaranteed by any feasible solution. The problem of choosing initial adjustment parameters \( k_i \) may be difficult to address without some experience on backstepping control. Classical approach of backstepping does not address this issue (see [1], [4], [6]). Some intuitive guidelines for tuning \( k_i \), based on our own experience: each \( k_i \) adjustment parameter is intimately related to the equivalent \( x_i \) state variable \( (i = 1 \ldots n) \). In other words, \( k_i \) depends on the dynamics of \( x_i \); faster time-response of \( x_i \) with respect to other state variables, implies higher values for this \( k_i \) with respect to the other adjustment parameters. In [7], the authors propose a more sophisticated backstepping synthesis, with saturation limits on control law.
C. Preprocess Input Data to Controller

When analyzing the final command $u$, continuous or discontinuous analytical form, one may notice the inconvenience of having some terms containing state variables $x_i$ at denominator. This is clearly due to $\alpha \in \mathbb{R}$ power numbers from (1): as long as $\alpha \neq 1$, the time-derivative:

$$ \frac{d}{dt}(x_i^{\alpha-1}) = \alpha \frac{dx_i}{dt} $$

leads to singularity problem, since multiplication terms $x_i^{\alpha-1}$ will appear at denominator within each virtual control $\phi_i^{\alpha-1}$. This problem does not arise in classical backstepping since all $\alpha = 1 \Rightarrow x_i^{\alpha-1} = 1$, $(i = 1,...,n)$.

For this reason, we should take special care, avoiding that multiplication for which $x_i \to 0 \Rightarrow u \to \infty$, with $(i = 1,...,n)$. We propose to preprocess the concerned state variables, using them as input to command $u = u(x_i)$:

$$ x_{i,es} = \begin{cases} 
  l_i \text{ sgn}_2(x_i) & \text{if } x_i \in B(0,l_i) \\
  x_i & \text{elsewhere} 
\end{cases} \quad (13) $$

with $B(0,l_i)$ a closed ball defined on $R$ and centered at the origin, of radius $l_i$ and as known

$$ \text{sgn}_2(x_i) = \begin{cases} 
  +1 & \text{if } x_i \geq 0 \\
  -1 & \text{elsewhere} 
\end{cases} \quad (14) $$

Remark: Signum nonlinearity function notation in (14) was preferred, with a subscript value of 2, to emphasize that this definition is different with respect to others found in the literature (see for instance [4]).

The filtered state variables will be used as input data for the command, instead of regular ones, namely $u = u(x_{i,es})$ instead of $u = u(x_i)$; $(i = 1,...,n)$.

This solution of pre-filtering input data to control block, proves itself to be very efficient from simulation point of view, avoiding undesired numerical instabilities. Next, we will apply the method schematically presented above to some systems that we are currently dealing with.

III. APPLICATION

The purpose of this section is to introduce an application example that may benefit from the proposed design control. Let us consider the nonlinear system

$$ \begin{align*}
  \dot{x}_1 &= x_2 \\
  \dot{x}_2 &= \frac{1}{m_1} \left[ -\beta x_2 + \beta x_3 - b_2 x_2 - \beta \frac{x_2}{x_1} \right] \\
  \dot{x}_3 &= \frac{1}{L(x_1)} \left[ -R x_1 - \frac{2L}{x_1^2} x_1 x_3 + U \right] 
\end{align*} \quad (15) $$

where where $\beta_i = \frac{\mu_i N_i S}{4}$, $L(x_i) = L_0 + \frac{L_1}{x_i}$, $L_0$, $L_1$, $\beta_i$ and $R$ are some given constant parameters. System (15) belongs to class (1) with $n = 3$, $\alpha_1 = 1$ and $\alpha_2 = 2$. This system may represent a model of physical one as for instance a levitation system [14, 15]. So $x_i$ can be a position variable, $x_2$ the speed and $x_3$ a current variable. $U$ is the control variable and then it can be the voltage signal.

B. Nonlinear Control: Stability

As presented in the previous section, the proposed extended backstepping procedure implies 3 steps: at the first step, we will be able to calculate virtual control $\phi_{i,j}$; at the second step $\phi_i^{\alpha_1}$ and at last step we will get the final command $U$.

For the first step of design procedure, let us consider first equation of (15) and replace $x_1$ by its virtual control $\phi_i = x_1 - x_{i,ref}$, chosen CLF is $V_i(x_i) = \frac{1}{2} \epsilon_i^T \epsilon_i$.

We impose $\dot{V}_i(x_i) = -k_i \epsilon_i^T \epsilon_i < 0$, with $k_i \in R^+$ in order to ensure global asymptotic convergence of $\epsilon_i \to 0$. After explicit calculations, we obtain $\phi_i = \dot{x}_{i,ref} - k_i \epsilon_i$.

For the second step, let us consider the subsystem consisting of first two equations of (15) and replace $x_1$ by its virtual control $\phi_{i,j}$; $\epsilon_i = x_2 - \phi_i$ and use the CLF:

$$ V_i(x_i) = \frac{1}{2} \epsilon_i^T \epsilon_i + \frac{1}{2} \epsilon_j^T \epsilon_j \quad (17) $$

We impose:

$$ \dot{V}_i(x_i) = -k_i \epsilon_i^T \epsilon_i - k_j \epsilon_j < 0, \quad k_i \in R^+ $$

and obtain $\phi_{i,j}$.

Remark: When calculating $V_j$, we obtain explicitly the term $-k_i \epsilon_i^T \epsilon_i = -k_i (x_1 - x_{i,ref})$ plus other terms. In other words, we force these other terms to be always negative and equal to $-k_i \epsilon_i^T \epsilon_i < 0$, according to (17).

We skip tedious calculations and show the results only:

$$ \phi_{i,j} = x_2 = \left( \frac{1}{\beta_i} \left[ \beta x_2 - \beta x_3 - b_2 x_2 - \beta \frac{x_2}{x_1} \right] - (k_i + k_j) (x_1 - x_{i,ref}) \right) $$

(18)

For the last step of design procedure, let us consider whole system (15): we define $\epsilon_i = x_i - \phi_i$ and use the CLF:

$$ V_i(x_i, x_2, x_3) = \frac{1}{2} \epsilon_i^T \epsilon_i + \frac{1}{2} \epsilon_j^T \epsilon_j + \frac{1}{2} \epsilon_k^T \epsilon_k $$

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As to desired explicit analytical form of control law $U$, whether continuous or discontinuous, one might choose according to the following subsections.

1) Continuous command: Let us use and impose:

$$\dot{V}_c(x_i,x_i,x_i) = -k_i x_i^2 - k_i x_i^2 - k_i x_i^2 < 0 \quad k_i \in R^+$$

to ensure GAS. By avoiding to write entangled calculations, we get explicitly control law $U_{exc} = U$ from equation (19).

Remark: Just as before, at the second step, when calculating explicitly $\dot{V}_c$, one may notice the presence of $-k_i x_i^2$ and $-k_i x_i^2$, plus other terms. Again, this means that we only need to impose that these other terms are always negative and equal to $-k_i x_i^2$.

$$U_{exc} = R_i x_i - 2L_i x_i x_i - \frac{1}{2} x_i x_i x_i$$

$$+ \left[ \frac{x_i}{\beta_i} - \frac{x_i}{\beta_i} + \frac{h_i}{m_i} (k_i + k_i) (x_i - x_{ir}) \right]$$

$$+ \left[ \frac{h_i}{m_i} (k_i + k_i) (x_i - x_{ir}) - \frac{x_i}{\beta_i} \right]$$

$$+ \left[ \frac{m_i x_i}{\beta_i} - \frac{m_i x_i}{\beta_i} \right]$$

(19)

2) Discontinuous command: Let us use and impose:

$$\dot{V}_c(x_i,x_i,x_i) = -k_i x_i^2 - k_i x_i^2 - k_i x_i^2 < 0 \quad k_i \in R^+$$

to ensure GAS. This will have as effect the introduction of discontinuous function $\text{sign}(.)$, inside control law $U$.

Due to space limitation, we avoid giving analytical formula.

C. Nonlinear Control: Constraints handling by using an optimization-based approach

If the system commonly presents hard limits for the position as:

$$x_i \in \left[x_{i_{\min}}, x_{i_{\max}}\right]$$

limits on current: $|x_i| \leq I_{\max}$ and on command $|U| \leq U_{\max}$

$I_{\max} \in R^+$ and $U_{\max} \in R^+$ as maximum allowed values.

All information from above paragraph translates itself into constraints, that can be easily expressed as inequalities (10), to be used within minimization problem (11). The goal of this minimization problem is to find optimal adjustment parameters $k_{i_{\text{opt}}}$, with $i \in \{1,2,3\}$, and thus achieve the goal of tuning command $U = U(k_{i_{\text{opt}}})$.

Optimal values for adjustment parameters $k_i$ ($i \in \{1,2,3\}$), have been calculated using fminsearch function (which is based on Nelder-Mead simplex search method [12]), in MATLABR2008b/ Optimization ToolboxTM.

C. Nonlinear Control: Constraints handling by using an optimization-based approach

The physical systems commonly present hard limits: on the position: $x_i \in \left[x_{i_{\min}}, x_{i_{\max}}\right]$ on current: $|x_i| \leq I_{\max}$ and on the command $|U| \leq U_{\max}$. All information from above paragraph translates itself into constraints, that can be easily expressed as inequalities (10), to be used within minimization problem (11). The goal of this minimization problem is to find optimal adjustment parameters $k_{i_{\text{opt}}}$ with $i \in \{1,2,3\}$, and thus achieve the goal of tuning command $U = U(k_{i_{\text{opt}}})$.

D. Preprocess Input Data to Controller

In equation (19), $x_i$ appears at denominator, so to avoid an eventual singularity, when $x_i \to 0$, we propose to preprocess this state variable, according to subsection II-C.

IV. SIMULATION RESULTS

A series of numerical simulations were performed in order to compare our proposed controller with other known ones, in terms of performance, and to point out the efficiency level for each method. To facilitate the explanations, we introduce the following acronyms, for each different control techniques used: EBC (Extended Backstepping Continuous control, according to II-A.1); EBD (Extended Backstepping Discontinuous control, according to subsection II-A.2); SMD (Sliding Mode Discontinuous control as in [5]); SMC (Sliding Mode Continuous control as in [5], CLF’s second derivative is chosen to be continuous); SFC (Static State feedback Control [4]); PFC (Predictive Functional Control [13, 23]); RHC (Receding Horizon Control [13]).

In Fig. 1-2, we are interested in comparing output results, when applying square waveform reference signal, for the unperturbed system, by using different control techniques: linear (RHC) and nonlinear (PFC, SFC, SMC, SMD, EBC, EBD). From the simulation results, one may observe that EBC exhibits good results with respect to all other techniques, although we have some peaks in control law. All simulation scenarios from Fig. 1 and Fig. 2 were performed using both centered uniform white noise and centered Gaussian, added to states, keeping the same standard deviation value. Results proved stable and robust. To conclude, according to our simulations, both EBC and EBD are stable ensuring asymptotic
convergence to zero of tracking error. EBC proves more
efficient and robust, with similar performance to sliding
mode. EBD is less accurate than EBC in Fig. 1,
suggesting that better adjustment parameters should be
looked for.

V. CONCLUSIONS
In this paper we propose a method for controlling a
special kind of pure-feedback nonlinear systems. This
recursive control technique is based on a kind of extension
of classical backstepping, applicable to a wider range of
nonlinear models than the so-called strict-feedback form
systems: first we calculate analytically a class of
commands ensuring global feedback asymptotic stability
and then treat structural constraints numerically, by
defining a non-convex optimization problem. We then
apply it to a nonlinear system, and end the paper with
some relevant simulations, comparing this approach to
other known control techniques.

Fig. 1. Normalized state results for square wave form reference signal,
when applying different linear and nonlinear control techniques.

Fig. 2. Normalized controllers results.

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