Abstract— A full bridge boost power converter is considered in order to convert 3-phase AC into DC voltage with power factor correction. Desired current trajectories are generated so that, if they are followed, the output voltage will have the desired value and the converter will have a unity power factor. Traditional as well as two types of second order (super-twisting sliding and prescribed convergence law algorithms) sliding mode controllers are used to control the converter. The controlled converters is simulated and studied for the effectiveness of the proposed control algorithms.

I. INTRODUCTION

The efficient conversion of power is a problem that is becoming very relevant in the modern world. With the focus of the government on global warming, dependence on foreign oil, and the focus on renewable energy, it is vital to have efficient energy systems. The power factor, one of the most important aspects in the field of AC/DC power conversion, characterizes the efficiency and quality of such process. The unity-value power factor converter does not introduce any distortion to the energy source and maximizes the performance of the power conversion [1-3]. Many schemes and solutions that are available in the field of power factor correction (PFC) are presented in the works [1-5]. The sliding mode control techniques [6, 7] attracted the engineers for controlling power converters due to the techniques robustness and capability to generate a control function in a high frequency switching format that is typical for controlling the switches in the AC/DC and DC/DC power converters. The power factor correction for a single-phase and a three-phase boost AC/DC boost power converter has been studied in [8, 9]. The problem was addressed by controlling the phase current so that the phase current follows a desired current profile by means of traditional sliding mode control. A non-minimum phase property of the mathematical model of the boost converter complicates addressing the output voltage control problem [10, 11].

This paper studies power factor correction (PFC) for a full bridge 3-phase AC/DC boost power converter. The purpose of this paper is to design and study the effectiveness of the traditional sliding mode controllers (SMC) and second order sliding mode (SOSM) controllers [12, 13] that convert AC power into DC power at a desired voltage level in a 3-phase AC/DC boost power converter while driving the power factor to a unity level.

In this paper, the proposed traditional SMC does not experience the control gain matrix singularity problem that is present in [9]. Also, to the best of our knowledge, this paper presents the first attempt of SOSM controller design for boost AC/DC power converters. The expected advantage of using SOSM control techniques is to obtain a higher accuracy [12, 13] of PFC and the output DC voltage stabilization.

II. PROBLEM FORMULATION

Fig. 1 shows the full bridge boost circuit that is used to convert from 3-phase AC power into DC power.

The circuit is comprised of the three main parts of the boost converter. The first part is the energy storage elements that are the inductors that are used to attain higher output DC voltages than could be obtained by a pure rectifier. The transistors with anti-parallel diodes form the needed switches and rectifier. The switches are bi-directional in that when the switches are closed, they short out the diode and thus let the current flow in either direction. The output is an RC filter that is used to smooth and reduce the output variations to a dc voltage level [1, 8].

The switches are arranged so that when the top switches (s1, s2, s3) are open the bottom switches (s4, s5, s6) are closed. Also, when the top switches are closed the bottom
switches are opened. For modeling, \( s_i = 0 \) corresponds to an open switch and \( s_i = 1 \) corresponds to a closed switch. For simplicity, the switches will be modeled:

\[
\begin{bmatrix}
U_1 \\
U_2 \\
U_3
\end{bmatrix} =
\begin{bmatrix}
s_1 \\
s_2 \\
s_3 \\
s_4 \\
s_5 \\
s_6
\end{bmatrix}
\]

(1)

where \( U_j \) can take the values \{−1,1\} for \( j = 1,2,3 \). For example, \( s_1 = 0 \) and \( s_4 = 1 \) when \( U_1 = −1 \) and \( s_1 = 1 \) and \( s_4 = 0 \) when \( U_1 = 1 \) [1,8].

The converter’s mathematical model is taken as [1,8]:

\[
\frac{di}{dt} = \frac{r}{L}i + \frac{1}{3L} \left( U_{g1} - U_{g2} - U_{g3} \right) - \frac{U_g}{6L} \left( U_1 - U_2 - U_3 \right)
\]

\[
\frac{dU_0}{dt} = \frac{U_o}{RC} + \frac{1}{2C} \left(i_1U_1 + i_2U_2 + i_3U_3\right)
\]

for \( i = \{i_1,i_2,i_3\}^T \) is the input phase currents, \( U_0 \) is the output voltage; \( U_g = \{U_{g1},U_{g2},U_{g3}\}^T \) is the input phase voltages, \( U = \{U_1,U_2,U_3\}^T \) is the control signals.

The gain matrix \( B \) is defined:

\[
B = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}
\]

The voltages, \( U_{gj} \), can have different voltage magnitudes but the same frequency. Thus it can be rewritten as [8]:

\[
U_g = \begin{bmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{bmatrix} \begin{bmatrix} \sin(\omega t) \\ \sin\left(\omega t + \frac{2\pi}{3}\right) \\ \sin\left(\omega t - \frac{2\pi}{3}\right) \end{bmatrix} = E^* \sin.
\]

(4)

\( E_j \) is the voltage magnitude of the \( j^{th} \) phase. The \( \sin \) can be obtained by the following:

\[
sin_j = \frac{U_{gj}}{E_j}.
\]

(5)

The phase voltage is divided by its magnitude in order to extract the sinusoidal wave.

The current and voltage that come out of an uncontrolled boost power converter is not in phase and does not have the correct harmonics. Furthermore, it does not boost the voltage to desired levels.

The estimation and correction of power factor (PF) value is a corner stone in the quality analysis of the designed control law. The power factor in a \( j^{th} \) phase can be computed as [8]:

\[
PF_j = PF_{ij} \cdot PF_{dj} = \frac{\text{RMS}(i_{j0})}{\text{RMS}(i_j)} \cos \phi, \quad j = 1,2,3
\]

(6)

The harmonic distortion term \( PF_{ij} \) characterizes the shape of the phase current: the more harmonics it has, the less the term value is. The displacement term, \( PF_{dj} \), is caused by a phase shift, \( \phi \) between phase current, \( i_j \), and a phase voltage: the bigger the phase shift, the smaller the term value. The best case of unity power factor corresponds to no harmonic distortion (phase current has only main harmonic \( i_{j0} \) and no phase shift between input current and phase voltage, \( U_{gj} \).

Therefore, the control objectives are as follows:

1. The DC portion of the output voltage should equal a desired constant output voltage value \( (V_d) \). The AC portion of the output voltage should be reduced to an acceptable range.

2. The input phase currents should be in phase with their corresponding phase voltage. The phase currents should only have the frequency corresponding to the frequency of the respective phase voltage.

III. CURRENT PROFILE DESIGN

In order to meet the control objectives, the phase currents will be controlled so that the phase currents follow their respective phase voltage. To do this, the input power must be balanced with the output power. This has been performed in [1,8] to obtain the following:

\[
I_d = \frac{E_i - \left(\frac{E_j}{2r} - \frac{2V_d^2}{3rR}\right)}{2r}
\]

(7)

\( I_{ij} \) is the desired current profile for each phase. \( V_d \) is the desired voltage level.

Furthermore, a condition for \( V_d \) in order to keep \( I_{ij} \) real is the following:

\[
V_d \leq E_i \sqrt{\frac{3R}{8r}}
\]

(8)

This corresponds to an upper limit for \( V_d \).

Similarly to the phase voltages, the desired phase currents can be written as the following:
The \( \sin \) term is the same as in (4) since the desired phase current curves need to be in phase with their corresponding phase voltage curves if the power factor is going to equal one.

IV. SLIDING MODE CONTROLLERS DESIGN

A. Assumptions

The following assumptions are assumed when studying controlling the 3-phase boost power converter. They are the following:
1. All of the phase voltages have the same frequency. The frequency can change as long as all of the voltages have the same frequency.
2. All of the phases have the same phase inductance.
3. The phase current, the phase voltage, the output voltage, and the control signals can be measured.
4. The phases evenly contribute to the output DC power.
5. The control vector \( U \) does not equal \( \{1,1,1\} \) or \( \{-1,-1,-1\} \) since this would cause the system to lose controllability.

B. Traditional Sliding Mode Control

The following sliding variable that are supposed to be driven to zero in finite time are introduced

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3
\end{bmatrix} = \begin{bmatrix}
i_1 - i_{d1} \\
i_2 - i_{d2} \\
i_3 - i_{d3} \\
\int (U_1 + U_2 + U_3) d\tau
\end{bmatrix}.
\]  

(10)

Remark 1. Due to Kirchoff’s current law, it is not necessary to track all three phase currents at the same time as in [8]. Also, tracking the third current caused a singularity that required a special treatment [8]. The third condition in (10) avoids the unnecessary requirement to track the third current and is used to demand control symmetry.

The sliding variable dynamics are obtained

\[
\begin{bmatrix}
\dot{\sigma}_1 \\
\dot{\sigma}_2 \\
\dot{\sigma}_3
\end{bmatrix} = \begin{bmatrix}
i_1 - i_{d1} \omega \cos(\omega t) \\
i_2 - i_{d2} \omega \cos(\omega t + \frac{2\pi}{3}) \\
i_3 - i_{d3} \omega \cos(\omega t - \frac{2\pi}{3}) \\
\end{bmatrix} = \Psi + BU,
\]  

(11)

where

\[
\Psi = \begin{bmatrix}
\frac{1}{3L} (2E_1 - E_2 - E_3 - 3r_i d_i) \omega \sin(\omega t) - I_{d1} \omega \cos(\omega t) + U_f \\
0 \\
\frac{1}{3L} (-E_1 + 2E_2 - E_3 - 3r_i d_i) \omega \sin(\omega t + \frac{2\pi}{3}) - I_{d2} \omega \cos(\omega t + \frac{2\pi}{3}) + U_f
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
-2U_{\text{ref}} \\
\frac{U_{\text{ref}}}{6L} \frac{U_{\text{ref}}}{6L} \\
0 \\
\frac{U_{\text{ref}}}{6L} \frac{U_{\text{ref}}}{6L} \\
\end{bmatrix}
\]

\[
B^{-1} = \begin{bmatrix}
-\frac{2L}{U_{\text{ref}}} & 0 & \frac{1}{3} \\
0 & -\frac{2L}{U_{\text{ref}}} & \frac{1}{3} \\
\frac{2L}{U_{\text{ref}}} & \frac{2L}{U_{\text{ref}}} & \frac{1}{3}
\end{bmatrix}
\]

The control functions then become:

\[
U_j = \text{sign}(\sigma_j), \quad j = 1,2,3.
\]  

(14)

C. Super-Twisting Control

The super-twisting control controls drives both \( \sigma \) and \( \dot{\sigma} \) to zero in finite time. This will increase the accuracy of the controller that is proportional to \( T^2 \), where \( T \) is a time increment of a DSP that is used to implement the control algorithm. In order to use this controller, the limits \( |\dot{F}| \leq F_i \), \( i = 1,2,3 \), in (12) need to be determined.

The vector-function \( F \) is calculated below:

\[
F_1 = -CA_1 \sin(\omega t) + \frac{2L}{U_{\text{ref}}} B_1 + \frac{1}{3} U_f,
\]

\[
F_2 = -CA_2 \sin\left(\omega t + \frac{2\pi}{3}\right) + \frac{2L}{U_{\text{ref}}} B_2 + \frac{1}{3} U_f,
\]

\[
F_3 = C \left( A_1 \sin(\omega t + \frac{2\pi}{3}) + A_2 \sin\left(\omega t + \frac{2\pi}{3}\right) - \frac{2L}{U_{\text{ref}}} (B_1 + B_2) + \frac{2}{3} U_f \right),
\]

where \( U_{\text{ref}} \) is assumed to not equal zero. In other words, the capacitor is already charged to some voltage. Furthermore, the control signals are neglected when the derivative of (11) is performed. This means that the limit that is obtained will be a minimum limit. Therefore, the gains will have to be tuned to a higher value than the limit. The resulting equations for the derivative of (11) are below:
\[
\begin{align*}
\dot{x}_1 &= -\frac{2\omega}{3U_0} A^* \cos(\omega t) - \frac{L}{U_0} D_1, \\
\dot{x}_2 &= -\frac{2\omega}{3U_0} A^* \cos\left(\omega t + \frac{2\pi}{3}\right) - \frac{L}{U_0} D_2, \\
\dot{x}_3 &= \frac{2\omega}{3U_0} \left(A \cos(\omega t) + A^* \cos\left(\omega t + \frac{2\pi}{3}\right)\right) + \frac{2L}{U_0} (D_1 + D_2), \\
A^* &= 2E_1 - E_2 - E_3 - 3rI_{d1}, \\
A = -E_1 + 2E_2 - E_3 - 3rI_{d2}, \\
D_1 &= I_{d1} \omega^2 \cos(\omega t), \\
D_2 &= I_{d2} \omega^2 \cos\left(\omega t + \frac{2\pi}{3}\right).
\end{align*}
\]

Setting the cosine and sine terms in (16) to one, the equations for the maximum values of (16) are:
\[
\begin{align*}
\dot{x}_1 &= -\frac{2\omega}{3U_0}(2E_1 - E_2 - E_3 - 3rI_{d1}) \frac{2L}{U_0} I_{d0} \omega^2, \\
\dot{x}_2 &= -\frac{2\omega}{3U_0}(-E_1 + 2E_2 - E_3 - 3rI_{d2}) \frac{2L}{U_0} I_{d0} \omega^2, \\
\dot{x}_3 &= \frac{2\omega}{3U_0}(E_1 - E_2 + 2E_3 - 3r(I_{d1} + I_{d2})) \frac{2L}{U_0} (I_{d1} + I_{d2}).
\end{align*}
\]

Denoting the disturbance limits:
\[
\left|\dot{x}_i^*\right| \leq \hat{|x}_i|, \quad i = 1,2,3,
\]
the super-twisting control law becomes[12,13]:
\[
U_j = -c_j \left|\dot{x}_j\right|^{\frac{1}{2}} \text{sign}(\dot{x}_j) - b_j \int \text{sign}(\dot{x}_j) dt,
\]
\[
c_j = 1.5 \sqrt{\lambda_j}, \quad b_j = 1.1 \lambda_j, \quad j = 1,2,3.
\]

**Remark 2.** The control signals (19) are continuous and must be implemented with Pulse-Width Modulation (PWM) if they are to be used to control the full bridge boost circuit in the real world.

**D. Prescribed Convergence Law**

Let’s consider the original problem of the power factor correction while taking into account the dynamics of the switching elements (Fig. 1). The transformed sliding variable dynamics in eq. (12) can be rewritten as:
\[
\begin{align*}
\dot{\sigma}^* &= F + U, \\
\tau \dot{U} &= -U + H,
\end{align*}
\]

where \( H = [H_1, H_2, H_3]^T \) are the control inputs of the switching elements and \( \tau \) is the time constant of the first order dynamics of the switches.

The vector-relative degree of the transformed sliding variable \( \sigma^* \) is \( [2,2,2] \). Therefore, neither traditional SMC (14) nor super-twisting control (19) can be directly applied in (20) to drive \( \sigma^* \to 0 \) in finite time. However, there exist a variety of HOSM control algorithms [13] that can accomplish this task. In particular, the prescribed convergence law [13] which is chosen to be:
\[
H_j = -\text{sign}\left(\sigma_j^* + \sigma_j^* + \frac{1}{2} \text{sign}(\sigma_j^*)\right), \quad j = 1,2,3.
\]

It’s worth noting that in order to implement the control law (21), \( \dot{\sigma}^* \) must be obtained with a differentiator.

**E. Sliding Mode Differentiator**

The prescribed convergence law requires that the derivative of the sliding variable be provided. This can be provided with a sliding mode differentiator. The sliding mode differentiator is created by creating the error equation:
\[
e_j = z_{0j} - \sigma_j^*.
\]

Therefore, the differentiator is obtained by the following:
\[
\dot{z}_{0j} = -\lambda_{0j} |z_{0j} - \sigma_j^*|^{\frac{1}{2}} \text{sign}(z_{0j} - \sigma_j^*) + z_{ij},
\]
\[
\dot{z}_{ij} = -\lambda_j \text{sign}(z_{0j} - \sigma_j^*),
\]
where \( \lambda_j = 1.5 \sqrt{\lambda_j}, \quad \lambda_j = 1.1 \lambda_j, \quad z_{0j} = \sigma, \quad z_{ij} = \dot{\sigma} \).

**V. SIMULATIONS**

Table 1 shows the parameters used for testing the control designs. The system uses a control rate that is slower than the simulation rate in order to test for the controller implementation. The load resistance changes after 1.5 seconds to test for varying loads. The frequency changes after 1.0 second to test the controller’s ability to handle varying frequencies. The phase voltages have different magnitudes to test the controller’s ability to handle phase voltages that are not equal.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method</td>
<td>Euler</td>
<td>Integration method</td>
</tr>
<tr>
<td>( \Delta T )</td>
<td>10^{-6}</td>
<td>Integration step size, sec.</td>
</tr>
<tr>
<td>( f_i )</td>
<td>10^6</td>
<td>Pseudo-analog simulation rate, Hz</td>
</tr>
<tr>
<td>( f_2 )</td>
<td>10^5</td>
<td>Control evaluation rate, Hz</td>
</tr>
<tr>
<td>( f_3 )</td>
<td>10^4</td>
<td>Pulse Width Modulator rate, Hz</td>
</tr>
<tr>
<td>( R )</td>
<td>40 ( \mapsto 30 )</td>
<td>Load resistance, ( \Omega )</td>
</tr>
<tr>
<td>( r )</td>
<td>0.02</td>
<td>Parasitic phase resistance, ( \Omega )</td>
</tr>
<tr>
<td>( L )</td>
<td>2</td>
<td>Phase inductor value, ( mH )</td>
</tr>
<tr>
<td>( C )</td>
<td>100</td>
<td>Output capacitance, ( \mu F )</td>
</tr>
<tr>
<td>( {E_1, E_2, E_3} )</td>
<td>{155,145,150}</td>
<td>Main voltages, ( V )</td>
</tr>
<tr>
<td>( f )</td>
<td>75 ( \mapsto 150 )</td>
<td>Main voltage frequency, Hz</td>
</tr>
<tr>
<td>( U_0(0) )</td>
<td>1</td>
<td>Initial output voltage, V</td>
</tr>
<tr>
<td>( V_d )</td>
<td>650</td>
<td>The desired output DC voltage, V</td>
</tr>
</tbody>
</table>
The limits that were used for the super-twisting control and the prescribed convergence law were 400 for the first two controls and 750 for the third control. They were obtained after the controller was tuned to improve performance.

Fig.2 – Fig.4 show the resulting relationship between the phase current and the phase voltage.

All of the controls result in the phase current being put relatively in phase with the phase voltage. However, they all result in higher harmonics that reduces the power factor. Furthermore, the prescribed convergence law had large higher harmonics and the feedback linearization had noticeable difference in phase angle that both control laws will have lower power factors than the other control laws.

Fig.5 – Fig.7 shows the output voltage produced by each control law.

The traditional SMC and the super-twisting control result in the voltage being close to the desired level but still a bit...
higher than the desired voltage level. The prescribed convergence law resulted in the correct voltage level, but it is not as smooth as the super-twisting control.

Fig. 8 – Fig. 10 shows the power factor produced by the different controls.

VI. CONCLUSIONS

The three sliding mode controllers were successfully designed for AC/DC boost power converter power factor correction and tested in realistic scenarios. All of the controls resulted in a system with high power factor and output voltage that was boosted to higher voltage levels. The traditional/classic sliding mode control with a modified sliding variable had an output voltage that was close to the desired voltage level but was unable to maintain the power factor at high levels all the time due to a change of the load. The prescribed convergence law was able to obtain the desired voltage level but was also unable to maintain the power factor. The super-twisting control produced voltage levels that were close to the desired voltage levels and smoother than the classic sliding mode control. Furthermore, the super-twisting control produced a high power factor that was able to withstand changes in frequency and load.

REFERENCES