Modeling Priority Analysis via Hybrid Petri Nets for an Internal Combustion Engine Management System

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Abstract—Nowadays, the Engine Management System is a collection of control strategies and adaptive corrections, necessary to uniform different algorithms, realized by various engineers in different periods. It results in a deep connection among the algorithms and a change in a portion of the code causes uncontrolled and undesired changes elsewhere.

This paper examines the modeling and analysis priority of an Electronic Control Unit and proposes an innovative methodology to design an Engine Management System for Internal Combustion Engine using a Hybrid Petri Net based approach, it shows how the Petri Net elements used can be adopted and optimized to better design the algorithms structure so to highlight and manage the different levels of priority of the strategies and, at the same time, to have the possibility to modify a single control procedure monitoring the undesired effects. The proposed model consists of the interconnections of a discrete logic layer, modeled via Petri nets, with a continuous variable layer, described by differential algebraic equations. The state changes of the discrete logic layer drive the operation of continuous variable systems. Hybrid Petri Nets represent the system evolution and running process of an Electronic Control Unit.

I. INTRODUCTION

The Internal Combustion Engine (ICE) is the technological innovation that has changed the world. It is considered both one of the greatest sources of benefits and one of the main causes of atmospheric pollution. Therefore, in order to improve performance and satisfy emission legislations, with particular attention to CO₂ production, the design of the engine has been continuously improved and new components have been introduced, such as sensors and complete subsystems. Considering that the generation of polluting substances, produced by the combustion, depends not only on mechanical components but also on the Engine Management System (EMS), it yields the control software as the main responsible for the engine behavior, and so for the emission levels. Moreover due to the introduction of new electronic components, which determine a strong increase of the degrees of freedom available during the engine operations, the design of control strategies became more complex and less intuitive.

Nowadays, the Engine Management System is a collection of control strategies and adaptive corrections, realized by various engineers in different periods. It results in a deep connection among the algorithms and a change in a portion of the code causes uncontrolled and undesired changes elsewhere. This control structure is usually based on experience and expertise of engineers dedicated to the calibration process transforming the goals, such as fuel consumption, performances of the vehicle, etc., in control references. This approach was acceptable at the beginning of the engine control system technology, when the targets in terms of performances and emissions were much simpler and there was a small number of tuning variables. Actually, considering the wide scale of production, it is not convenient to use this methodology for various reasons, and mainly because it is not feasible to optimize each subsystem independently from the others. Therefore, the final solution will be far from the optimal one.

This motivation leads to realize a robust control strategy, able to operate in an acceptable way in all operative conditions, instead of requiring the control system to adapt to new conditions in order to obtain the best performances by the engine. The increment of the software structure, due to the addition of new and independent control loops for each subsystem, implies a greater number of parameters to be tuned. As a direct consequence of this approach, the calibration effort is enormous (and therefore the time of production): each variation requires a new calibration.

The EMS is a complex dynamic system that includes both discrete events and continuous variables; it must and control the system based on events through a growing numbers of real-time continuous and discrete transactions. These new characteristics have great influence on real-time scheduling system performance for which is necessary a priority organization [1]. This paper proposes a well-known Hybrid Petri Net (HPN) methodology to re-organize the EMS, aiming at improving the Internal Combustion Engine (ICE) performance (during its lifetime) and reducing calibration efforts.

Petri nets have been used extensively as tools for the modeling, analysis and synthesis of discrete event systems [2], [3] and [4]. Petri nets offer advantages over finite automata, particularly when the issues of model complexity and concurrency of processes are of concern. Such advantages are also present when Petri nets are used to model hybrid systems. Several other approaches using Petri nets to model hybrid systems have also been reported in the literature [5], [6]. As an example, high-level hybrid Petri nets proposed in [9], [10], [11] are characterized by the use of structured individual tokens (e.g., colors) in the discrete part of the net, and provide a rich modeling formalism which takes advantage of the modular structure of Petri nets. They have...
be widely used in manufacturing automation to model and
analyze distributed discrete event dynamic systems [12] and
for modeling and analyzing fault diagnosis systems for gas
turbines, transformers, communication safety and manufac-
turing systems [13], [14]. Whereas [15] considered a new
extension of Petri nets, called differential Petri nets. Through
the introduction of the differential place, the differential
transition, and suitable evolution rules, it is possible to model
concurrently discrete-event processes and continuous-time
dynamic processes, represented by systems of linear ordinary
differential equations.

The classical timed Petri net approach with its discrete
state space is well suited for discrete event system. In [7]
and [8] the continuous Petri net is introduced, which offers a
continuous state space modeling approach. By combination
of continuous and discrete Petri nets the author would be
able to model the complex behavior of hybrid dynamical
systems with one graphical description language. A hybrid
system is a dynamic system, which consists of a continuous
variable system interacting with a discrete event system.
Such system has a combination of discrete-event evolution
and continuous-time state evolution, which can not be char-
acterized only with a continuous-time model or a discrete
model. In this paper discrete modeling adopts Petri nets
theory, while the continuous model makes use of Differential
Algebraic Equations to describe the continuous dynamic
behavior.

The authors aim is to describe an example on EMS,
formed by two kind of control algorithms, respectively
presented in [16] and [17], with an approach based on HPN.
Certainly, the authors are conscious that a classical EMS is
more complex but the target of this work is to introduce
a modeling priority analysis via Hybrid Petri Nets for an
ICE management system. The model includes two layers:
discrete logic layer and continuous dynamic layer connected
with a state estimation interface. The continuous dynamic
layer directly regulates the control variables calculation. The
interface estimates the system state according to continuous
variables and produces results for discrete layer. According
to the estimated results of interface, the discrete logic layer
evolves and coordinates the regulators of continuous dynam-
ic layer. When states of controlling or controlled system
change, the identified results of interface as well as logic
variables of discrete logic layer representing control models
change accordingly. The control models switch in response
to the system state. Hybrid Petri nets implement both the
supervision of the differential algebraic equations of control
strategies and the evolution of Petri net, and in turn, the
result of integration fires a transition. By the combination
of continuous and discrete description, system features can be
researched in a more close to reality method.

II. DIFFERENTIAL ALGEBRAIC EQUATIONS

In this paper, the authors adopt classical Petri Nets theory
to describe the EMS decisonal level, while the continuous
model, present in HPN, makes use of differential algebraic
equations to describe the continuous dynamic behavior. In
particular the differential algebraic equations, presented in
this work, describe the control strategy applied to the engine
and respectively realized in [16], [17] to the warm-up and
idle controls.

A. Idle Control Strategy

In [17] a Linear Quadratic Optimization strategy acting
on Air/Fuel (A/F) ratio is applied to the Idle Speed Control
(ISC) problem. It is supposed to dispose of a A/F ratio
controller, neglecting as a consequence the fuel dynamic.
Therefore, besides the throttle valve and spark advance, a
control input \( \lambda_{af} \) is considered. The adopted objective
function is reported in the following

\[
V = \frac{1}{2} \int_{t_o}^T \left[ q_1(\omega - \hat{\omega})^2 + q_2(p_m - \hat{p}_m)^2 + 
+ p_1(\lambda_{af} - \hat{\lambda}_{af})^2 + p_2(\theta - \hat{\theta})^2 + 
+ p_3(\alpha - \hat{\alpha})^2 \right] dt,
\]

where \( \hat{\omega} \) is the idle speed reference fixed at 700 rpm,
corresponding to 73.3 rad/s. The state variables: engine
speed \( \omega \) and intake manifold pressure, \( p_m \), are measured,
so observers are not required, the input variables are air/fuel
density \( \lambda_{af} \), spark advance \( \theta \) and throttle valve angle \( \alpha \).
The weight coefficients \( q_i (i = 1, 2) \) and \( p_j (j = 1, 2, 3) \), and
the remaining references are parameters to be set in order to
reduce the fuel consumption but guaranteeing the idle speed
regulation. To this aim, minimizing the cost function

\[
J = \int_{t_o}^T \text{dmf}_{cyl} dt = \int_{t_o}^T \frac{\text{dmf}_{cyl}}{\lambda_{af}} dt.
\]

where \( \text{dmf}_{cyl} \) is the quantity of fuel injected into the cylin-
ders and \( \lambda_{af} \) is the stoichiometric value equal to 14.6. the
following values are obtained via purposely designed genetic
algorithms

- a small value for the reference throttle valve \( \hat{\alpha} = 5^\circ \), and consequently a low intake manifold pressure \( \hat{p}_m = 23500 \text{Pa} \), corresponding to a minimum air flow
aspirated;
- a spark advance angle corresponding to an optimal
combustion \( \hat{\theta} = 15^\circ \);
- a slightly lean \( \lambda_{af} \) ratio reference equal to 0.85.

Regarding the weight coefficients, higher importance has been
added at engine speed while, conversely, a lower weight has been used both for throttle valve and air/fuel ratio
objective. The control strategy applied to the ISC problem
is described in following. Let the nonlinear system be describe
by \( \dot{x} = f(x, u) \) and \( \pi \) be generic points in the state
space and in the input space, and linearize system at this
point

\[
\delta \dot{x} = A \delta x + B \delta u + d,
\]

where

- \( \delta x \) and \( \delta u \) are the deviations from the chosen fixed point
(\( \pi, \pi \)), respectively, of the state and the inputs

\[
\delta x = x - \pi \quad \delta u = u - \pi
\]
A and B are the jacobian matrices

\[ A = \frac{\partial f(x,u)}{\partial x} \mid x = \bar{x}, u = \bar{u} \]

\[ B = \frac{\partial f(x,u)}{\partial u} \mid x = \bar{x}, u = \bar{u} \]  (5)

d = f(\bar{x}, \bar{u}) is the nonlinear function calculated at the linearization point (\(\bar{x}, \bar{u}\)).

The optimal control for the problem can be obtained by solving the equation (see [18]) for Hamiltonian function H

\[ \frac{\partial H(\delta x, \delta u)}{\partial \delta u} = 0 \Rightarrow \delta u^* = \delta \ddot{u} - R^{-1}B^T\lambda \]  (6)

Assuming \(\lambda = P\delta x + b\), the following is obtained:

\[ \dot{\lambda} = P\dot{\delta x} + P\dot{\delta} + b = P\dot{\delta} + P[A\dot{x} + B\delta u - BR^{-1}B^T(P\delta x + b)] + d \]

\[ \dot{b} = -Q\dot{\delta} + Q\dot{\delta} - A^T(P\delta x + b) \]

Applying simple algebraic manipulation, we arrive at

\[ [P\dot{\delta} + PAp + A^TP\delta x - PBR^{-1}B^TP\dot{x} + Q\dot{x}] + [b + A^Tb - PBR^{-1}B^Tb + Pd + Pb\delta u - Q\dot{\delta}] = 0 \]

and, hence, resulting in the following relationships

\[ -\dot{P} = PA + A^TP - PBR^{-1}B^TP + Q \]

\[ -\dot{b} = (A - BR^{-1}B^TP)^Tb + Pd + Pb\delta u - Q\dot{\delta} \]  (7)

with

\[ P(T) = 0, \quad b(T) = 0. \]

Now, the optimal control is given by

\[ \delta u^* = \delta \ddot{u} - R^{-1}B^TP\dot{x} - R^{-1}B^Tb \]  (8)

where it is possible to distinguish two different components

\[ \delta u_{fb} = -R^{-1}B^TP\dot{x}, \]  (9)

\[ \delta u_{ff} = -R^{-1}B^Tb, \]  (10)

respectively, a feed-back action (9), trying to regulate the states to zero, and a feed-forward action (10), solving the tracking problem. Finally, substituting the input deviation and reference into equation 8, we obtain the final control law

\[ u^* = \ddot{u} - R^{-1}B^TP\dot{x} - R^{-1}B^Tb. \]

The control scheme is illustrated in Figure 1.

Moreover, in order to reduce the computational complexity of the proposed solution, the differential equations 11 have been substituted with the algebraic equations,

\[ PA + A^TP - PBR^{-1}B^TP + Q = 0 \]

\[ (A - BR^{-1}B^TP)^Tb + Pd + Pb\delta u - Q\dot{\delta} = 0 \]  (11)

obtaining so a suboptimal control law.

B. Warm-Up Control Strategy

In [16] warm-up control strategy commands the engine in order to minimize the polluting emissions at the Three Way Catalyzer (TWC) outlet. Here the controller mainly acts on the spark advance angle, \(\theta\), to rapidly increase the feedgas temperature: by reducing spark advance angle, combustion efficiency decreases, consequently heat losses are higher and which contributes to the feedgas warming. Obviously, this strategy reduces the effectively produced torque and, in order to ensure driver requirement, this has to be compensated by increasing the air mass flow rate supply to cylinders.

The peculiar use of spark angle and air mass flow rate generates an extra-pollution, that should be compensated by earlier activation of catalytic reactions. It is so crucial to balance these two tendencies, increasing feedgas temperature vs. extra-pollution, in order to optimize the results of the control strategy so to minimize the total amount of outlet catalyst emissions. Moreover the controller also regulates the injection duration in order to guarantee a stoichiometric mixture and, thus maximum TWC conversion efficiency.

Since on-line pollutants analyzer are not available on commercial cars, no feed-back loop is closed around the TWC. Thus, the TWC dynamic model is only used for tuning the controller parameters and for strategies validation and comparison.

On a commercial engine, different controllers with different priorities simultaneously run in order to ensure the optimal working over all operating points. In particular idle speed control is activated when the engine speed is lower than a fixed threshold to avoid extinction. Usually idle speed control has higher priority than emission control, thus our controller is deactivated during idle and the commercial controller drives the engine. The engine control problem was treated as an optimal control problem, where the authors suppose the spark angle (\(\theta\)) to be control input, and air mass flow rate (\(\dot{m}_a\)), air/fuel ratio (\(\lambda\)), engine speed (\(\omega\)) and coolant temperature (\(T_{cool}\)) to be not control inputs. More details about this control algorithms are reported in [16].

III. HYBRID PETRI NET METHODOLOGY

An Engine Management System is a dynamic hybrid control system, which consists of a continuous variable system interacting with a discrete event system [19]. Such
system has a combination of discrete-event and continuous-time state evolutions, which cannot be characterized only with a continuous-time model or a discrete-event model. Different modeling approaches for hybrid system have been proposed [20], and the hierarchical model showed in Figure 2 is used extensively. The hierarchical model consists of a Discrete Event System (DES) supervisor, a Continuous Variables System (CVS) and an interface which includes control switch and event recognizer. DES supervisor determines discrete states according to specific rules and controls the process of continuous variable system. Control switch is executive body of DES supervisor, which chooses different continuous regulators according to DES supervisor orders. The continuous regulators act on controlled system in real-time. The event recognizer reflects the real-time state of controlled system to DES supervisor in the form of logic value, and drive the changes of DES state. In this work, the common methods of modeling for CVS are differential equation based on Linear Quadratic algorithms, the selected method to model the relationship between DES and CSV is the Hybrid Petri Nets [21]. The coupling of discrete variable and continuous variable in hybrid system is difficult to model. This problem can be solved by Hybrid Petri Nets which represent both continuous and discrete parts of a hybrid system in the same context.

A. Definition of Hybrid Petri Net

Hybrid Petri Net is defined by $HPN = \{P,T,I,O,M,D,S\}$, verifying the following conditions [22]

- $P = P_c \cup P_d = \{P_1, \ldots, P_n\}$ is a finite set of places, where $P_c$ is a finite set of continuous places and $P_c$ is a finite set of discrete places, represented by symbols reported in Figure 3.
- $T = T_c \cup T_d = \{T_1, \ldots, T_n\}$ is a finite set of transitions, where $T_c$ is a finite set of continuous transitions and $T_d$ is a finite set of discrete transitions, represented by symbols reported in Figure 3.

- $I(P, T_j)$ is a function that defines arcs from a transition to a place. $I : T \times P \rightarrow R, T \times P_d \rightarrow N^+$, where $R$ and $N^+$ are real number and positive integers.
- $O(P, T_j)$ is a function that defines arcs from a place to a transition. $O : P_c \times T \rightarrow R, P_d \times T \rightarrow N^+$, The input and output place sets of transitions are represented as $T_*$ and $T^*$, respectively. The incidence matrix is defined as $C(P, T) = I(P, T) - O(P, T)$, where $C_{xy}$ describes the application of $C$ to $P_x$ and $T_y$ with $(X,Y \in c,d)$.
- Marking of place $M : P_d \rightarrow N^+, P_c \rightarrow R$, is a function that specifies to each discrete place a nonnegative number of token (represented by $\bullet$) and assigns to each continuous place fluid volume.
- $D$ is the function associated to discrete transitions specifies the firing delay time of $T_d : D : T_d \rightarrow R^+$.
- $S$ is the function associated to continuous transitions, $S(T) = \nu(h)$, where $\nu$ specifies the firing speed, $h$ specifies the firing delay associated to the transition.
- $E$ is the characteristic vector defined as a firing vector for discrete transitions, $E_j : T_j \rightarrow \{0,1\}, E(T_j) = 1$ denoted that $T_j$ is firing; otherwise is not firing. $E(\tau)$ is the firing vector at time $\tau$, which dues to output data from G mentioned below.
- $G$ is guard expression in form of Boolean functions, $G(T_j)$ associated to discrete transitions $T_j$ is the firing condition of $T_j$, if $T_j$ meets the condition, $G(T_j)$ output is equal to 1, otherwise 0.

B. Enabling condition and firing rule of Hybrid Petri Net

A discrete transition $T_j$ is enabled at time $\tau$ if

$$\forall P_i \in T_j, M_i(\tau) \geq O(P_i, T_j). \quad (12)$$

An enabled discrete transition can be fired, and the firing state is dependent on firing vector $E(\tau)$. When a discrete transition is fired at time $\tau$, marking of the places in HPN change:

$$M(\tau) = M(\tau^-) + [C_{cd} C_{dd}] E(\tau). \quad (13)$$

A continuous transition is enabled at time $\tau$ if

$$\forall P_i \in T_j, P_j \in T_d : M_i(\tau) \geq O(P_i, P_j). \quad (14)$$

A continuous transition will be fired without restraint of Boolean functions as long as it is enabled. When continuous
transitions are fired at time $\tau$, the markings of the places in HPN change:

$$M_d(\tau + dh) = M_d(\tau)$$  \hspace{1cm} (15a)

$$M_c(\tau + dh) = M_c(\tau) + C_c v_k(\tau) dh.$$  \hspace{1cm} (15b)

Moreover, it is important to underline that the enabled continuous transition depends on the marking of its pre-discrete and the firing of continuous transition change the marking of the discrete place.

The hybrid system model based on HPN is shown in Figure 4. The discrete part of HPN model, referred as Discrete Petri Net (DPN) consists of discrete place $P_d$ and discrete transition $T_d$. $P_d$ denotes state of regulator and $T_d$ denotes the transfer of control modes. The continuous part, referred as Continuous Petri Net consists of continuous places $P_c$ and continuous transition $T_c$. $P_c$ denotes continuous dynamic variables being regulated and $T_c$ denotes the regulator operation on continuous and discrete places. According to the continuous variable including the controlled system variable $X$ and Continuous Petri Net place marking $M_c$, Boolean functions of interface $G$ identify the system state and generate a Boolean vector $E$ to Discrete Petri Net. Starting from initial discrete marking $M_{d0}$ of DPN, in according to 12 and 13, the present regulator state is determined. $T_d$ continuously drives $M_c$ of $P_c$ and $P_d$ according to condition 14 and rule 15. So the continuous variables are regulated in real time.

**IV. ENGINE MANAGEMENT SYSTEM**

The role of the EMS is to drive all the actuators present in the commercial ICE in order to obtain the optimum engine operation in terms of fuel consumption, performance, exhaust gas emissions and driving smoothness. To do it, a large number of operating parameters have to be estimated by algorithms that process sensor data. The rules obtained by the control algorithms take the form of sequences of signals which are used to drive the actuators, as described by Petri Net in Figure 5. The control sequence starts with Key On event in transition $T_1$ and brings the EMS in the following state Power On modeled by $P_2$. In this state can be verified two possible actions:

- **Key Off($T_4$)** event brings the EMS in Power Latch($P_3$) state, where the adaptive variables last value are memorized in ECU RAM memory through transition $T_5$ and after it brings the system in Power Off($P_1$) state.
- **Synchronizing RPM($T_3$)** event brings the EMS in a new state $P_4$, where three possible transition can occur:
  - **Key Off($T_5$)** event brings the EMS in Power Latch state, and after transition $T_5$, it brings the system in Power Off state.
  - if the ECU receive diagnostic signals from sensor or actuators, recovery action are executed in transition $T_6$, it brings the ECU in Power On($P_2$) state to repeat the sequence and verify the sensors and actuators correctness.
  - $T_7$ drives measures acquisition by the temperature, pressure sensors to leads the system in $P_5$ and $P_6$ states.

In $P_5$ the Electronic Control Unit is able to operate in satisfactory way in normal and idle condition in order to obtain the best performances by the engine both for warm-up strategy and stabilized ^1 condition. From this state, it is possible to fire two different transitions, depending on the idle condition is required:

- **Idle condition:** the control strategy described in [17] is applied, through the calculus of the three control inputs, spark advance, lambda and throttle valve.
- **normal condition:** in this event reference control inputs are applied to guarantee all engine functionalities, starting from the three control inputs.

The two transition are mutually exclusive, in each one is possible to disable the other with an inhibitory arc that has unit capacity.

The continuous set of transitions $T_c = \{T_{11}, T_{12}, T_{13}, T_{15}, T_{16}, T_{17}\}$ describe the control strategy

^1By stabilized condition we refer to the engine when it reaches a water temperature of 90° and an air temperature of 40°.
applied to the engine, whereas $T_c = \{T_{11}, T_{12}, T_{13}\}$ is responsible of the idle control strategy reported in [17] for calculus on spark advance, injection time and throttle valve, in $T_c = \{T_{15}, T_{16}, T_{17}\}$ is reported a standard control to obtain all engine functionalities. After, the control inputs are forwarded to the engine, respectively during the aspiration phase modeled by $P_{18}$, in order to execute a good air/fuel mixture the throttle valve opening and fuel injection are applied and in combustion phase described by $P_{20}$ to obtain more satisfying combustion, the spark advance choice is calculated. Moreover, the compression and exhaust engine phases are modeled in $P_{19}$ and $P_{21}$, after the transitions $T_{18}$ and $T_{19}$, where the piston goes from Bottom Dead Center (BDC) to Top Dead Center (TDC). Next, new operation is executed, starting from ECU ready state modeled by $P_{0}$.

In the Petri Net reported in Figure 5 the other strategy has been modeled to allow the best performance of the engine in warm-up condition. Starting from a waiting transition $T_{22}$, the ECU goes to a new state $P_{6}$, where the measures of engine temperature and RPM are known. If the engine is cold, the ECU goes to place $P_{7}$ where is analyzed the lambda sensor state, if his state is suitable to guarantee the warm-up control, that is its temperature then light off $^2$ temperature, the control algorithm reported in [16] is applied to the engine, in particular to determination of the spark advance. Naturally, in warm-up condition described by $P_d = \{P_6, P_7\}$ the other control strategies operating on spark advance calculus are disabled, though inhibitory arcs from $P_d = \{P_6, P_9\}$ to transitions $P_d = \{T_{11}, T_{15}\}$. This kind of organization allows to respect a high priority level for control strategies that act on the same control inputs.

V. CONCLUSION

According to hybrid characteristics, in this paper the HPN formalism has been used to model and analyze the Engine Management System, with particular attention for priority analysis between two different control strategy operating on the same control inputs. Moreover, the functional structure EMS has been discussed, which is generally a hybrid dynamic system of discrete event dynamic system interacting with continuous variable dynamic system and an unified hybrid description has also been proposed. The main advantage of using a HPN is the ability to model the entire of system consisting continuous dynamic and discrete event behaviors with an optimized structure to better design the control algorithms interconnection so to respect the different levels of priority of the strategies and, at the same time, to have the possibility to modify a single control procedure monitoring the undesired effects on the others.

REFERENCES


$^2$Light off temperatures of oxidation catalysts are considered as one of the important parameters for catalyst performance evaluation.